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Optimal pairs trading strategies in a cointegration framework

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Abstract

Statistical arbitrage is based on pairs trading of mean-reverting returns. We used cointegration approach and ECM-DCC-GARCH to construct 98 pairs of 152 stocks of 3 currencies. Stocks trading is done by Contract for Difference, a financial derivative product which facilitates short selling and provides a leverage up to 25 times. To measure the performance of a leveraged strategy, we introduced the profit factor which is the annualized return rate per unit risk. And the historical risk is measured by maximum drawdown. We compared three main strategies: percentage, standard deviation of cointegration long term residuals and Bollinger Bands (dynamic standard deviation), with and without double confirmation of short term standard deviation modeled by ECM-DCC-GARCH. Each of the three main strategies is optimized by two optimizers: absolute profit and profit factor. The optimization period goes from 2012-01-01 to 2014-12-31, and validation period is from 2015-01-01 to 2016-06-01. Our results showed that the USD Bollinger Bands strategy without double confirmation and optimized by profit factor, outperformed other strategies and provided the highest annualized return rate per unit risk.

Keywords: Pairs trading, Cointegration, GARCH Model, Bollinger bands, Back-testing, Market efficiency

JEL Classification: G11

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1. Introduction

Pairs trading is a strategy of statistical arbitrage. It has been popular among major investment banks and hedge funds since its birth in the 1980s with an average annualized return up to 11%. It benefits from temporary divergence between the prices (also referred as spread) of two long-term related assets. When such divergence happens, the overvalued asset will be sold short with the undervalued one bought long. Positions will be closed when such divergence disappears. And the profit would be the sum of winning position and losing one. As the profit is independent from the market trend, pairs trading is also regarded as a market-neutral strategy.

To implement pairs trading, three main approaches are often used: distance approach, cointegration approach and stochastic approach.

The distance approach is model-free and therefore avoids misestimating of parameters. The 15th percentile of distance distribution was used by Nath (2003) as a trigger for trading and the 5th percentile as the stop-loss barrier. Huck (2013) demonstrated the high sensitivity of the return to changes in the length of formation period. This approach was also used by Gatev et al. (2006). Disadvantage of this approach is its lack in forecasting ability according to Do et al. (2006).

The forecasting framework was developed by Vidyamurthy (2004) on the co-integration approach by analyzing the residuals' mean reversion. Lin et al. (2006) further developed it by adding a stop-loss. Puspaningrum et al. (2010) estimated the trade duration and optimized accordingly the trading signals on this approach. Pierdzioch et al. (2015) used Residual Augmented Least Squares test for non-cointegration to study conintegration of gold and silver prices. Huck et al. (2015) compared co-integration approach with the distance one and found out that the distance method generated insignificant excess returns while the co-integration one provided a high, stable and robust return.

The stochastic approach models the spread as an Ornstein-Uhlenbeck process. Elliott et al. (2005) used hidden OU process to forecast and decide trading signals. Ekström et al. (2011) analyzed parameters' sensitivity and optimized the trading liquidation based on OU process. This approach was also used by Vladislav (2004) and Boguslavsky et al. (2004).

Besides these common approaches above, other studies have been performed. Engleberg et al. (2009) analyzed the US stock market data and found that market making contributes to the profit of pairs trading. Fabozzi et al. (2015) exploited the mean-reverting properties of prices, and developed dynamic factor models of prices. Yang, J.-W., et al. (2016) combined the Markov regime-switching model and the Vasicek model to test pairs trading on S&P 500 stock. Jacobs et al. (2015) analyzed determinants of pairs trading profitability and indicated that news and dynamics of investor attention as well as limits to arbitrage are important factors. Mori et al. (2011) examined the performance of pairs trading in the US real estate market compared with that in the US stock market from 1987 to 2008. Bogomolov (2013) proposed a new non-parametric approach based on two Japanese charting indicators renko and kagi. Chiu et al. (2015) introduced a closed-form explicit solution from a nonlinear Hamilton-Jacobi-Bellman partial differential equation to find the optimal strategy. Huck (2010) introduced multi-step-ahead forecasts which led to major changes in the trading system and raises new empirical and methodological questions.

This paper is concentrated on pairs trading strategies in a cointegration framework. We followed the work of Huck and Afawubo (2015) and put above all the superiority of cointegration criteria as stock selection tool adapted to pairs trading. Our work is concentrated on the identification of optimal entry and exit strategies of selected stock pairs. To be more exact, the technical elements of our approach are as follow:

We examine using Engle and Granger tests (1987) the cointegration relationships between the stock pairs' prices. We have a sample of 152 stocks listed in 3 currencies (USD, EUR, and GBP), which are qualified to trade via Contract for Difference (CFD). In order to find out the optimal strategy of pairs trading, we exploit at maximum the information provided for each stocks' pair by the Error Correction Model (ECM) which is associated to the long term relationship between two stocks' prices. Particularly we calculate the short term conditional variance of the stationary linear combination of the two stocks' prices, from a bivariate DCC-GARCH version of the Error Correction Model (ECM-DCC-GARCH). We check if this information is useful to confirm the moment to install mean-return strategies of the long term relationship residual.

To measure the performance of a leveraged strategy, we introduced the profit factor which is the annualized return rate per unit risk. And the historical risk is measured by maximum drawdown. We compared three main strategies: percentage, standard deviation of cointegration long term residuals and Bollinger Bands (dynamic standard deviation), with and without double confirmation of short term standard deviation modeled by ECM-DCC-GARCH. Each of the three main strategies is optimized by two optimizers: absolute profit and profit factor. The optimization period goes from 2012-01-01 to 2014-12-31, and validation period is from 2015-01-01 to 2016-06-01. Our results showed that the USD Bollinger Bands strategy without double confirmation and optimized by profit factor, outperformed other strategies and provided the highest annualized return rate per unit risk.

This paper is organized in the following manner. The section 2 presents the selection principals of stock pairs by cointegration tests. We also present the construction of ECM-DCC-GARCH model and the short term variance calculation of cointegration residual. We also list in this section the pairs qualified to perform our trading strategies on. The section 3 shows the different trading strategies and the obtained performance indicators to evaluate strategies. The strategy back testing results are given in the section 4. The section 5 in the end resumes our principal results and suggests potential improvements in the future.

2. Cointegration framework

2.1. Principals

According to Engle and Granger (1987), the cointegration relationship between two stocks' prices are identified by a non-stationarity test (table of Engle and Yoo 1987) on the residual of the model below estimated by OLS

$$P_{a,t} = \alpha + \beta P_{b,t} + \varepsilon_t \tag{1}$$

We would like also to use information provided by an error correction model (ECM) carrying the price dynamic of each stock a and b. The representation's theorem of Granger (1987) showed that two error correction models, one for each stock, should be associated to the integration relationship between the stocks' prices. We say that the error correction model of stock prices' first difference represents the short term dynamics of price by integrating the necessity of adjustment towards long term equilibrium which is given by the cointegration relationship or by the inter-attraction between the two stocks' prices a and b.

The two proposed models integrate an elementary dynamic with one single delay on the prices' variation and a GARCH(1,1) specification on the random variables in order to take into consideration the conditional hetero-skedasticity effects. The two ECM are as below:

$$\Delta P_{a,t} = \mu_1 + \alpha_1 \Delta P_{a,t-1} + \beta_1 \Delta P_{b,t-1} + \lambda_1 \left[P_{a,t-1} - \left(\hat{\alpha} + \hat{\beta} P_{b,t-1}\right)\right] + \varepsilon_{1,t} \tag{2}$$

$$\Delta P_{b,t} = \mu_2 + \alpha_2 \Delta P_{a,t-1} + \beta_2 \Delta P_{b,t-1} + \lambda_2 \left[P_{a,t-1} - \left(\hat{\alpha} + \hat{\beta} P_{b,t-1}\right)\right] + \varepsilon_{2,t} \tag{3}$$

We pose $\Sigma_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{1,t} \end{bmatrix}$ and suppose that $\Sigma_t / \Sigma_{t-1} \backsim N(0, H_t)$ where H_t is the conditional variance-covariance matrix of Σ_t with as components:

$$h_{ii,t} = V(\varepsilon_{i,t} / \varepsilon_{1,t-1}, \varepsilon_{2,t-1}), \; i = 1,2 \tag{4}$$

$$h_{12,t} = Cov(\varepsilon_{1,t}, \varepsilon_{2,t} / \varepsilon_{1,t-1}, \varepsilon_{2,t-1}), \; i = 1,2 \tag{5}$$

The ECM-GARCH models are estimated by the method of maximum likelihood in admitting a ECM-DCC-GARCH representation of the matrix H_t. For our pairs trading strategy optimization, the ECM-GARCH can provide us with at least two types of information.

They allow at first place to measure the significativeness and intensity of the restoring force towards long term equilibrium in the dynamics of stock prices a and b. If a coefficient lambda is not significative, then the stock price does not contribute on average to the return to long term price equilibrium. This property reveals more fundamentally the weak exogenesis of the considered stock price. Remind that we are waiting here a coefficient λ_1 négative if $P_{a,t}$ contributes to return to the equilibrium and a coefficient λ_2 positive if $P_{b,t}$ also contributes to the long term equilibrium.

Please also note that the value of parameters λ_1 and λ_2 control the return speed of $\widehat{\varepsilon_t} = \left[P_{a,t-1} - \left(\hat{\alpha} + \hat{\beta} P_{b,t-1}\right)\right]$ towards its equilibrium value which is null by construction.

In the following part of article, this information about restoring forces is not used to construct the pairs trading strategies but to identify ex-post the most performant strategies. We can particularly suppose that the pairs trading strategies formed on the return to equilibrium of $\widehat{\varepsilon_t}$ would be as performant as that the two prices participate effectively in the return to equilibrium and that the parameters of restoring force have high absolute values.

The ECM-GARCH models provide a second type of information which could be potentially useful for pairs trading strategies. They could be used as forecasting instruments of prices $P_{a,t}$ and $P_{b,t}$ to forecast in the end the trajectory of $\widehat{\varepsilon_t} = \left[P_{a,t-1} - \left(\hat{\alpha} + \hat{\beta} P_{b,t-1}\right)\right]$ when this variable stays near a critical threshold allowing to initialize a strategy or on the contrary to get out of one. In fact we gave up the directional forecast of $\widehat{\varepsilon_t}$ so that to concentrate us on the uncertainty evaluation of the $\widehat{\varepsilon_t}$ trajectory given by the ECM-GARCH model. We search more precisely to evaluate at each date *t*, the conditional variance of $\widehat{\varepsilon_t}$ given Σ_{t-1} (V ($\widehat{\varepsilon_t}$ / Σ_{t-1})).

The evaluation is performed in the following way:

$$\mathrm{V}(\widehat{\varepsilon_t} / \Sigma_{t-1}) = V\left[P_{a,t-1} - \left(\hat{\alpha} + \hat{\beta} P_{b,t-1}\right) / \Sigma_{t-1}\right] \tag{6}$$

Which gives

$$\mathrm{V}(\widehat{\varepsilon_t} / \Sigma_{t-1}) = V\left(P_{a,t-1} / \Sigma_{t-1}\right) + \hat{\beta}^2 V\left(P_{b,t-1} / \Sigma_{t-1}\right) \text{-} 2\hat{\beta} Cov\left(P_{a,t-1}, P_{b,t-1} / \Sigma_{t-1}\right) \tag{7}$$

We then use the elements of variance-covariance matrix H_t to evaluate $V\left(P_{a,t-1} / \Sigma_{t-1}\right), V(P_{b,t-1} / \Sigma_{t-1})$ and $Cov(P_{a,t-1}, P_{b,t-1} / \Sigma_{t-1})$.

We use the fact that

$$\mathrm{V}\left(\Delta P_{a,t} \,/\, \Sigma_{t-1}\right) = V\left(\varepsilon_{1,t} \,/\, \Sigma_{t-1}\right) = h_{11,t} \tag{8}$$

$$\mathrm{V}\left(\Delta P_{b,t} \,/\, \Sigma_{t-1}\right) = V\left(\varepsilon_{1,t} \,/\, \Sigma_{t-1}\right) = h_{22,t} \tag{9}$$

$$\mathrm{Cov}\left(\Delta P_{a,t}, \Delta P_{b,t} \,/\, \Sigma_{t-1}\right) = Cov(\varepsilon_{1,t}, \varepsilon_{2,t} \,/\, \Sigma_{t-1}) = h_{12,t} \tag{10}$$

And at the date t, with $P_{a,t-1}$ and $P_{b,t-1}$ being given,

$$\mathrm{V}\left(\Delta P_{a,t} \,/\, \Sigma_{t-1}\right) = \mathrm{V}\left(P_{a,t} - P_{a,t-1} \,/\, \Sigma_{t-1}\right) = \mathrm{V}\left(P_{a,t} \,/\, \Sigma_{t-1}\right) = h_{11,t} \tag{11}$$

$$\mathrm{V}\left(\Delta P_{b,t} \,/\, \Sigma_{t-1}\right) = \mathrm{V}\left(P_{b,t} - P_{b,t-1} \,/\, \Sigma_{t-1}\right) = \mathrm{V}\left(P_{b,t} \,/\, \Sigma_{t-1}\right) = h_{22,t} \tag{12}$$

$$\mathrm{Cov}\left(\Delta P_{a,t}, \Delta P_{b,t} \,/\, \Sigma_{t-1}\right) = Cov\left(P_{a,t} - P_{a,t-1}\,, P_{b,t} - P_{b,t-1} \,/\, \Sigma_{t-1}\right) = Cov\left(P_{a,t}\,, P_{b,t} \,/\, \Sigma_{t-1}\right) = h_{12,t} \tag{13}$$

So in the end we have

$$\mathrm{V}(\widehat{\varepsilon_t} \,/\, \Sigma_{t-1}) = h_{11,t} + \hat{\beta}^2 h_{22,t}\text{-}2\hat{\beta}\, h_{12,t} \tag{14}$$

This conditional variance allow us to construct at all dates a confidence interval around $\widehat{\varepsilon_t}$. The confidence interval and its upper limit are then potentially useful for example to confirm the break-through of a critical threshold which switches on a trading strategy.

2.2. Results

We performed Engle and Granger cointegration test on 152 stocks without considering their fundamental information, because even if two stocks are from totally different domains, they could be held by the same investing institute whose trading strategy could generate cointegration. And this phenomenon has been observed with an example of Federal Express (logistic) and Novartis (pharmacy).

268 pairs have a P-value <= 2%. After eye check, 98 pairs (37%) were selected. 10 pairs have neither of the 2 stocks participating in the return to equilibrium, 83 have 1, and 5 have 2. A hypothesis to explain why most pairs have only one stock participating in the return to equilibrium, is that multivariate cointegration has been found out, and we might have only tested part of a multivariate cointegration. However there is no evidence showing that a multivariate cointegration is better to trade than a bivariate one, as it is more difficult to find a common multiple and manage positions of multivariate cointegration.

Start date: cointegration started from this date

Ratio: if one share of stock A is paired to X shares of stock B, then the X here is the ratio. The CFD allows to trade at least 10 shares per lot with an increment of 1 share, so the X is rounded to one decimal and therefore 10 shares of stock A will be paired to an integer number shares of stock B.

PA: P-value of restoring force test for stock A

PB: P-value of restoring force test for stock B

PaRt: number of stocks that participate in the return to equilibrium of long term cointegration residual

	Name A	Ticker A	Name B	Ticker B	Start Date	ratio	PA	PB	PaRt
1	3M	MMM	Oracle	ORCL	1/1/2012	5.5	22.3%	0.2%	1
2	3M	MMM	Tiffanys	TIF	5/15/2012	1.5	32.6%	0.0%	1
3	3M	MMM	WellFargo	WFC	1/1/2012	3	38.4%	0.0%	1
4	3M	MMM	YumFoods	YUM	1/1/2012	7.5	4.9%	0.9%	2
5	Ab&Fitch	ANF	PhilMorris	PM	3/1/2012	5.2	5.0%	17.9%	1
6	Adobe	ADBE	Comcast	CMCSA	7/1/2012	1.9	6.4%	0.1%	1
7	Adobe	ADBE	Qualcomm	QCOM	1/1/2012	2.4	66.7%	0.7%	1
8	Adobe	ADBE	Schwabb	SCHW	1/1/2012	2.5	65.5%	0.0%	1
9	Adobe	ADBE	Tiffanys	TIF	5/1/2012	1	45.1%	0.1%	1
10	Adobe	ADBE	YumFoods	YUM	1/1/2012	4.7	54.5%	0.9%	1
11	AIG	AIG	GenElec	GE	1/1/2012	3.8	40.7%	2.1%	1
12	AIG	AIG	YumFoods	YUM	1/1/2012	3	97.8%	1.5%	1
13	Amazon	AMZN	YumFoods	YUM	1/1/2012	20.5	4.4%	19.1%	1
14	AmericanEx	AXP	YumFoods	YUM	1/1/2012	4.3	39.6%	0.2%	1
15	Baidu	BIDU	YumFoods	YUM	3/1/2012	19.7	73.7%	19.4%	0
16	Boeing	BA	HarleyDav	HOG	1/1/2012	2.8	6.2%	2.6%	1
17	Boeing	BA	YumFoods	YUM	1/1/2012	8	50.8%	0.5%	1
18	BofAmerica	BAC	YumFoods	YUM	1/1/2012	1.1	72.6%	1.8%	1
19	BristlMyer	BMY	BofAmerica	BAC	2/1/2012	2.8	3.3%	20.3%	1
20	BristlMyer	BMY	Schwabb	SCHW	1/1/2012	1.4	8.8%	1.9%	1
21	Chevron	CVX	YumFoods	YUM	2/1/2012	3.6	17.0%	8.8%	0
22	Cisco	CSCO	YumFoods	YUM	2/1/2012	1.2	45.5%	7.7%	0
23	Comcast	CMCSA	GenElec	GE	1/1/2012	3.6	12.6%	8.8%	0
24	Comcast	CMCSA	YumFoods	YUM	1/1/2012	3	87.5%	3.0%	1
25	ConPhillip	COP	YumFoods	YUM	4/1/2012	3	19.1%	0.5%	1
26	Costco	COST	HomeDepot	HD	1/1/2012	1	1.4%	2.5%	2
27	Costco	COST	YumFoods	YUM	1/1/2012	5.6	64.8%	8.2%	0
28	Disney	DIS	Adobe	ADBE	1/1/2012	1.1	0.7%	36.3%	1
29	Disney	DIS	MerkCo	MRK	1/1/2012	2.4	55.9%	0.0%	1
30	Disney	DIS	Novartis	NVS	1/1/2012	1.3	15.4%	1.8%	1
31	Disney	DIS	Oracle	ORCL	1/1/2012	3.9	93.4%	0.5%	1
32	Disney	DIS	Tiffanys	TIF	5/15/2012	1	75.4%	0.1%	1
33	Disney	DIS	WellFargo	WFC	1/1/2012	2.2	44.2%	0.0%	1
34	Disney	DIS	YumFoods	YUM	1/1/2012	5.4	33.4%	1.8%	1
35	E.bay	EBAY	Verizon	VZ	1/1/2012	2.2	0.9%	7.0%	1
36	FedEx	FDX	YumFoods	YUM	1/1/2012	9.7	32.5%	3.9%	1

37	GoldmSachs	GS	P&G	PG	1/1/2012	3.5	0.0%	3.3%	2
38	Haliburton	HAL	YumFoods	YUM	1/1/2012	4	97.2%	0.9%	1
39	HarleyDav	HOG	Qualcomm	QCOM	1/1/2012	1.2	18.0%	1.1%	1
40	HarleyDav	HOG	YumFoods	YUM	1/1/2012	2.7	69.7%	0.4%	1
41	HomeDepot	HD	Travelers	TRV	1/1/2012	1.1	3.9%	7.0%	1
42	Honeywell	HON	AIG	AIG	1/1/2012	1.6	30.5%	1.5%	1
43	Honeywell	HON	BofAmerica	BAC	1/1/2012	4.5	15.6%	3.5%	1
44	Honeywell	HON	BristlMyer	BMY	1/1/2012	1.7	35.0%	0.8%	1
45	HPackard	HPQ	Nvidia	NVDA	1/1/2012	2.5	19.6%	1.3%	1
46	HPackard	HPQ	YumFoods	YUM	3/1/2012	2.7	92.0%	2.1%	1
47	Intel	INTC	Dupont	DFT	1/1/2012	2.3	85.0%	0.2%	1
48	J&J	JNJ	AIG	AIG	3/1/2012	1.7	11.2%	0.7%	1
49	J&J	JNJ	YumFoods	YUM	1/1/2012	4.7	39.6%	0.7%	1
50	JPMorgan	JPM	BofAmerica	BAC	1/1/2012	2.4	1.1%	94.1%	1
51	JPMorgan	JPM	P&G	PG	1/1/2012	1	0.0%	32.1%	1
52	L.V.Sands	LVS	YumFoods	YUM	1/1/2012	3.9	87.9%	0.7%	1
53	Linkedin	LNKD	Toyota	TM	1/1/2012	2.9	0.0%	99.6%	1
54	M.Stanley	MS	YumFoods	YUM	1/1/2012	2.4	16.7%	1.5%	1
55	Mastercard	MA	PhilMorris	PM	3/1/2012	141.6	5.2%	21.5%	0
56	MerkCo	MRK	Qualcomm	QCOM	1/1/2012	1.2	3.9%	12.5%	1
57	MerkCo	MRK	YumFoods	YUM	1/1/2012	2.3	39.3%	2.2%	1
58	MGMResorts	MGM	YumFoods	YUM	1/1/2012	1.8	74.7%	0.7%	1
59	Microsoft	MSFT	Nvidia	NVDA	1/1/2012	2.6	0.5%	8.9%	1
60	Microsoft	MSFT	YumFoods	YUM	1/1/2012	2.2	78.0%	2.8%	1
61	MoodysCorp	MCO	Adobe	ADBE	1/1/2012	1.3	4.7%	8.6%	1
62	MoodysCorp	MCO	Tiffanys	TIF	6/1/2012	1.3	22.9%	0.7%	1
63	MoodysCorp	MCO	WellFargo	WFC	1/1/2012	2.6	70.7%	0.4%	1
64	MoodysCorp	MCO	YumFoods	YUM	1/1/2012	6.6	35.6%	1.6%	1
65	Motorola	MSI	GenElec	GE	1/1/2012	2.3	1.3%	21.9%	1
66	Motorola	MSI	YumFoods	YUM	1/1/2012	2.2	39.6%	2.5%	1
67	Netflix	NFLX	AmericanEx	AXP	1/1/2012	10	11.2%	1.2%	1
68	Netflix	NFLX	GenElec	GE	1/1/2012	58.7	5.2%	4.8%	1
69	Netflix	NFLX	Qualcomm	QCOM	1/1/2012	17.5	19.4%	0.5%	1
70	Netflix	NFLX	YumFoods	YUM	1/1/2012	46.6	91.1%	0.5%	1
71	Novartis	NVS	YumFoods	YUM	1/1/2012	4.4	72.9%	1.7%	1
72	Oracle	ORCL	YumFoods	YUM	1/1/2012	1.6	79.3%	2.9%	1
73	P&G	PG	YumFoods	YUM	1/1/2012	2.9	68.1%	1.9%	1
74	Pepsico	PEP	YumFoods	YUM	1/1/2012	3.5	98.8%	5.7%	0
75	PhilMorris	PM	AT&T	T	5/10/2012	3.8	0.0%	23.8%	1

76	Qualcomm	QCOM	MGMResorts	MGM	1/1/2012	1.1	0.0%	92.9%	1
77	Qualcomm	QCOM	YumFoods	YUM	1/1/2012	2.1	17.9%	0.5%	1
78	RalpLauren	RL	Cisco	CSCO	1/1/2012	2.1	0.1%	74.5%	1
79	Starbucks	SBUX	BristlMyer	BMY	1/1/2012	1.3	15.4%	0.7%	1
80	Starbucks	SBUX	YumFoods	YUM	1/1/2012	3.8	53.1%	6.6%	0
81	TeslaMotor	TSLA	Qualcomm	QCOM	1/1/2012	14.4	38.0%	1.1%	1
82	TeslaMotor	TSLA	YumFoods	YUM	1/1/2012	28.7	57.8%	1.2%	1
83	Tiffanys	TIF	M.Stanley	MS	1/1/2012	2.1	0.4%	35.0%	1
84	Tiffanys	TIF	U.P.S	UPS	1/1/2012	1.4	0.0%	97.6%	1
85	Tiffanys	TIF	WellFargo	WFC	5/1/2012	2.1	0.2%	17.6%	1
86	Tiffanys	TIF	YumFoods	YUM	2/1/2012	4.8	29.9%	0.3%	1
87	TimeWarner	TWX	JPMorgan	JPM	1/1/2012	1.9	60.2%	0.1%	1
88	TimeWarner	TWX	Pepsico	PEP	1/1/2012	1.6	0.5%	2.4%	2
89	TimeWarner	TWX	YumFoods	YUM	1/1/2012	5.3	34.6%	9.2%	0
90	Travelers	TRV	YumFoods	YUM	1/1/2012	4.5	54.6%	7.4%	0
91	TrpAdvisor	TRIP	YumFoods	YUM	1/1/2012	7.5	22.9%	1.5%	1
92	U.P.S	UPS	YumFoods	YUM	1/1/2012	3.6	22.6%	0.7%	1
93	UnitedTech	UTX	HarleyDav	HOG	1/1/2012	1.7	37.6%	0.5%	1
94	UnitedTech	UTX	YumFoods	YUM	1/1/2012	4.5	58.6%	0.3%	1
95	WellFargo	WFC	MerkCo	MRK	1/1/2012	1.1	27.8%	1.0%	1
96	WellFargo	WFC	YumFoods	YUM	1/1/2012	2.3	28.7%	0.6%	1
97	YumFoods	YUM	Ford	F	1/1/2012	1.1	0.0%	68.3%	1
98	YumFoods	YUM	GenElec	GE	1/1/2012	1.2	0.0%	2.1%	2

Table 1 list of all testing pairs

3. Performance indicators and trading strategies

3.1. Performance indicators

In order to measure the performance of our strategies, we use several indicators divided into two groups. The first contains indicators for pairs, which measure the performance of each pair in the training and testing sample:

Absolute net profit: A pair is composed of 1 stock A and x stock B, with x as the cointegration coefficient. The absolute net profit measures the net gain of a pair in its own currency.

Pair maximum drawdown: A maximum drawdown (MDD) is the maximum loss from a peak to a trough of a portfolio, before a new peak is attained. Maximum Drawdown (MDD) is an indicator of downside risk over a specified time period. Here it reflects the max floating loss that a pair has suffered in the sample period, which is also the minimum capital required to complete the test without a margin call for leveraged trading. The MDD will be used as initial capital to calculate profit factor.

Profit factor: For leveraged trading, the initial capital is the initial margin required by broker. For the same investment value, the margin could vary according to different leverage, but net profit remains the same. This could cause a problem to calculate the return rate using the traditional method of profit/capital. E.g. to invest a value of 10,000 USD with a leverage of 20:1, only 500 USD margin is needed. Now if we increase the capital beyond 500 USD but keep the same investment value of 10,000 USD, the final profit will not change however our return rate will decrease. So we need to find a new method to measure the performance.

The profit factor is the risk normalized annual return rate:

$$\text{profit factor} = \frac{net\ profit}{Max\ Drawdown} \times \frac{365}{days} \times 100\% \tag{15}$$

It tells that with the risk of losing 100 currency unit, how much net profit would be made in one year.

As the max drawdown only varies with investment value and does not depend on the capital, the profit factor turns out to be a normalized comparable indicator to measure the performance of each pair.

Sell interval: This is the average holding time in days of a sell order (selling stock A and buying stock B) between its open and close

Buy interval: This is the average holding time in days of a buy order (buying stock A and selling stock B) between its open and close.

Wait interval: This is the average waiting time in days between two orders.

In the second group, we have indicators for strategies:

Each strategy corresponds to a virtual portfolio. These indicators below measure the global performance of each portfolio.

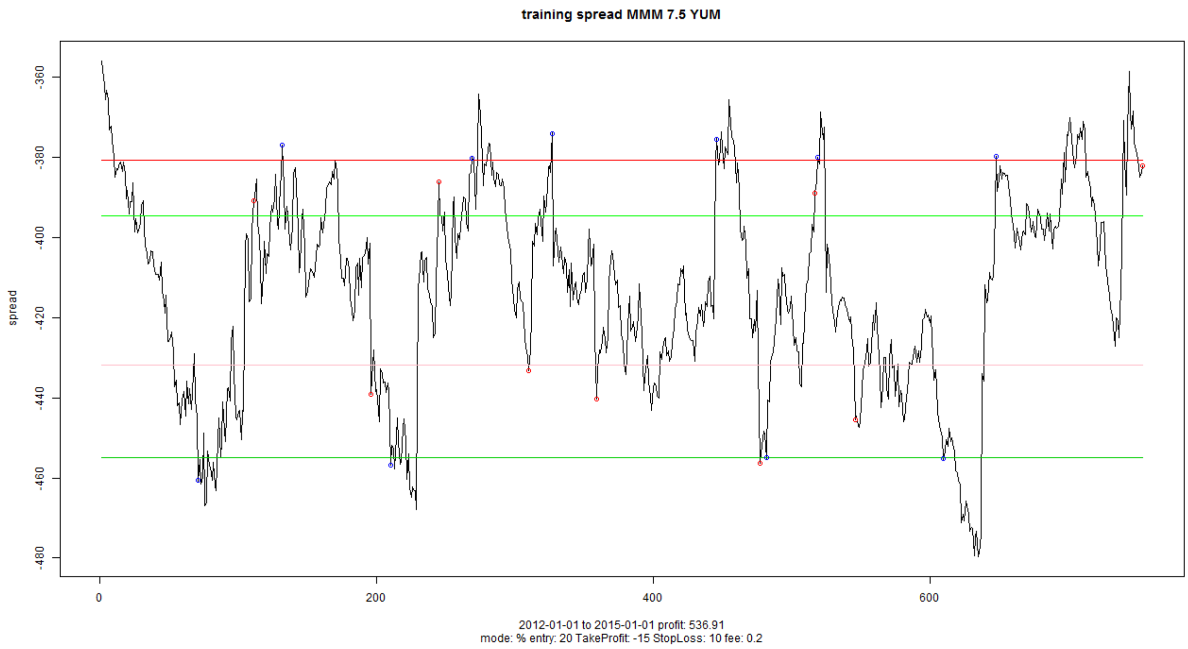
$$\text{Success rate} = \frac{profitable\ trades}{total\ trades} \times 100\%$$

$$\text{Maximum profit loss ratio} = \frac{\max profit\ per\ trade}{|\max loss\ per\ trade|} \times 100\% \tag{17}$$

$$\text{Average profit loss ratio} = \frac{average\ profit}{|average\ loss|} \times 100\% \tag{18}$$

3.2. Trading strategies

3.2.1. Percentage strategy



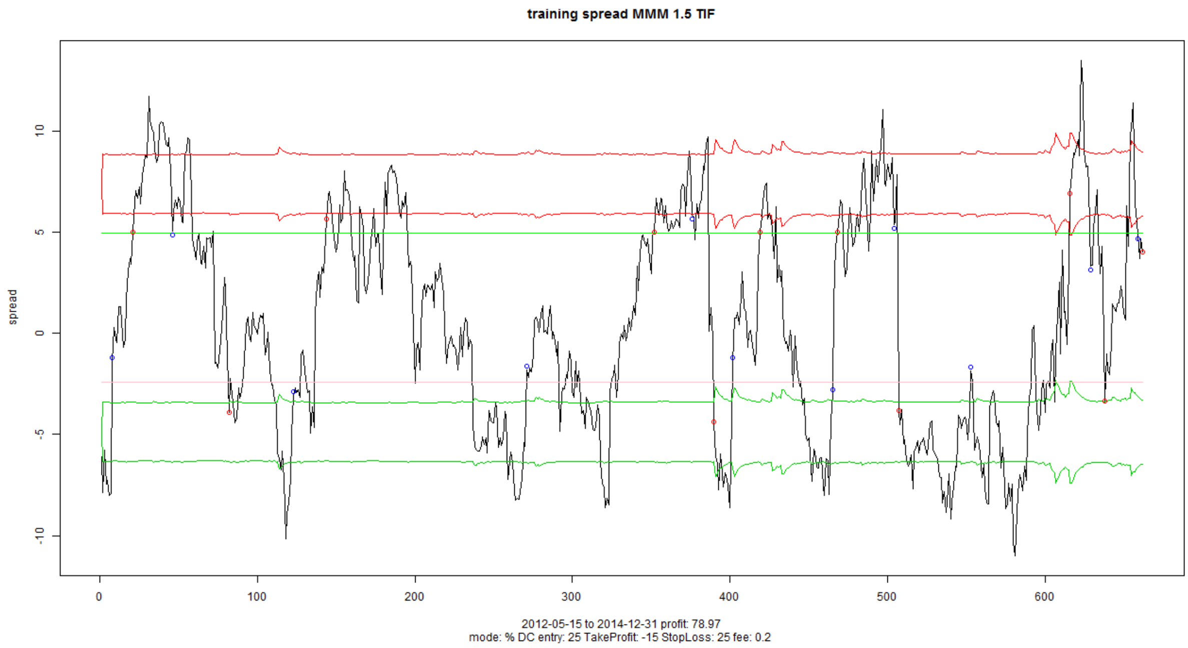
Graphic 1 residuals of percentage strategy on 1MMM-7.5Yummy Foods

This strategy sets the minimum of residual as 0% and the maximum as 100%. It has 6 levels: sell-entry, sell-stop-loss, sell-take-profit, buy-entry, buy-stop-loss and buy-take-profit, each corresponds to a certain percentage which would be optimized later in the article.

When the residual goes cross the entry level (sell or buy), a corresponding trade is opened. This trade will eventually be closed either at stop-loss level or at take-profit level. All open trades will be forced to close at the end of test period.

E.g. a buy order is opened if the residual falls below 10%, and if it goes on to fall below -10%, order will be stopped-loss. If the residual rises and goes beyond 80%, order will be taken-profit.

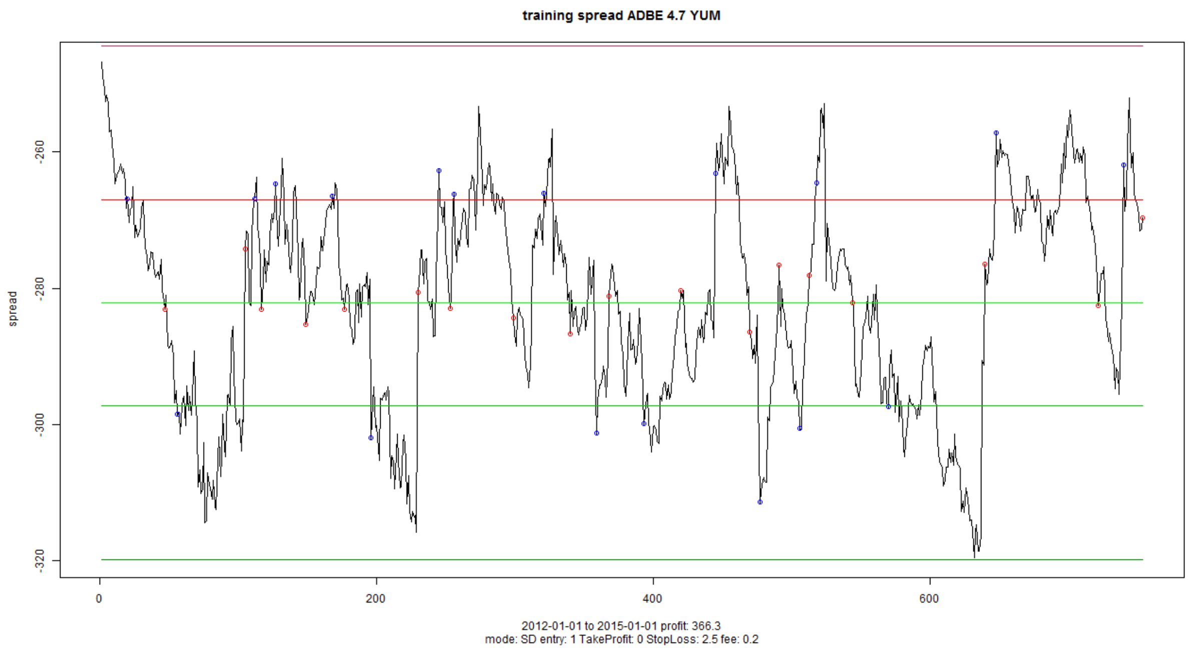
- Option of double confirm using GARCH short-term standard deviation



Graphic 2 residuals of percentage strategy with double confirmation on 1MMM-1.5Tiffany

This option uses bivariate GARCH model to form two bands of short term estimated standard deviation, and adds them on the two sides of entry level. The residual needs to first cross the far-end band then return and cross the near-end band in order to open a trade.

3.2.2. Strategy of long term standard deviation



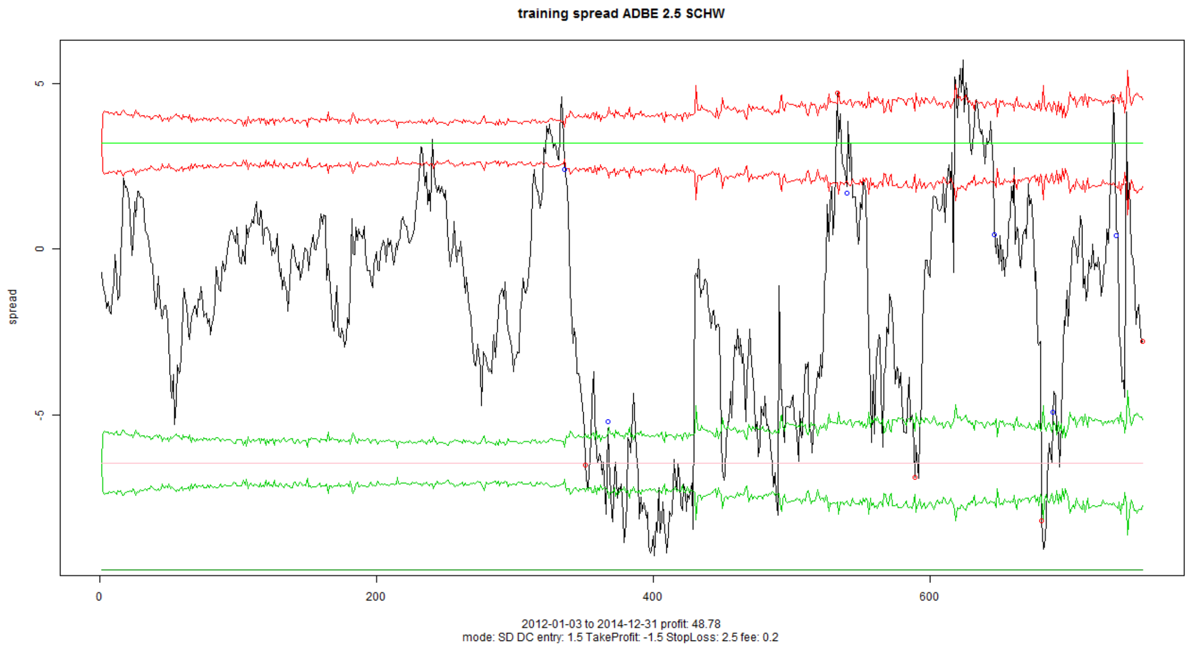
Graphic 3 residuals of standard deviation strategy on 1Adobe-4.7Yummy Foods

This strategy is similar with the percentage one. It uses different multiples of long term standard deviation (SD) to take place of percentage levels, with the mean of residual as 0 SD.

E.g. A sell order is opened if the residual goes beyond 2 SD. If the residual goes on and breaks 3 SD, then this sell order will be stopped-loss. If it returns and falls below -1 SD, this order will be taken-profit.

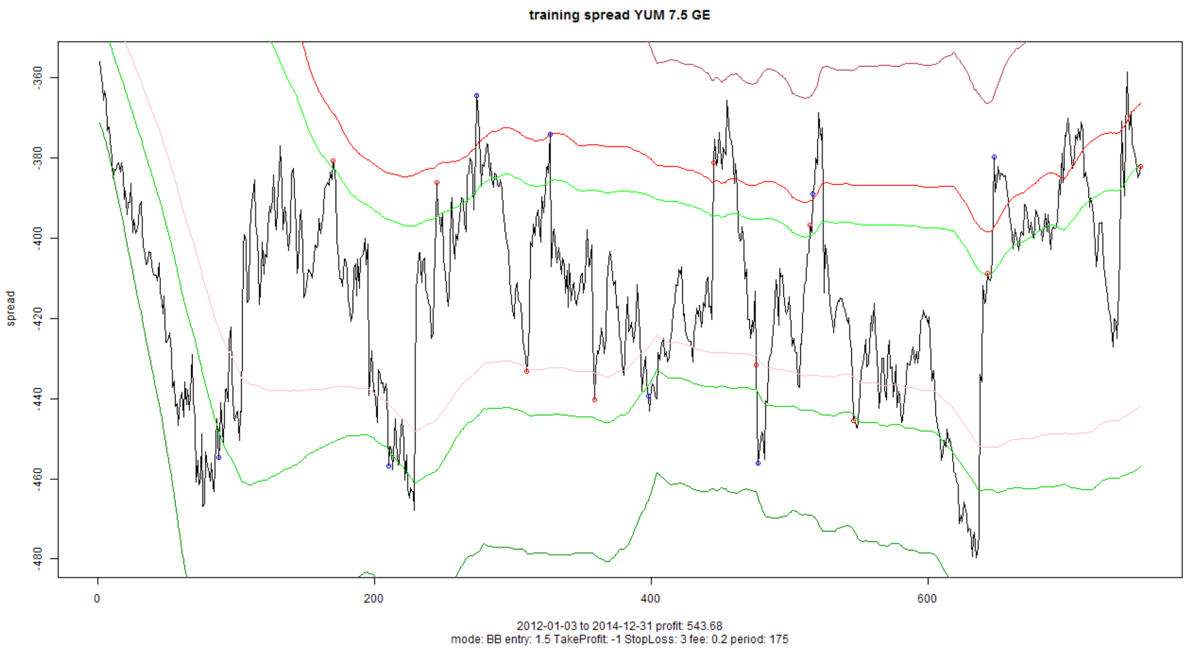
- Option of double confirm using GARCH short-term standard deviation

Similarly, two GARCH modelled short-term standard deviation bands could also be added to the entry level of SD multiples. The trigger of an order will then need double confirmation.



Graphic 4 residuals of standard deviation strategy with double confirmation on 1Adobe-2.5Schwab

3.2.3. Bollinger Bands strategy



Graphic 5 residuals of Bollinger Bands strategy on 1Yummy Foods-7.5General Electric

Bollinger Bands is a tool invented by John Bollinger in the 1980s as well as a term trademarked by him in 2011. Having evolved from the concept of trading bands, Bollinger Bands and the related indicators %b and bandwidth can be used to measure the "highness" or "lowness" of the price relative to previous trades. Bollinger Bands are a volatility indicator similar to the Keltner channel.

Bollinger Bands consist of:

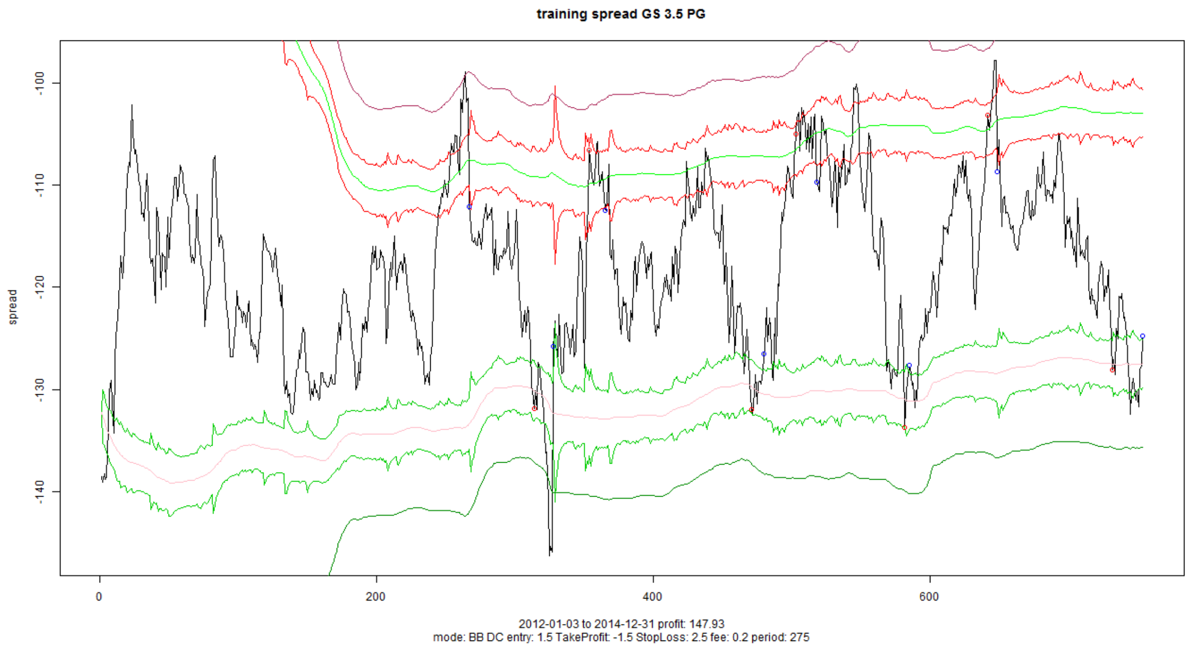
An N-period moving average (MA)

An upper band at K times an N-period standard deviation above the moving average (MA + Kσ)

A lower band at K times an N-period standard deviation below the moving average (MA − Kσ)

The difference with the strategy of long term standard deviation is that, in the Bollinger Bands strategy, the SD levels are no longer fixed but re-estimated for the last N-period and are based on the N-period moving average.

- Option of double confirm using GARCH short-term standard deviation



Graphic 6 residuals of Bollinger Bands strategy with double confirmation on 1GoldmanSachs-3.5P&G

Similarly, the GARCH modelled short-term standard deviation bands could also be added to the entry level of Bollinger Bands. A double confirmation will then be necessary to trigger an order.

3.2.4. Parameter optimization

We divide the sample into two sections. The training sample lasts from 2012-01-01 to 2014-12-31, and the testing sample lasts from 2015-01-01 to 2016-06-01. We optimize the parameters on the training sample and then test the performance on the testing sample.

Entry, Stop-loss and Take-profit levels are optimized for all strategies. Periods are additionally optimized for Bollinger Bands strategy.

Two optimizers are used to evaluate the results:

- The absolute net profit does not take historic risk into consideration
- The profit factor as introduced above, is the risk normalized annual return rate which takes the Max Drawdown into consideration.

Percentage Strategy	Start	End	Increment
Entry Buy	0%	30%	5%
Entry Sell	100%	70%	5%
Take-profit buy	140%	75%	5%
Take-profit sell	-40%	25%	5%
Stop-loss buy	-20%	25%	5%
Stop-loss sell	120%	75%	5%

Table 2 Optimization setting for percentage strategy

Standard Deviation	Start	End	Increment
Entry Buy	-0.5	-4	0.5
Entry Sell	0.5	4	0.5
Take-profit buy	2	-1	0.5
Take-profit sell	-2	1	0.5
Stop-loss buy	-1	-4	0.5
Stop-loss sell	1	4	0.5

Table 3 Optimization setting for standard deviation strategy

Bollinger Bands	Start	End	Increment
Entry Buy	-0.5	-4	0.5
Entry Sell	0.5	4	0.5
Take-profit buy	2	-1	0.5
Take-profit sell	-2	1	0.5
Stop-loss buy	-1	-4	0.5
Stop-loss sell	1	4	0.5
Periods	155	365	10

Table 4 Optimization setting for Bollinger Bands strategy

4. Back testing Results

4.1. Performance ranking

The Bollinger Bands strategy on USD stocks optimized by profit factor has given the highest profit factor in the out-of-sample test. But the highest net profit in out-of-sample test was generated by The Bollinger Bands strategy on USD stocks optimized by net profit. Higher Max Drawdown came along with higher profit, which reduced its performance of profit factor.

Detailed ranking table is shown below:

Pairs: number of pairs

Max: net profit of the best pair

Min: net profit of the worst pair

Ratio: max divided by absolute value of min

Mean: average profit of all pairs

Median: the median profit of all pairs

Win%: the percentage of profitable pairs among all pairs

Lose%: the percentage of losing pairs among all pairs

Intervals:

Sell: average holding days of a selling order

Buy: average holding days of a buying order

Wait: average waiting days between two open orders

Equity max: the highest value of the virtual account's equity during the out-of-sample test

Min: the lowest value of the virtual account's equity during the out-of-sample test

The absolute net profit is not the average profit of all pairs. During the out-of-sample test, a virtual account was created for each strategy, with its equity value recorded every day. And the absolute net profit is the equity value of this virtual account at the end of test

The strategy rank number 1: Bollinger bands without GARCH confirmation and optimized by profit factor, has given an absolute net profit of $4024.97. If we entered at the worst time during back testing, we would have suffered from a floating loss of $1453.49 which is the global maximum drawdown. From this we could calculate a profit factor of 283.92%, which means if we risk to lose $100 in total, we will make a net annual profit of $283.92. 53.54% of the 99 pairs ended up profitable and 32.32% were lost, with the rest breaking even, mostly never having triggered trading. The mean profit of 99 pairs under this strategy is $40.66, while the median is only $2.95, which implies that we had many small winning and losing pairs together with a few very profitable pairs that raised the total gain. It took on average 70 days for a selling pair to take profit or to stop loss, 42 days for a buying pair. Waiting days between two trades of the same pair are averagely 78.

We don't use double confirmation of GARCH to close positions for the following reasons:

1-The double confirmation works in a mean-return manner. The residual should first hit the farther limit and then hit back the nearer one to satisfy a double confirmation. For a position closed by stop-loss, the farther limit is against profit with the nearer one pro profit. If a double confirmation is satisfied for a stop-loss, we should on the contrary not close the position because the residual is now likely to return.

2-The stop-loss should be definitive in order to avoid extreme loss. And a double confirmation could result in void stop-loss when only the farther limit is hit and residual never returns to hit the nearer one.

3-As for take-profit, our trading rule allows only one live position per pair at any given time. If we use double confirmation for take-profit, the round trip position might not be able to open even if its entry conditions are met, because the current position cannot be closed before the nearer limit of double confirmation is hit.

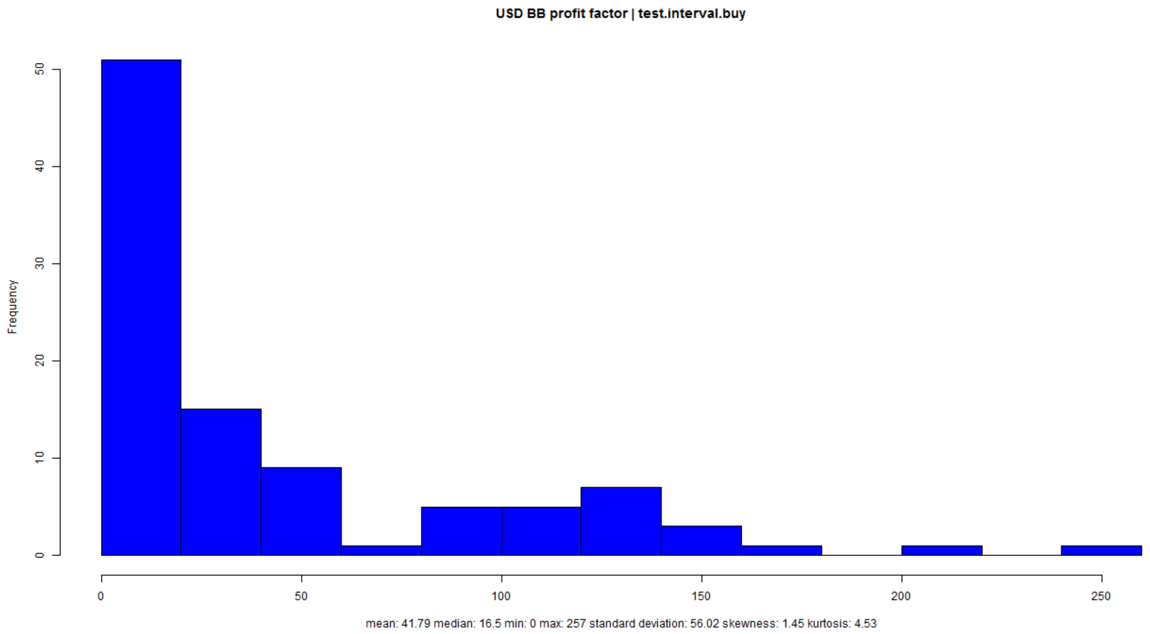
4-If we allow more than one position per pair to implement double confirmation for take-profit, then new problem will appear. Multiple positions could be triggered and result in much bigger loss.

currency	strategy	GARCH	optimizer	profit factor %	absolute net profit	pairs	max	min	ratio	mean	median	win%	lose%	interval sell	buy	wait	MDD	equity max	min
USD	BB		profit factor	283.92	4024.97	99	1108.63	-757.94	1.46	40.66	2.95	53.54	32.32	70	42	78	1453.49	4949.67	-755.51
USD	BB		profit	159.17	5934.79	99	985.88	-1048.65	0.94	59.95	20.01	70.71	28.28	86	64	27	3822.79	6318.36	-2721.27
EUR	BB		profit	145.58	1207.77	24	166.42	-90.83	1.83	50.32	53.11	79.17	20.83	61	91	11	818.43	1427.4	-574.98
USD	percentage		profit factor	126.37	2661.16	99	1128.17	-462.3	2.44	26.88	-0.46	45.45	50.51	34	22	82	2159.17	3408.94	-748.7
USD	BB	DC	profit	81.13	2256.61	98	719.83	-1003.72	0.72	23.03	10.58	59.18	40.82	92	60	40	2851.96	2757.62	-2367.65
EUR	BB		profit factor	79.28	389.21	24	75.51	-45.12	1.67	16.22	12.23	62.5	25	44	90	102	484.32	436.85	-355.14
USD	percentage	DC	profit factor	79.22	914.92	98	335.61	-607.05	0.55	9.34	0	44.9	41.84	29	26	104	1184.14	1438.96	-861.66
USD	BB	DC	profit factor	58.03	1713.73	98	464.63	-2300.55	0.2	17.49	12.85	62.24	33.67	64	63	75	3027.95	3083.51	-2451.46
USD	percentage		profit	57.11	1614.49	98	1033.46	-770.93	1.34	16.31	1.18	51.52	45.45	41	33	61	2898.28	3486.98	-851.96
USD	SD		profit	54.92	1334.19	99	518.25	-796.59	0.65	13.48	4.46	52.53	44.44	38	31	63	2490.59	3014.2	-1527.83
USD	SD	DC	profit factor	54.01	886.57	98	1812.01	-670.3	2.7	9.05	-1.68	36.73	52.04	24	21	91	1682.86	1812.01	-356.93
USD	SD		profit factor	48.53	538.84	99	209.88	-411.51	0.51	5.44	-3.1	37.37	57.58	22	17	79	1138.4	1300.33	-892.19
EUR	SD		profit	-2.82	-19.5	24	170.19	-56.39	3.02	-0.81	-9.16	33.33	66.67	17	23	66	681.12	523.54	-157.58
GBP	BB		profit	-5.55	-1955131.82	12	132969.3	-1940063.15	0.07	-146523	1555.16	66.67	33.33	72	47	7	34928812	11430377	-23498435.4
USD	percentage	DC	profit	-16.42	-409.98	98	335.61	-1149.7	0.29	-4.18	0	47.96	37.76	37	29	88	2559.97	1524.4	-1419.09
GBP	percentage		profit	-34.61	-5323996.09	12	123448	-5314816.55	0.02	-428963	1370.56	66.67	33.33	31	8	43	15255965	9931969	-5323996.09
USD	SD	DC	profit	-35.81	-1118.69	98	386.23	-1031.21	0.37	-11.42	0	44.9	39.8	32	23	86	3202.52	695.86	-2506.66
GBP	SD		profit	-39.02	-6007789.32	12	156786.4	-6001811.1		0.03	-481788	66.67	33.33	18	17	50	15273119	9265329	-6007789.32
EUR	percentage		profit	-54.53	-665.05	24	18.81	-110.08	0.17	-27.71	-23.62	12.5	87.5	20	27	63	1207.02	360.04	-846.99
EUR	SD		profit factor	-64.03	-444.05	24	92.89	-139.48	0.67	-18.5	-11.84	20.83	79.17	7	16	44	684.15	75.63	-608.52
EUR	percentage		profit factor	-66.01	-729.36	24	33.35	-181.55	0.18	-30.39	-27.3	25	75	12	26	48	1089.98	33.67	-1056.31
GBP	BB		profit factor	-69.02	-13263.2	12	3682.32	-13243.7	0.28	-627.3	0	33.33	41.67	8	68	87	19060.96	1966.05	-17094.91
GBP	SD		profit factor	-88.34	-8896834.34	12	17534.27	-8894364.3	0	-739478	44.92	50	50	8	12	75	9988979	1092145	-8896834.34
GBP	percentage		profit factor	-94.46	-11079141.01	12	34577.11	-11075775.6	<0.005	-917999	134.59	50	50	20	20	71	11633283	554141.3	-11079141.6

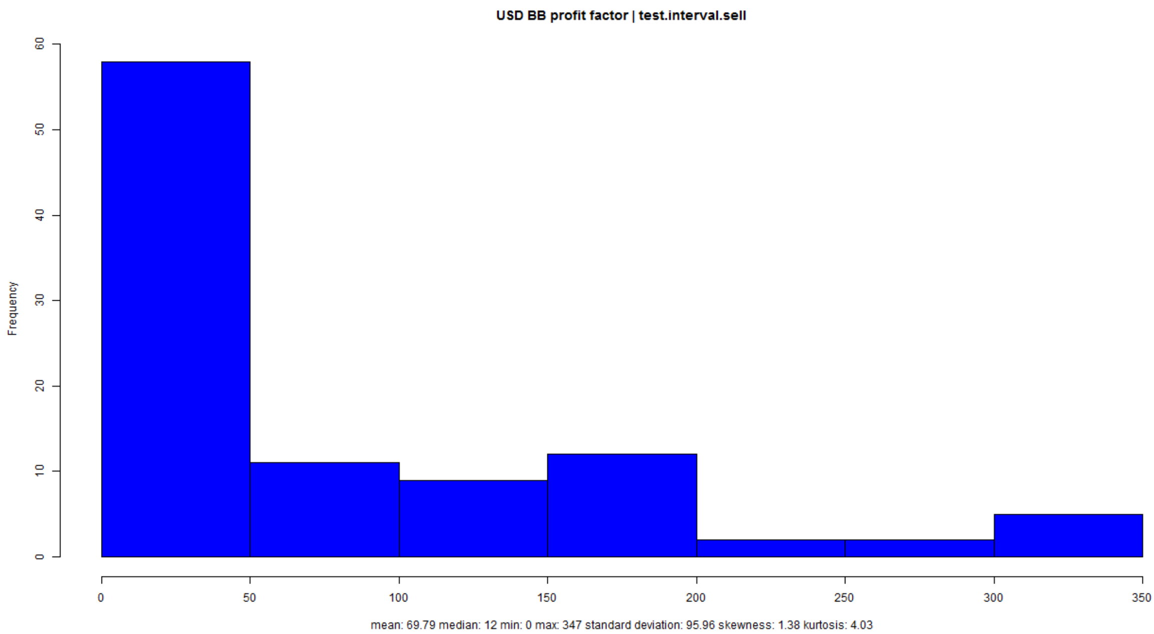
Table 5 strategies' performance ranking

4.2. The frequency histograms of indicators

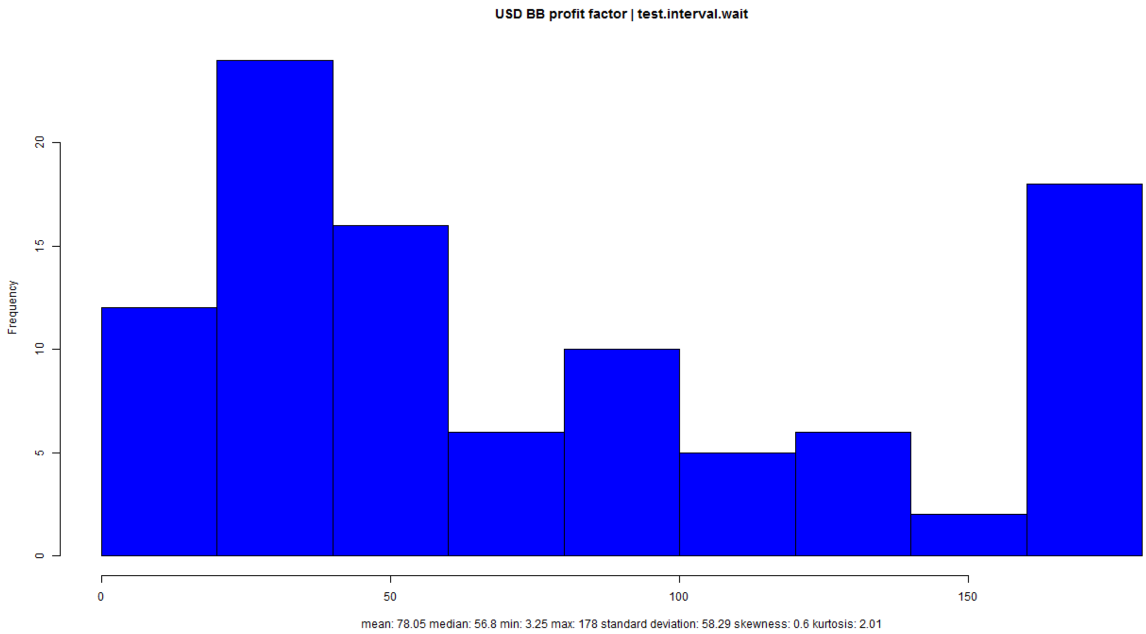
Here we would like to take the best strategy in sense of profit factor as an example, which is the Bollinger Bands strategy without GARCH confirmation on USD stocks optimized by profit factor.



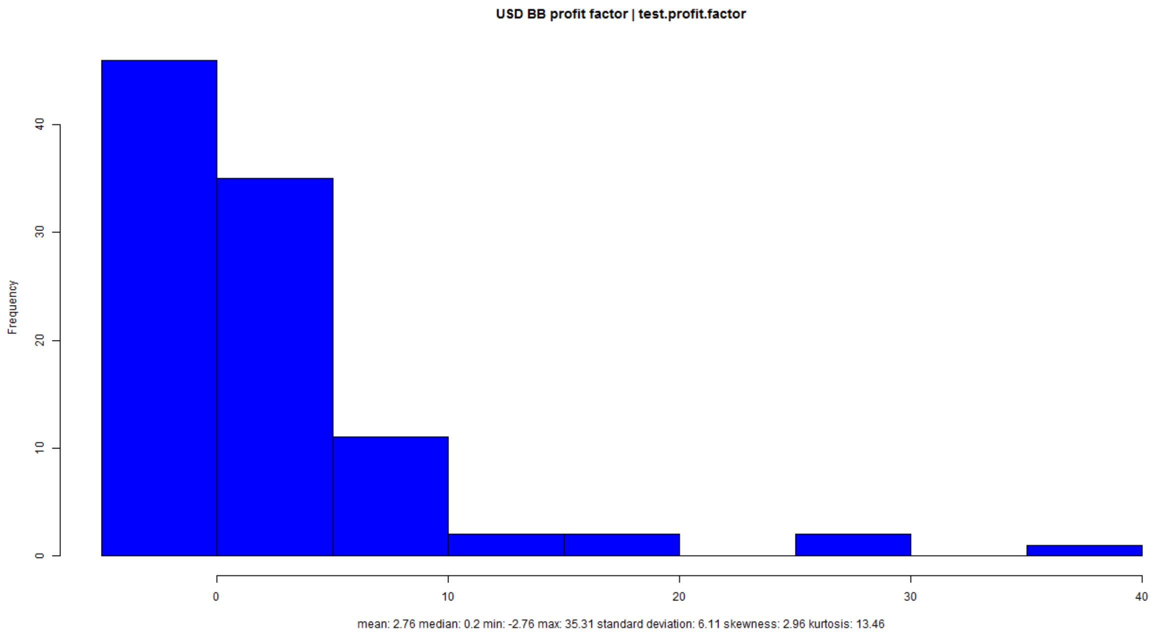
Graphic 7 histogram of buy interval of all pairs



Graphic 8 histogram of sell interval of all pairs



Graphic 9 histogram of wait interval of all pairs



Graphic 10 histogram of profit factor of all pairs

Graphic 7 - 9 shows that most orders last less than 50 days, and most of the time we need to wait less than 50 days to trigger one trading order. Some of the pairs waits more than 150 days, mostly because they actually never generated a single trading signal.

Graphic 10 shows that the most common profit factor is from -5 to 5, meaning that by risking 1 currency unit, we can lose or gain 5 currency units per year. Extremely high profit factors are observed while the extremely negative ones are not generated, because of the stop loss protection.

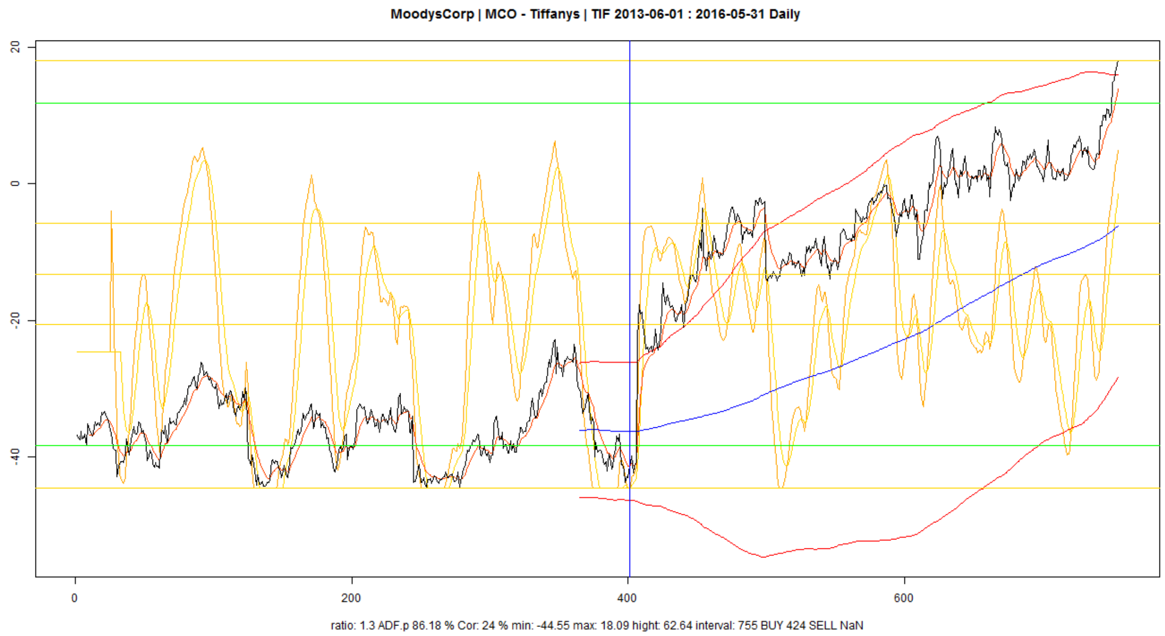
4.3. Retest stationarity of failed pairs

We selected all the pairs resulting in loss in the Bollinger Bands without double confirmation strategy optimized by profit factor, and performed ADF test again from 2013-06-01 to 2016-06-01. Here is the result:

TickerA	TickerB	NameA	NameB	ratio	ADF p.value
MCO	WFC	MoodysCorp	WellFargo	2.6	3.24
ADBE	CMCSA	Adobe	Comcast	1.9	4.37
MSI	YUM	Motorola	YumFoods	2.2	12.76
HPQ	YUM	HPackard	YumFoods	2.7	17.94
UTX	HOG	UnitedTech	HarleyDav	1.7	27.74
SBUX	YUM	Starbucks	YumFoods	3.8	31.15
DIS	ORCL	Disney	Oracle	3.9	34.15
QCOM	YUM	Qualcomm	YumFoods	2.1	36.05
BMY	SCHW	BristlMyer	Schwabb	1.4	39.68
MA	PM	Mastercard	PhilMorris	141.6	40.13
DIS	MRK	Disney	MerkCo	2.4	40.43
DIS	WFC	Disney	WellFargo	2.2	40.77
JNJ	AIG	J&J	AIG	1.7	41.94
TIF	UPS	Tiffanys	U.P.S	1.4	60.7
BA	HOG	Boeing	HarleyDav	2.8	61.74
TSLA	QCOM	TeslaMotor	Qualcomm	14.4	63.8
DIS	NVS	Disney	Novartis	1.3	64.44
QCOM	MGM	Qualcomm	MGMResorts	1.1	64.69
JPM	BAC	JPMorgan	BofAmerica	2.4	65.25
MRK	QCOM	MerkCo	Qualcomm	1.2	65.55
SBUX	BMY	Starbucks	BristlMyer	1.3	66.67
TIF	WFC	Tiffanys	WellFargo	2.1	67.54
MCO	ADBE	MoodysCorp	Adobe	1.3	71.31
HD	TRV	HomeDepot	Travelers	1.1	71.37
BMY	BAC	BristlMyer	BofAmerica	2.8	76.73
HON	BAC	Honeywell	BofAmerica	4.5	79.14
MCO	TIF	MoodysCorp	Tiffanys	1.3	86.18
ADBE	QCOM	Adobe	Qualcomm	2.4	91.7
INTC	DFT	Intel	Dupont	2.3	91.85
ADBE	TIF	Adobe	Tiffanys	1	98.48
MSFT	NVDA	Microsoft	Nvidia	2.6	99
HPQ	NVDA	HPackard	Nvidia	2.5	99

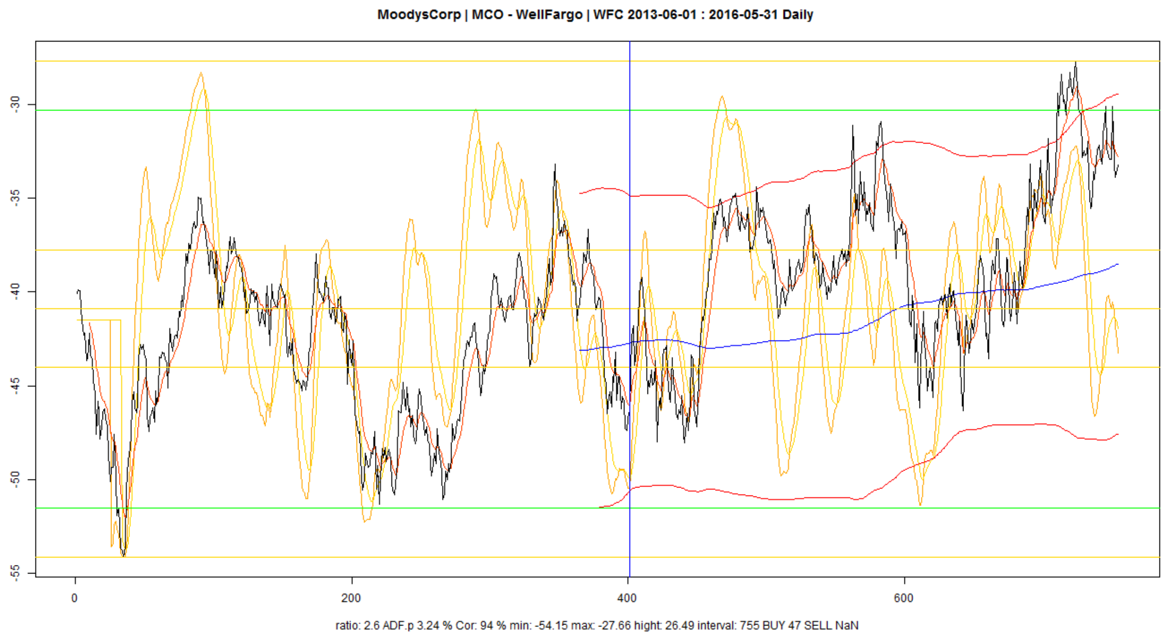
Table 6 ADF test result of failed pairs in the Bollinger Bands without double confirmation strategy optimized by profit factor, from 2013-06-01 to 2016-06-01

We can see that only first two pairs still remain stationary during this test period.



Graphic 11 Example of a failed pair who lost its stationarity in test period

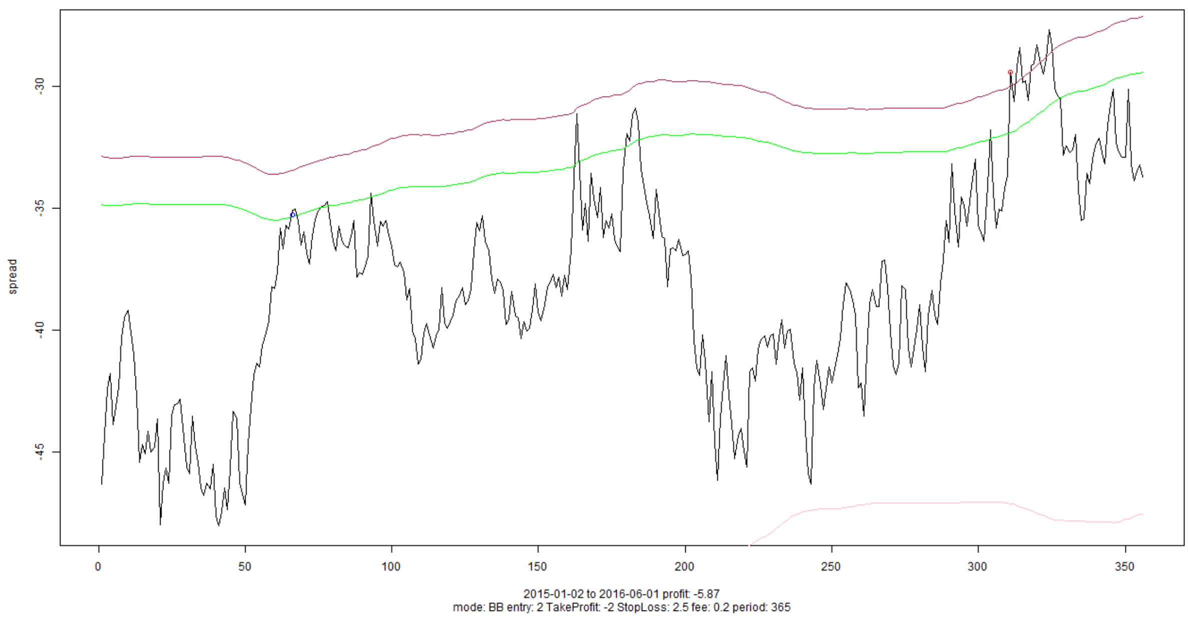
The graphic 11 shows the example of Moody's Corporation – Tiffany's. This pair lost its stationarity almost exactly at the beginning of test period.



Graphic 12 Example of a failed pair who kept its stationarity in test period

The graphic 12 shows the example of Moody's Corporation – Wells Fargo. This pair still kept its stationarity during test period. From the graphic 13, we can see that this pair resulted in loss mainly due to change of optimal entry & exit levels.

Now we know that two reasons could result in loss: loss of stationarity and change of optimal entry & exit levels.



Graphic 13 trading chart of Moody's Corporation – 2.6 Wells Fargo in test period

5. Conclusion

This paper looks for optimal pairs trading strategies in the econometrics framework of the cointegration theory. We have therefore exploited at maximum all the information provided by the bivariate ECM-DCC-GARCH model associated to long term relationship between the stock pairs' prices and in particular the short term conditional variance of the stationary linear combination between the prices.

The best strategy with the highest profit factor is Bollinger Bands without GARCH confirmation. It generated $4024.97 as the absolute net profit, with a global maximum drawdown of $1453.49, which gave a profit factor of 283.92%. By taking a global risk of $100, we will be able to make a profit of $283.92 every year. 53.54% of all pairs are winning, 32.32% are lost, and the rest breaks even because they never started trading. With a median profit of $2.95, this strategy has a mean profit of $40.66. This suggests that small winning and losing pairs makes the majority with a few extremely profitable pairs raising the total gain. An open selling order lasts on average 70 days, while a buying one 42 days. We wait on average 78 days between two trades of the same pair.

Some of our results shows that for certain pairs neither of the two stocks participates in the return to equilibrium. This fact invites to investigate hidden variables that contribute to the mean-return mechanism. If it involves other stocks' prices, we will need to look into pairs trading strategies of more than two stocks. It is also possible that the linear specifications of error correction model (ECM) are not able to estimate correctly the value of parameters λ_1 and λ_2 associated to restoring force. It is reasonable to think that the mechanism of return to equilibrium obeys non-linearity, threshold effect, and switch of regime… We have the intuition that the prices' dynamic i.e. that of the variable $\widehat{\varepsilon_t}$ may be very different from the interior and its distribution tail. The identification of a nonlinear version of the ECM model could therefore be aim of future research. The question of link between pairs trading's profitable strategies and markets' efficiency makes also part of our future thinking.

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