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# Cultural Dynamics, Social Mobility and Urban Segregation

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# Cultural Dynamics, Social Mobility and Urban Segregation\*

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## Abstract

We consider the relationship between intergenerational mobility and urban segregation. To this end, we develop a model of neighbourhood formation and preference transmission. The key feature here is that the incentives the parents have to transmit their trait to their children depend on the endogenous social composition of the neighbourhood. When the urban equilibrium that emerges at each date is segregated, some urban areas are characterized by better social-mobility prospects than others. Segregation also generates some persistence of socio-economic status within dynasties. We show that there exist multiple history-dependent steady-states in the joint dynamics of segregation and the distribution of culture traits. Further, segregation has ambiguous effects for long run efficiency. We show that depending on the degree of substitutability between the two instruments of socialization (i.e, individual effort and residential choice), integration may emerge endogenously and be efficient. This suggests public policies that would produce neighbourhood socio-economic compositions that are more favourable to the transmission of particular cultural traits, such as for instance group-based policies.

Keywords: cultural transmission, peer effects, residential segregation, human capital inequality.

JEL Classification: D31, I24, R23.

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# 1 Introduction

The neighbourhoods in which children grow up are particularly important for their socio-economic success. Neighbourhoods influence children's economic opportunities via both institutional channels, including the decentralized funding of local services and public goods (e.g. school resources), and the social channels of role models, peer effects, identity formation and job contacts. The concentration of social problems such as criminal activity, unemployment, school drop-out rates and teenage childbearing in some urban areas, for instance inner-city areas in the US and the suburbs in Europe, is a striking manifestation of the deleterious effects that living in a deprived neighbourhood can have on children's destinies. Recent work on intergenerational mobility in the US has emphasized the role of neighbourhoods for individual prospects of social mobility (see Chetty, Hendren, Kline and Saez, 2014, Chetty and Hendren, 2015). In particular, Chetty, *et al.* (2014) use data from Federal income tax records over the 1996-2012 period to show the striking spatial variation in social mobility. They also show that high-mobility areas have less residential segregation and income inequality, better primary schools, and greater social capital and family stability. Using the same database, Chetty and Hendren (2015) identify a causal effect of the place of residence on intergenerational mobility. They show, for example, that for children with parents at the 25th percentile of the national income distribution, growing up in a one standard deviation better neighbourhood during the first twenty years of childhood increases future income by 10%. These results suggest that the pattern of urban segregation plays a major role in the intergenerational transmission of economic status.<sup>1</sup>

It is well-documented that urban segregation interacts with the transmission of preferences, beliefs and social norms that are key for socio-economic success (e.g. educational investment, labour-market decisions, and marriage decisions). Various empirical contributions have shown that urban segregation affects attitudes and values such as ethnic identity (see Bisin, Patacchini, Verdier and Zenou, 2011a, Constant, Schuller and Zimmerman, 2013) and subcultures acting against socio-economic integration (see, for instance, Crane, 1991, for neighbourhood effects in dropping-out and teenage childbearing, and Gaviria and Raphael, 2001, for peer-group effects at the school level in alcohol and

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<sup>1</sup>See, for instance, Durlauf (2004) and Topa and Zenou (2014) for surveys of the empirical analysis of neighbourhood effects. We should mention here that there is not yet a consensus about the size of neighbourhood effects on educational outcomes. Some experimental or quasi-experimental work finds little evidence of neighbourhood effects on educational outcomes (see Kling *et al.* 2007, Oreopoulos, 2003). Topa and Zenou (2014) provide an interesting discussion of why these experimental analyses may lead to insignificant treatment effects on economic outcomes. A growing literature in sociology emphasises the duration of exposure to neighbourhoods, which helps explain why experimental work finds little evidence of neighbourhood effects (see Sharkey and Elwert, 2011, Wodtke *et al.* 2011). Chetty, Hendren and Katz (2015) consider the Moving To Opportunity Experiment and show that treatment effects are substantial when considering the duration of exposure to a better neighbourhood.

drug use).<sup>2</sup> Urban segregation also influences parental involvement in their children's education. Using the UK National Child Development Study, Patacchini and Zenou (2011) find that parents invest more in the education of their child when they live in a neighbourhood of high quality in terms of education. Other work has also stressed the parental choice of the social arenas where their children interact (e.g, schools, neighbourhoods, friendship networks, and places of worship) as one way of transmitting the desired cultural traits. In particular, Ioannides and Zanella (2008) analyse the determinants of location choices using PSID data. Their findings suggest that parents with children search for neighbourhoods with attributes that are favourable to human-capital production and the transmission of parents' cultural traits.<sup>3</sup>

Understanding the interdependency between urban segregation and cultural transmission is crucial for the design of policies promoting social integration. First, it is not clear how segregation helps or hinders the transmission of cultural traits in promoting individual socio-economic success. Second we do not know how the existence of opposite cultures regarding personal achievement affects segregation.

In this paper, we consider the relationship between intergenerational cultural transmission, spatial socio-economic segregation and inequality dynamics. To this end, we develop a theoretical setup that formalizes the interdependency between neighbourhood quality and parental involvement in the intergenerational transmission of cultural traits.

More specifically, we consider overlapping generations of individuals who live two periods. Children decide how much educational effort to exert. Two key factors influence this educational effort: preferences toward education, which differ across children (section 2 provides the empirical evidence supporting this assumption), and peer effects. Preferences are transmitted following the cultural transmission mechanism *à la* Bisin and Verdier (2001). A child acquires some cultural trait either from her/his parent's socialization investment (direct vertical socialization) or by encountering role models in the neighbourhood (oblique socialization). There are also peer effects generated by the neighbourhood which affect the cost of education. A key feature is that these peer effects in education vary spatially, so that incentives to exert educational effort differ across space. As parents, individuals decide how much to invest in socializing their children to their own taste for education. The incentive to transmit one's own preferences comes from imperfect altruism, meaning that parents

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<sup>2</sup>See also the work of Wilson (1987) and Anderson (1999), who document how living in poor inner-city communities may produce a culture of poverty opposed to mainstream culture.

<sup>3</sup>It has also been shown that school choice by parents is motivated by their desire to choose peers who will best transmit their preferred cultural traits (see Tinker and Smart, 2012, for Muslim schools in Britain, and Sikkink and Emerson, 2008, for the effect of school choice on racial segregation in the US).

are able to correctly assess the optimal choices of their child, but only through the filter of their own preferences. The incentives behind the choice of socialization investment depend on socio-economic composition in two ways. On the one hand, peer effects influence the child's future well-being, which is taken into consideration by parents. Peer effects make parents with a taste (distaste) for education more (less) willing to exert socialization effort in order to transmit their trait. On the other hand, there is an opposite effect generated by the oblique socialization mechanism. The more people with the same cultural trait in the neighbourhood, the more efficient is oblique socialization and the less incentive a parent has to transmit her/his trait. This is called the cultural substitution effect. The decision to exert socialization effort depends on the trade-off between peer effects and cultural substitution. Due to peer effects, the incentive for parents to invest in socialization rises with the fraction of agents with similar preferences. However, due to cultural substitution, socialization investment is lower whenever agents with the same preferences are a majority.

Furthermore, the socio-economic composition of each neighbourhood is endogenous, implying that the location decision is one way of socializing children. Parents choose where to live, and thus optimally decide the level of oblique transmission. The urban equilibrium results from different forces. As agents are imperfectly altruistic, they want to have a child of their own type. This cultural intolerance effect creates incentives for parents to live with agents of the same type and is a force for segregation. However, the location decision (and so oblique transmission) is not the only socialization instrument. Parents can also exert effort in order to directly transmit the trait to their child. Depending on whether these two socialization instruments are complements or substitutes there will be further incentives to segregate or new incentives to live in a mixed urban area. The two instruments are complements if the gain from socialization effort is high in areas where oblique transmission is high. In this case, parents exert a higher socialization effort in areas where their trait prevails. This occurs when peer effects are substantial (as peer effects increase the return to the socialization effort where agents of the same type are more numerous). Complementarity between both instruments thus reinforces the incentive to live in areas where the trait prevails and so pushes towards segregation. The location decision and socialization effort are substitutes when the gain from socialization effort is high in areas where oblique transmission is low. This leads parents to exert higher socialization effort in areas where their trait is rare. This is the case when the cultural substitution effect is high (as the latter reduces the efficiency of the socialization effort where there are more agents with the same cultural trait). Substitutability thus reduces the incentive for parents to live in areas where their trait is a majority. More precisely, some parents may be willing to live in

lower quality neighbourhoods (i.e. where their trait is a minority) so as to save the rent but offset low oblique transmission by exerting a high socialization effort. Substitutability is a force toward integration.

In a first step, we assume that segregation forces outweigh integration forces meaning that the socialization effort is relatively inefficient and cultural intolerance of agents is high. Hence, in each period the urban equilibrium is segregated. There are four main features. First, as individuals with a taste for education cluster in some urban areas, there is inequality across the city in terms of education. Second, and more interestingly, depending on the population fraction of individuals with a taste for education, the equilibrium is characterized by some degree of substitutability between location decision and socialization choice. For some ranges of the population of agents with preferences for education, the segregated equilibrium is such that peer effects are strong in the urban area where agents with preferences for education are in the majority. In equilibrium, location choice and socialization effort are complements in the transmission of cultural traits. For other ranges of the population with preferences for education, the segregated equilibrium is such that cultural substitution is strong in neighbourhoods where agents with preferences for education are in the majority. In equilibrium, location choice and socialization effort are substitutes in the transmission technology<sup>4</sup>. Third, intergenerational mobility differs between urban areas. Children living in areas where the education rate is higher are more likely to acquire the education trait whatever their parent's preferences. Fourth, at the city level, urban segregation makes socio-economic status within dynasties be persistent. This result relies on the fact that parents with preferences for education are more willing to pay in order to inhabit areas with more education, implying that their offspring have a greater chance of interacting with the desired role models.

We analyse the joint dynamics of segregation and the distribution of preferences for education over the whole population. We show that there are multiple history-dependent steady states. When the initial fraction of agents with preferences for education is low (i.e, the level of education is low) cities are trapped in low education (where some neighbourhoods are completely deprived of education). In such a case, peer effects are so low that agents who value education do not have incentives to actively transmit these traits. This negatively affects the dynamics of these cultural traits and thus the long run level of education. We call these cities 'socially immobile', as the number of children experiencing upward and downward social mobility is exactly the same. When initial values of the fraction

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<sup>4</sup>Note that when we assume that segregation forces outweigh integration forces, this does not prevent the segregated equilibrium from emerging.

of people with a taste for education are higher (and takes intermediate values), the distribution of traits within the city and notably the concentration of agents who value education generates strong peer effects which provide incentives to transmit the trait favouring economic success. This positively affects the dynamics of education. The number of people experiencing upward social mobility outweighs those with downward mobility. We call these cities ‘socially mobile’. Nonetheless, expansion of education is halted once a threshold fraction of individuals with a taste for education has been reached. This is because, for higher fractions of agents with a taste for education, cultural substitution outweighs peer effects, removing the incentive to exert socialization effort in urban areas. Above this threshold, the distribution of traits is stationary and the city becomes ‘socially immobile’.

We address the fundamental issue of efficiency, and ask whether segregation is desirable for the long-run level of education in a city. We show that segregation has ambiguous effects on the long run level of human capital.

On the one hand, segregation may generate strong peer effects by concentrating one population in some neighbourhood favouring transmission of traits which favour economic success. On the other hand, concentration of individuals with the same trait is not always desirable since cultural substitutability implies that some individuals rely on oblique transmission within the neighbourhood thus lowering their socialization effort. This negatively affects the dynamics of cultural traits which favour education and so the long run level of human capital.

Also, we show that when the socialization effort and the location choice are sufficiently substitutable (i.e, the socialization effort is efficient and/or cultural intolerance is low), integration arises endogenously and is efficient. The intuition is that, for some distribution of traits in the population, at equilibrium some parents with preferences for education are willing to live in the lowest quality neighbourhood (in terms of the prevalence of their cultural trait) where they save the rent but offset low oblique transmission by a high socialization effort, while others parents have incentives to live in the best quality neighbourhood where they pay the rent but save the cost of socialization. This integrated equilibrium is efficient. Unlike segregation, this integrated equilibrium pushes agents who like traits that favour economic success to actively transmit these traits to their child (rather than relying on oblique transmission in the best quality neighbourhood).

These results allow us to emphasize the role of public policies so as to make the socio-economic composition of urban areas provide incentives for the transmission of the taste for education. We show that urban policies affecting location decisions such as housing-subsidies programs can be designed to make the integrated city emerge. Another way to restore efficiency is to increase the substitutability

of socialization instruments. This may be the object of appropriate group-based policies.

Our paper belongs to two strands of literature. It is first related to the literature on neighbourhood effects and endogenous socio-economic segregation explaining how local interactions drive spatial segregation and persistent inequality (see for instance, Loury, 1977, Bénabou, 1993, 1996a,b, Durlauf, 1996). In this work, the dynamics of income inequality relies on human-capital accumulation, and individual human capital is determined by that of their parents as well as local spillovers. The choice of parental investment in their children's education is not considered in this literature. We depart from this work as our preference-transmission set-up allows us to highlight the role played by cultural substitution and substitutability between instruments of socialization in the emergence and efficiency of the urban equilibrium. Our paper is also related to the literature on cultural transmission launched by Bisin and Verdier (2001). The transmission of the traits such as identities, time preferences or beliefs, which account for heterogeneous preferences toward education among children in our model, has been analysed theoretically (see Bisin, Patacchini, Verdier, and Zenou, 2011b, for oppositional identities, Doepke and Ziliboti, 2008, for time preferences and the spirit of capitalism, Guiso, Sapienza and Zingales, 2008, for beliefs and trust in other people, Lindebck and Nyberg, 2006, for the transmission of working norms). Bisin, Patacchini, Verdier, and Zenou (2011b) is relatively close to our paper, as they consider the impact of segregation on the persistence of oppositional values. In their framework, however, socio-economic segregation is exogenous. By introducing location decisions, we are able to characterize the socio-economic composition of the neighbourhood that emerges in equilibrium, which drives the particular pattern of preferences transmission. We thus identify conditions on the size of the segregation and integration forces leading to the urban configuration that best promotes long-run human capital in the city. We also highlight conditions under which the existence of opposite cultures give rise to an integrated city.

The remainder of our paper is as follows. The next section provides empirical evidence which motivates our hypothesis of heterogeneous preferences toward education. In section 3, we develop the model. In section 4, we characterize the urban equilibrium that emerges at each date  $t$ . Section 5 looks at the dynamics of urban segregation and the cultural traits. Section 6 addresses the issue of efficiency in the urban equilibrium. Section 7 concludes.



## 2 Evidence of Heterogeneous Preferences over Education and their Intergenerational Transmission

The first crucial assumption of our model that individuals do not value similarly education reflects a variety of phenomena. These may first be embodied in deep preferences: some work has revealed that children develop various identities in schools, which then produce particular educational behaviours (see Akerlof and Kranton, 2002, for a review of ethnographic work and social experiments documenting the influence of social categories on behaviour). For example, the literature on racial inequality emphasizes that ethnic minorities impose costs on members who adopt majority behaviours (see Fryer and Torelli, 2010, Battu and Zenou, 2010, and Battu *et al.* 2007 for empirical evidence on oppositional identities). In particular, Fryer and Torelli (2010) provide an empirical analysis of ‘Acting White’. Using the Addhealth database which covers a sample of junior high and high school students, they find that Black and Hispanic students with better grades have less same-race friends within their school. Precisely, a black, respectively hispanic, student with a 4.0 has, on average, 1.5, respectively 3, fewer same-race friends than a white student with a 4.0. Moreover, studies on school choice indicate that parents do not search for the same school characteristics supporting the view that preferences over education are heterogeneous. For example, Hastings *et al.* (2009) use data from a natural experiment in the Charlotte-Mecklenburg School District on parental school-choice behaviour and find that high-income parents of high-achieving students place the greatest weights on test scores. They also show that the willingness to commute to attend high test-score schools increases with preferences for school test scores. Jacob and Lefgren (2007) also suggest that parents have different values regarding education. In particular, they find that parents in low-income and minority schools strongly value student achievement and are indifferent to teacher ability to promote student satisfaction, with these results being reversed for parents in higher-income and non-minority schools.

Different preferences over education may also reflect that children differ with respect to time-preferences and do not discount similarly the expected benefits of their human capital investment decisions. Conducting field experiments, Castillo *et al.* (2011) show that discount rates greatly vary across children. They also find that discount rates are correlated with schooling achievement. In the same vein, a study by Cadena and Keys (2014), using the National Longitudinal Survey of Youth, suggests that impatient people are more prone to suboptimal investment behaviors in human capital. The literature on human capital formation suggests that this heterogeneity of time-preferences could

be explained by the variability of non-cognitive abilities (see Cunha and Heckman, 2009).

Subjective expectations of the returns to schooling can also explain why individuals have different views regarding the benefits of education. Using data from a household survey on Mexican junior and senior high-school graduates, Attanasio and Kauffman (2010) find different expectations between children from rich and poor families and show that these are crucial for schooling decisions. There are also well-documented cross-country differences in beliefs about the determinants of socio-economic success. For example, Alesina and Angeletos (2005) write that

“According to the *World Values Survey*, 60 per cent of Americans versus 29 per cent of Europeans believe that the poor could become rich if they just tried hard enough; and a larger proportion of Europeans than Americans believe that luck and connections, rather than hard work, determine economic success.” (p. 960).

The second crucial modelling assumption is that preferences over education are transmitted across generations as the result of deliberate socialization actions (e.g. time spent for their offspring-rearing, choice of family’s friends and other social environments). This approach has also much empirical support. For instance, Mulligan (1997) conducts a study on the Panel Study of Income Dynamics on the 1968-1989 waves and finds a significant correlation between parents and children’s willingness to work. A recent study by Dohmen, Falk, Huffman and Sunde (2012) examines specific attitudes relevant for educational attainment, i.e. willingness to take risk and trust in others. Their work provides evidence for these attitudes being transmitted across generations due to socialization actions by parents (e.g. marriage decisions) and transmission by peers. As well, there is empirical evidence that parents play an important role in reproducing both the cognitive and noncognitive skills of their children (see Carneiro and Heckman, 2003, Cunha *et al.*, 2006, Grönqvist *et al.*, 2014). Using military enlistments records for cohorts of Swedish men that provide evaluation of fathers’ and sons’ abilities, Grönqvist *et al.* (2014) find a strong intergenerational correlation of non-cognitive abilities. Their studies also suggests that educational outcomes of children are strongly related to their parents cognitive and non-cognitive abilities. Moreover, a large sociological literature on the culture of poverty has documented that parents’ attitudes toward education are transmitted to their children. In their review of the literature, Lamont and Small (2008) refer to Lareau (2004)’s findings:

“Lareau’s (2004) *Unequal Childhoods* shows that middle class parents on the one hand, and working class and poor parents on the other, manage the extracurricular activities of their children differently. This provides them with different endowments or assets of

cultural capital. Drawing on ethnographic fieldwork in a small number of poor, working class, and middle class families, Lareau found that middle class people she interviewed favored ‘purposeful cultivation’ and organize numerous extracurricular activities for their children. By contrast, her working class and poor interviewees favored ‘natural growth’ and were much less involved in managing their children’s lives than their middle class counterparts are [...]. The leisure time of the working class and poor in her study was relatively unstructured and did not contribute to teaching children skills that middle class children learned and that would prepare them for professional life.” (p. 87)

A large literature in sociology, anthropology, psychology and economics provides evidence that parents play a role in the formation of their children’s ethnic identity by transmitting information, values and perspectives about ethnicity and race (see Alba, 1990, Boyd and Richerson, 1985, Cavalli-Sforza and Feldman, 1981, Phinney, 1990, Hughes *et al.*, 2006, Casey and Dustman, 2010). In particular, Casey and Dustman (2010) use a longitudinal dataset for Germany looking at second generation immigrants population. They emphasize a strong and significant association between parents’ and children’s home and host countries identities.

## 3 The Set-up

### 3.1 The City

The city is comprised of two residential areas indexed by  $j = 1, 2$ . We consider that there are no landowners, and without loss of generality normalize the opportunity cost of building a house to 0. Houses are identical across the city. The inelastic supply of houses within a residential area is of mass  $L$ . This land-market is a closed-city model where the population of the city is a continuum of families of mass  $M$ . Each family, comprised of a parent and a child, lives in one and only one house. The city can accommodate the entire population and we assume for the sake of simplicity that  $L = M/2$ . Agents live two periods. As a child, the individual faces a discrete educational choice. As an adult, the individual has to decide in which neighbourhood her family will live, and the effort to exert to transmit her cultural trait.

## 3.2 Children's Educational Choices

**Preferences.** Children have to decide whether they exert high educational effort denoted by  $\bar{e}$  or low effort  $\underline{e} < \bar{e}$ , depending on their preferences for education. Children of type  $a$ , who value education, derive extra benefit from exerting high effort, which is equal to  $a$ . By contrast, children of type  $b$  derive benefit  $b$  from exerting low education effort<sup>5</sup>. Let  $U_t^{ij}$  denote the preferences of a child with trait  $i = \{a, b\}$  living in area  $j = 1, 2$  at date  $t$ . Preferences are defined as follows:

$$U_t^{aj} = p(e)w_r + (1 - p(e))w_p + ap(e) - (1 - \lambda_t^j)c\mathbf{1}_{e=\bar{e}} \quad (1)$$

and

$$U_t^{bj} = (e)w_r + (1 - p(e))w_p + (1 - p(e))b - (1 - \lambda_t^j)c\mathbf{1}_{e=\bar{e}}. \quad (2)$$

When exerting effort  $e$ , a child is rich and earns income  $w_r$  with probability  $p(e)$ . Otherwise, he/she is poor with income  $w_p < w_r$ . We assume for the sake of simplicity that  $p(\bar{e}) = 1$  and  $p(\underline{e}) = 0$ . Exerting effort  $\bar{e}$  incurs an education cost of  $(1 - \lambda_t^j)c$ , with  $c \in \mathbb{R}_+$  and  $\lambda_t^j$  the fraction of children exerting effort  $\bar{e}$  in neighbourhood  $j$ . We thus allow for peer effects in education by assuming that the cost falls with the fraction of educated children in the neighbourhood,  $\lambda_t^j$ . We assume that exerting effort  $\underline{e}$  incurs no cost. Finally, this cost function is the same for children of type  $a$  and  $b$ .<sup>6</sup>

We denote by  $N_t$  ( $N_t^j$ ) the number of agents with trait  $a$  in the whole population (in area  $j$ ) at time  $t$ , and by  $Q_t$  (resp.  $q_t^j$ ) the fraction  $N_t/L$  ( $N_t^j/L$ ) of these agents at time  $t$ .

**Educational Choice.** At date  $t$ , a child with trait  $a$  residing in area  $j$  chooses to become educated if and only if

$$w_r - (1 - \lambda_t^j)c + a > w_p$$

<sup>5</sup>As can be seen from equations (1) and (2) this formulation more broadly captures heterogeneity expected returns to education.

<sup>6</sup>In the literature, it is usually assumed that the derivative of the cost with respect to  $\lambda_j$  is not the same between different type of agents in order to obtain segregated equilibria (see Bénabou, 1993). Here we choose the same cost function so as to emphasize the new mechanisms driving segregation.

while a child with trait  $b$  residing in area  $j$  exerts high effort if and only if

$$w_r - (1 - \lambda_t^j)c > w_p + b.$$

We make the following assumptions

**Assumption 1** For any  $w_r$  and  $w_p$ ,

(i)  $a$  is such that  $w_r - c + a > w_p$ ,

(ii)  $b$  is such that  $w_r < w_p + b$ .

This implies that, for any  $\lambda_t^j \in [0, 1]$ , all type- $a$  individuals exert high effort whereas all type- $b$  individuals exert low effort. Given that  $p(\bar{e}) = 1$  and  $p(\underline{e}) = 0$ , we have that any child with trait  $a$ , ( $b$ ,) becomes rich (poor) as an adult. We can deduce that the fraction of children who become educated is

$$\lambda_t^j = q_t^j.$$

Assumption 1 is not crucial for our results but does allow us to obtain straightforwardly the rate of education. The only restriction we impose is that there is no corner solution, such that all children choose high effort. We could have assumed that children differ with respect to innate ability, and only the smarter individuals become educated (so a higher fraction of type- $a$  children choose education). This would not change our results.

### 3.3 The parents' transmission decision and location choice

At any date  $t$ , parents who differ with respect to both income and preferences make two decisions. They choose both the location  $j$  where they pay the land rent  $\rho_t^j$  and their socialization effort.

**The Dynamics of Preferences.** The transmission of preferences follows the lines of the model introduced by Bisin and Verdier (2001). The intergenerational transmission of trait  $i$  is the result of social interactions which arise at two levels. The child is first exposed to vertical socialization by his/her parents. The probability that the latter directly transmits her/his trait is  $d^i$ . If not socialized within the family (with probability  $1 - d^i$ ), the child is obliquely socialized picking the trait of a role

model chosen randomly in the neighbourhood  $j$ . The probability of being obliquely socialized into trait  $a$  (resp.  $b$ ) in neighbourhood  $j$  is  $q_j$  (resp.  $1 - q_j$ ), the fraction of agents with trait  $a$  (resp.  $b$ ) in this neighbourhood. The transition probabilities are given by

$$P_{aat}^j = d^a + (1 - d^a)q_t^j \text{ and } P_{abt}^j = (1 - d^a)(1 - q_t^j) \quad (3)$$

$$P_{bbt}^j = d^b + (1 - d^b)(1 - q_t^j) \text{ and } P_{bat}^j = (1 - d^b)q_t^j. \quad (4)$$

In particular,  $P_{aat}^j$  denotes the probability that a child from a type- $a$  family be socialized into type  $a$  at time  $t$ .

In this model, direct transmission is the result of a choice. Parents exert a discrete effort  $d^i \in \{0, \tau\}$  in order to transmit their preferences.<sup>7</sup> The incentive to transmit one's own preferences comes from imperfect empathy: parents are able to correctly assess the optimal choices of their child but through the lens of their own preferences. We denote by  $V_{ii'}^{j,t+1}$  the gain to a parent of type  $i$  of having a child of type  $i'$  in neighborhood  $j$  at date  $t + 1$ . Moreover, we assume that parents are myopic, so that  $V_{ii'}^{j,t} = V_{ii'}^{j,t+1}$ ,  $\forall i \in \{a, b\}$ .<sup>8</sup>

**Preferences.** We denote by  $U_t^{i,z}(\rho_t^j, d^i)$ , the utility at date  $t$  of a parent with trait  $i$  and income  $w_z$ ,  $z = r, p$ , who lives in neighbourhood  $j$  and exerts socialization effort  $d^i$ . Omitting the time index for transition probabilities and gains  $V_{ii'}^j$ , we have for a trait- $a$  parent who is rich

$$U_t^{a,r}(\rho_t^j, d^a) = w_r - \rho_t^j + P_{aa}^j V_{aa}^j + P_{ab}^j V_{ab}^j - \Theta(d^a), \quad (5)$$

where  $\Theta(\cdot)$  is the socialization cost with  $\Theta(0) = 0 < \theta = \Theta(\tau)$ . The socialization gains are given by

$$\begin{aligned} V_{aa}^j &= w_r - (1 - \lambda_t^j)c + a \\ V_{ab}^j &= w_p. \end{aligned}$$

Note that the fact that parents are myopic implies that they consider that the peer effects affecting their child's educational effort come from the rate of education in their own generation,  $\lambda_t^j$ .<sup>9</sup>

<sup>7</sup>The general version of the model developed in the Appendix considers a continuous choice of socialization effort and shows that our main results remain under this extension.

<sup>8</sup>The latter assumption allows us to ignore self-fulfilling expectations equilibria (see Bisin and Verdier, 2000, for a treatment of rational expectations equilibria). This would complicate the analysis without introducing any additional key insights.

<sup>9</sup>In any case, the child's effort is a positive function of the rate of education  $\lambda_t^j$ .

If we let  $\Delta V_a^j \equiv V_{aa}^j - V_{ab}^j$  and  $\Delta w \equiv w_r - w_p$ , and given that  $\lambda_t^j = q_t^j$ , we have

$$\Delta V_a^j = \Delta w - (1 - q_t^j)c + a. \quad (6)$$

We thus see that  $\Delta V_a^j > 0$ , given Assumption 1, which amounts to saying that there is cultural intolerance.

A parent with trait  $b$  and income  $w_p$  who lives in neighbourhood  $j \in \{1, 2\}$  has utility

$$U_t^{b,p}(\rho_t^j, d^b) = w_p - \rho_t^j + P_{bb}^j V_{bb}^j + P_{ba}^j V_{ba}^j - \Theta(d^b), \quad (7)$$

with

$$V_{bb}^j = w_p + b$$

$$V_{ba}^j = w_r - (1 - \lambda_t^j)c.$$

Letting  $\Delta V_b^j \equiv V_{bb}^j - V_{ba}^j$ , we have

$$\Delta V_b^j(q_t^j) = -(\Delta w - (1 - q_t^j)c - b). \quad (8)$$

Given Assumption 1, type- $b$  parents also feel cultural intolerance, i.e.  $\Delta V_b^j > 0$ .

**The socialization choice of type- $a$  parents.** At date  $t$ , a type- $a$  parent exerts effort in neighbourhood  $j$  if and only if

$$U_t^{a,r}(\rho_t^j, \tau) \geq U_t^{a,r}(\rho_t^j, 0).$$

Given (3), (5) and (6), the above inequality can be expressed as follows

$$(1 - q_t^j)\tau (\Delta w - (1 - q_t^j)c + a) - \theta \geq 0. \quad (9)$$

The choice of socialization effort involves a trade-off between the gain from socialization, denoted by  $(1 - q_t^j)\tau (\Delta w - (1 - q_t^j)c + a)$ , and the cost of transmission  $\theta$ . In particular, the socialization gain depends on the fraction of  $a$  agents via two opposing effects. The cultural substitution effect is negative and captured by the term  $(1 - q_t^j)$ . As the fraction of agents with the same trait increases, oblique transmission becomes more effective, so that the incentive to exert high socialization effort is reduced. Peer effects are positive and captured by  $-(1 - q_t^j)c$ . All else equal, a child with preferences

for education is better-off as the fraction of the educated in the neighbourhood rises.

We assume the following

**Assumption 2** *The parameters  $c, w_r, w_p, \tau, \theta$  are such that equation (9) has two positive roots,  $\tilde{q}_1$  and  $\tilde{q}_2$ , such that (i)  $\tilde{q}_1 < 1/2 < \tilde{q}_2 < 1$  and (ii)  $\tilde{q}_1 + \tilde{q}_2 < 1$ .*

The Appendix provides a more detailed analysis of (9). Under Assumption 2, we consider intermediate values of peer effects relative to the cost of socialization. In particular, when peer effects are relatively low, Assumption 2 implies that there is no incentive to socialize children into trait  $a$ . However, when peer effects are higher and outweigh cultural substitution, type- $a$  parents have an incentive to make a positive socialization effort.

We deduce the optimal socialization choices of type- $a$  parents in neighbourhood  $j$

$$d^{j,a} = \begin{cases} 0 \Leftrightarrow q^j < \tilde{q}_1 \quad \text{or} \quad q^j > \tilde{q}_2, \\ \tau \Leftrightarrow q^j \in [\tilde{q}_1, \tilde{q}_2]. \end{cases} \quad (10)$$

The optimal effort function of type- $a$  agents is a non-monotonic step function of  $q^j$ , the fraction of these agents in the neighbourhood. For low values of  $q^j$ , i.e.  $q^j < \tilde{q}_1$ , peer effects prevail. In this case, the incentive to exert effort is an increasing function of  $q^j$ : there are social complementarities. Hence optimal effort is positive if and only if peer effects are sufficiently strong, i.e.  $q^j > \tilde{q}_1$ . For higher values of  $q^j$ , the cultural substitution effect becomes stronger and can outweigh the peer effects. The incentive to exert effort then becomes a decreasing function of  $q^j$ : there is social substitutability. Optimal effort is positive if and only if cultural substitution does not outweigh the peer effects, i.e.  $q^j < \tilde{q}_2$ . It is zero when cultural substitution is strong, i.e.  $q^j > \tilde{q}_2$ .<sup>10</sup>

**The socialization choice of type- $b$  parents.** At time  $t$ , a type- $b$  parent with income  $w_p$  exerts effort in neighbourhood  $j$  if and only if

$$U_t^{b,p}(\rho_t^j, \tau) \geq U_t^{b,p}(\rho_t^j, 0)$$

which is equivalent to

$$q_t^j \tau \Delta V_b^j(\lambda_t^j) - \theta \geq 0.$$

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<sup>10</sup>The empirical literature has provided evidence of such non-monotonic patterns in socialization effort. In particular, Bisin, Topa and Verdier (2004), find an inverted-U relationship between the socialization rates of transmitting some religious trait and the share of this religious trait in the US population.



Given (8), we obtain

$$-q_t^j \tau (\Delta w - (1 - q_t^j)c - b) - \theta \geq 0. \quad (11)$$

The gain from socialization is now given by  $-q_t^j \tau (\Delta w - (1 - q_t^j)c - b)$  which is positive given Assumption 1. It depends on a cultural substitution effect captured by the first term  $q_t^j$ . Parents with trait  $b$  have less incentive to socialize their child when there are more type- $b$  individuals in the neighbourhood, i.e. when  $q_t^j$  is lower. The gain from socialization also depends on peer effects that are captured by the term  $-(1 - q_t^j)c$ .

We assume the following

**Assumption 3** *The parameters  $\tau$ ,  $\Delta w$ ,  $c$ ,  $b$  and  $\theta$  are such that type- $b$  parents do not exert any socialization effort whatever the value of  $q^j$ .*

This assumption amounts to considering that education is profitable enough so that type- $b$  parents never transmit their trait.<sup>11</sup>

It turns out that the dynamics of the fraction of the population with trait  $a$  in neighbourhood  $j \in \{1, 2\}$  are given by

$$q_{t+1}^j = \begin{cases} q_t^j, & \text{if } q_t^j < \tilde{q}_1, \\ q_t^j + q_t^j (1 - q_t^j) \tau, & \text{if } \tilde{q}_1 \leq q_t^j \leq \tilde{q}_2 \\ q_t^j, & \text{if } \tilde{q}_2 < q_t^j. \end{cases} \quad (12)$$

**Parents' Location Choice.** At any date  $t$ , parents choose where to live by solving the following program

$$\text{For type-}a \text{ parents, } \max_j u(w_z - \rho_t^j) + P_{aa}^j V_{aa}^j + P_{ab}^j V_{ab}^j - \Theta(d^{j,a}), \quad \text{with } d^{j,a} \text{ given by (10).}$$

$$\text{For type-}b \text{ parents, } \max_j u(w_z - \rho_t^j) + P_{bb}^j V_{bb}^j + P_{ba}^j V_{ba}^j - \Theta(d^b), \quad \text{with } d^b = 0.$$

$Q_t$  is the fraction of trait- $a$  agents in the whole population, so that  $q_t^1 + q_t^2 = Q_t$ . Without loss of generality, we impose that  $q_t^1 \geq q_t^2$  and  $\rho_t^2 = 0$ . The urban equilibrium is defined as follows:

**Definition 1** *At any date  $t$ , given  $Q_t$ , the urban configuration  $[\rho_t^*, q_t^{1*}, d^{*1,a}, d^{*2,a}]$  is an equilibrium if no one wants to move and change their socialization choice.*

<sup>11</sup>The general version of the model presented in the Appendix allows for the case  $d^a < d^b$ . However, our main results are present in this simple version of the model.

## 4 Urban Equilibria in the Short Run

To obtain the urban equilibria, we characterize the willingness to pay to live in urban area 1, denoted by  $\bar{\rho}_t^i$  for  $i \in \{a, b\}$ . This is such that a type- $i$  parent is indifferent between the two neighbourhoods. For a type- $a$  individual, given that  $P_{ab}^j = 1 - P_{aa}^j$ , we have

$$\bar{\rho}_t^{a,1} = P_{aa}^1 \Delta V_a^1 - P_{aa}^2 \Delta V_a^2 + V_{ab}^1 - V_{ab}^2 - (\Theta(d^{1,a}) - \Theta(d^{2,a}))$$

with  $d^{j,a}$ ,  $j \in \{1, 2\}$ , given by (10). For a type- $b$  individual, we obtain

$$\bar{\rho}_t^{b,1} = P_{bb}^1 \Delta V_b^1 - P_{bb}^2 \Delta V_b^2 + V_{ba}^1 - V_{ba}^2.$$

Given (3), (4), (6), (8), Assumption 3, and after some manipulation, the bid-rent differential can be expressed as follows:

$$\bar{\rho}_t^{b,1} - \bar{\rho}_t^{a,1} = -(q_t^1 - q_t^2)(b + a) - (d^{1,a}(1 - q_t^1)\Delta V_a^1 - \Theta(d^{1,a})) + d^{2,a}(1 - q_t^2)\Delta V_a^2 - \Theta(d^{2,a}). \quad (13)$$

The social composition of the neighbourhood reflected in  $q_t^j$  affects the bid-rent differential. Location choice turns out to be one way of socializing children, as parents' choice of place of residence determines the strength of oblique transmission. Both segregation and integration forces are at play.

A first effect is captured by the term  $-(q_t^1 - q_t^2)(b + a)$ . Due to cultural intolerance, parents with trait  $a$  (resp.  $b$ ) prefer to have type- $a$  (resp.  $b$ ) children. They hence find it profitable to live in area 1 (resp. 2) since this provides a greater probability of transmitting their trait by oblique transmission. Cultural intolerance is a force toward segregation.

Second, the location decision interacts with socialization effort. Depending on whether these two instruments are complements or substitutes, a segregation or integration force emerges. This is captured by the second term,  $-(d^{1,a}(1 - q_t^1)\Delta V_a^1 - \Theta(d^{1,a})) + d^{2,a}(1 - q_t^2)\Delta V_a^2 - \Theta(d^{2,a})$ .

Location decision and direct socialization effort are complements if the gain of the socialization effort is higher in area 1, where agents of type  $a$  are more present, i.e. where oblique transmission is higher. This case holds when peer effects prevail, so as to generate social complementarities (peer effects raise the benefit of the socialization effort where the fraction of agents  $a$  is higher). Looking at (13), if the benefit from socialization effort is high in area 1 and low in area 2, then the second term in the expression turns out to be negative and equal to  $-(\tau(1 - q_t^1)\Delta V_a^1 - \theta)$ . There hence

exists an additional incentive to segregate: type- $a$  parents find it profitable to live in area 1 since the relative cost of the socialization effort is low while oblique transmission is high.

The two instruments are substitutes if the benefit from socialization effort is higher in area 2 where the fraction of type- $a$  agents is low, i.e. where there is little oblique transmission. This is true when the cultural substitution effect is high and generates social substitutability (meaning that the benefit from socialization effort falls with the fraction of agents with similar preferences). If the cost of socialization effort is low (resp. high) in area 2 (resp. 1), the second term in (13) turns out to be positive and equal to  $(\tau(1 - q_t^2)\Delta V_a^2 - \theta)$ . There is thus a new force encouraging agents to live in mixed urban areas: parents make a trade off between living in area 1, where they enjoy high oblique transmission and save the cost of socialization, and living in area 2, where they compensate for low oblique transmission by exerting high socialization effort.<sup>12</sup>

In what follows we assume that both segregation forces prevail.<sup>13</sup> This formally requires that

**Assumption 4** *The parameters  $a, b, c, \theta, \tau$  and  $\Delta w$  are such that*

$$4(a + b)^2 > \tau (\tau(a + \Delta w)^2 - 4c\theta).$$

This amounts to saying that cultural intolerance is high compared to the gains from socialization effort, which notably increase with both  $\tau$ , the efficiency of socialization effort, and  $\Delta w$ , the returns to education, and fall with both the cost of education  $c$ , and the cost of effort  $\theta$ .

**Proposition 1** *At any date  $t$ , there is a unique stable equilibrium. This is characterized as follows*

- (i) For any  $Q_t \in [0, \tilde{q}_1]$ ,  $[\rho_t^* = \bar{\rho}_t^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = 0, d^{*2,a} = 0]$ .
- (ii) For any  $Q_t \in [\tilde{q}_1, \tilde{q}_2]$ ,  $[\rho_t^* = \bar{\rho}_t^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = \tau, d^{*2,a} = 0]$ .
- (iii) For any  $Q_t \in [\tilde{q}_2, 1]$ ,  $[\rho_t^* = \bar{\rho}_t^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = 0, d^{*2,a} = 0]$ .
- (iv) For any  $Q_t \in [1, 1 + \tilde{q}_1]$ ,  $[\rho_t^* = \bar{\rho}_t^{a,1}, q_t^{1*} = 1, d^{*1,a} = 0, d^{*2,a} = 0]$ .
- (v) For any  $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$ ,  $[\rho_t^* = \bar{\rho}_t^{a,1}, q_t^{1*} = 1, d^{*1,a} = 0, d^{*2,a} = \tau]$ .
- (vi) For any  $Q_t \in [1 + \tilde{q}_2, 2]$ ,  $[\rho_t^* = \bar{\rho}_t^{a,1}, q_t^{1*} = 1, d^{*1,a} = 0, d^{*2,a} = 0]$ .

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<sup>12</sup>Note that segregation and integration forces would also occur in a model with continuous socialization effort, with the only difference that they would be continuous functions of  $q$ . For instance, for some values of  $q$ , type- $a$  parents would exert greater effort in area 1 due to peer effects, thus favouring segregation. If type- $b$  parents were to exert high socialization effort in area 1, a new integration force would be at play. Considering the case where segregation outweighs integration would require similar assumptions on the parameters.

<sup>13</sup>We later on consider the case where the integration force is large relative to segregation.

According to this Proposition, the segregated equilibrium, that is the emergence of a culturally homogenous urban area, is the unique stable equilibrium. However, the characteristics of the segregated equilibrium given by social composition  $q_t^1$  and the socialization choices in urban areas 1 and 2 vary, depending on the number of trait- $a$  individuals in the population measured by  $Q_t$ .<sup>14</sup>

When  $Q_t \in [0, 1]$ , in equilibrium, all trait- $a$  individuals locate in urban area 1. There is an educational divide among urban areas. Urban area 1 includes all trait- $a$  agents who all exert educational effort while urban area 2 is deprived of any education incentives. When  $Q_t \in [1, 2]$ , in equilibrium, trait- $a$  individuals locate in both urban areas: education spreads out in the city. Urban area 2 is now characterized by some positive education rate which is still lower than that in urban area 1, where all inhabitants are of type  $a$  and invest in education. Such cities are depicted in Figure 1. It is

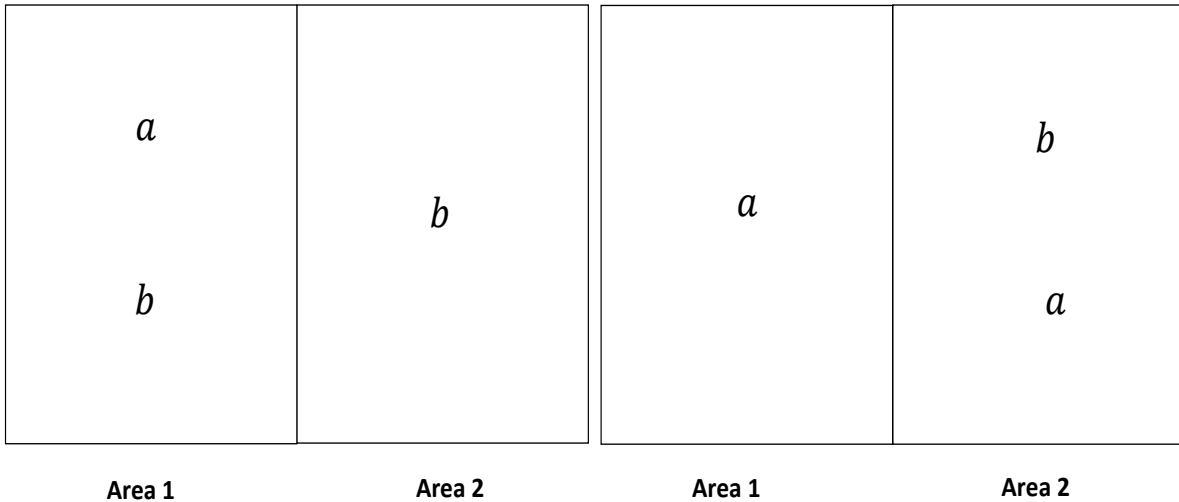


Figure 1: A segregated city when  $Q_t \in [0, 1]$  (at the left-hand-side) and when  $Q_t \in [1, 2]$  (at the right-hand-side).

also worth stressing that the urban equilibrium is characterized by a given degree of substitutability between the location decision and socialization effort for the transmission of trait  $a$ . Consider for example item (ii) when  $Q_t \in [\tilde{q}_1, \tilde{q}_2]$ . According to Assumption 2, peer effects outweigh the cultural substitution effect in area 1 while they are low in area 2 ( $q_t^2 = 0$ ). Hence, the relative cost of effort is low in area 1 so that trait- $a$  parents have an incentive to exert socialization effort in this area

<sup>14</sup>In order to obtain the urban equilibrium given a certain  $Q_t$ , we must characterize the bid-rent differential for any  $q_t^1$  and  $Q_t$ . In other words, the sorting condition that provides a ranking of bid-rent slopes is no longer sufficient to characterize the urban equilibrium. The reason is that the transition probabilities differ with respect to trait, regardless the socialization effort is discrete or continuous.

where oblique transmission is high. Both location and socialization choices are complements for the transmission of trait  $a$ . Now consider item (v) when  $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$ . In area 1, the cultural substitution effect is high compared to peer effects, while in area 2, peer effects are higher. The relative cost of socialization is thus high in area 1 and low in area 2. Type- $a$  agents have an incentive to exert socialization effort in area 2 where they are a minority (where oblique transmission is low). The location decision and socialization effort are thus substitutes. As a consequence, integration and segregation forces are at play in the location decision. However, given Assumption 4, segregation forces prevail.

This result can help understand why the empirical evidence is not conclusive about the degree of substitutability the two socialization instruments. For instance, Patacchini and Zenou (2004) find that parents exert greater socialization effort when they live in high-quality neighborhoods, consistent with complementarity. Work on identity and segregation finds that the choice of oppositional identities may be more prevalent in racially-mixed areas (see Fryer and Torelli, 2005, Bisin, Patacchini, Verdier et Zenou, 2011a). Our model rather suggests that the degree of substitutability of the two socialization instruments is endogenously determined by the equilibrium social composition of urban areas.

Whatever the characteristics of the segregated equilibrium, the social-mobility prospects for children living in urban area 1 are better than for those living in urban area 2. It is easy to show that the probability that a child of trait- $a$  parents be socialized into this trait is always higher in urban area 1, i.e.  $P_{aat}^1 = d^{*1,a} + (1 - d^{*1,a})q_t^{*1} > P_{aat}^2 = d^{*2,a} + (1 - d^{*2,a})q_t^{*2}$ . Furthermore, the prospect of upward social mobility for children of trait- $b$  parents is better in urban area 1, i.e.  $P_{bat}^1 = q_t^{*1} > P_{bat}^2 = q_t^{*2}$ . This result is consistent with the empirical findings of Chetty *et al.* (2014) regarding the spatial variation of intergenerational mobility.

Finally, there is some persistence of socio-economic status attainment within dynasties. The probability that a child in a trait- $a$  family acquire trait  $a$  is higher than that of a child born in a trait- $b$  family, whatever the segregated equilibrium.<sup>15</sup> The odds of investing in education and becoming rich are thus higher for children from trait- $a$  families. The intuition is that, due to segregation, children from trait- $a$  families always have a greater opportunity to meet neighbourhood role models with trait  $a$  and adopt this trait due to oblique transmission. The symmetric equilibrium with  $q_t^{*1} = Q_t/2$

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<sup>15</sup>Formally, for any segregated equilibrium, the probability of a child in a trait- $a$  family to acquire trait  $a$ ,  $(q_t^{*1}/Q_t)P_{aat}^1 + ((Q_t - q_t^{*1})/Q_t)P_{aat}^2$ , is higher than the probability of a child in a trait- $b$  family to acquire trait  $a$ ,  $((1 - q_t^{*1})/(2 - Q_t))P_{bat}^1 + ((1 - Q_t + q_t^{*1})/(2 - Q_t))P_{bat}^2$ . When  $Q_t \leq 1$  and  $q_t^{*1} = Q_t$ , we have  $d^{*1,a}((1 - Q_t)/Q_t) + 1 > (1 - Q_t)/(2 - Q_t)$ , for  $d^{*1,a} = 0, \tau$ . When  $2 \geq Q_t > 1$  and  $q_t^{*1} = 1$ , we have  $1 > (Q_t - 1)[1 + d^{*2,a}(Q_t - 2)]$  for  $d^{*2,a} = 0, \tau$ .

prevents trait- $a$  families from choosing the social arenas where their children interact, providing, in some cases, a probability of acquiring trait  $a$  that is independent of family background.<sup>16</sup> This result is consistent with the negative correlation between segregation and social mobility found in Chetty *et al.* (2014).

## 5 ‘Socially-Immobile’ versus ‘Socially-Mobile’ Cities

Each urban equilibrium characterized by Proposition 1 produces particular cultural dynamics. Given (12), we then have

**Proposition 2** *At any date  $t$ , population dynamics are characterized as follows:*

- (i) For any  $Q_t \in [0, \tilde{q}_1[$ ,  $Q_{t+1} = Q_t$ .
- (ii) For any  $Q_t \in [\tilde{q}_1, \tilde{q}_2]$ ,  $Q_{t+1} = Q_t + Q_t(1 - Q_t)\tau$ .
- (iii) For any  $Q_t \in ]\tilde{q}_2, 1 + \tilde{q}_1[$ ,  $Q_{t+1} = Q_t$ .
- (iv) For any  $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$ ,  $Q_{t+1} = Q_t + (Q_t - 1)(2 - Q_t)\tau$ .
- (v) For any  $Q_t \in ]1 + \tilde{q}_2, 2]$ ,  $Q_{t+1} = Q_t$ .

This proposition highlights the interplay between urban segregation and cultural dynamics. These dynamics straightforwardly result from (12) and Proposition 1. Over the interval  $[0, 2]$ , the fraction of the cultural trait  $a$  in the whole population either increases over time or is stationary. There are multiple history-dependent steady states. This multiplicity relies on the interaction between peer effects and cultural substitution, which generates particular socialization patterns depending on the segregated equilibrium. These dynamics are depicted in Figure 2.

When the fraction of trait- $a$  agents is low, i.e.  $Q_t \in [0, \tilde{q}_1[$ , the population dynamics is stationary as, due to low peer effects, no socialization choice is exerted in the city. The number of children of type- $a$  parents who experience downward social mobility, i.e. who acquire trait  $b$  and end up poor, exactly equals the number of children of type- $b$  parents who experience upward social mobility, i.e. who acquire trait  $a$  and become rich. The city is then called ‘stationary mobile’. Further, the stationary fraction of cultural trait  $a$  is low. The city is thus trapped in a state of low education. When the fraction of trait- $a$  individuals is higher, i.e.  $Q_t \in [\tilde{q}_1, \tilde{q}_2]$ , the urban equilibrium is such that urban area 1 provides incentives to exert socialization effort (due to strong peer effects in this area). The fraction of cultural trait  $a$  in the whole population increases over time. This type of city

<sup>16</sup>In a symmetric equilibrium with  $q_t^{*1} = Q_t/2$ , the probability differential of acquiring trait  $a$  between both types of children is  $(1 - (Q_t/2))(d^{*1,a} + d^{*2,a})$ , which is zero when  $d^{*1,a} = d^{*2,a} = 0$ .

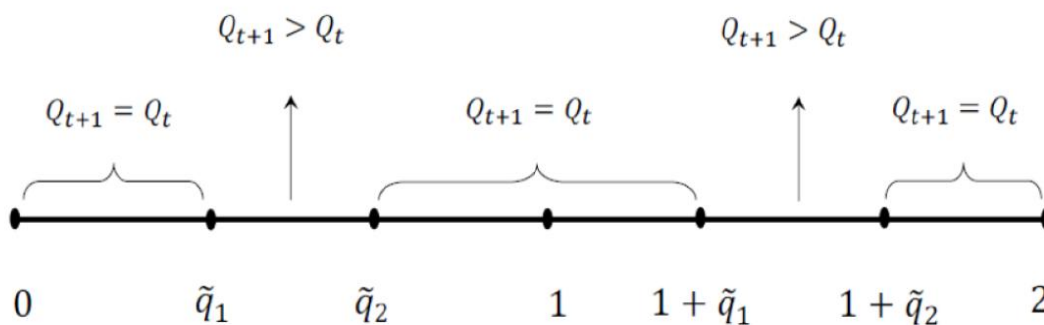


Figure 2: Dynamics of cultural traits in the segregated city.

is called ‘socially mobile’ as the number of children of type- $b$  parents who experience upward social mobility thanks to oblique transmission, i.e. who acquire trait  $a$ , exceeds the number of children of type- $a$  parents who experience downward social mobility. Education thus spreads out in the city.

Nonetheless, this expansion of the fraction of trait- $a$  agents comes to an end once it reaches the threshold  $\tilde{q}_2$ . When  $Q_t \in [\tilde{q}_2, 1]$ , urban segregation is such that all parents with trait  $a$  live in urban area 1 and no longer find it profitable to exert socialization effort due to the strong cultural substitution effect. The city is ‘socially immobile’. Overall, any population starting with  $Q_0 \in [\tilde{q}_1, \tilde{q}_2]$  ends up with intermediate education  $Q_\infty = \tilde{q}_2$ .

When  $Q_t \in [1, 1 + \tilde{q}_1[$ , the city is ‘stationary mobile’. Whatever the urban area they inhabit, trait- $a$  individuals have no incentive to exert socialization effort. For larger fractions of the type- $a$  population, i.e.  $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$ , the city is ‘dynamically mobile’ as  $a$  parents living in urban area 2 socialize their offspring. Again, cultural trait  $a$  spreads out in the population and education increases. However, this expansion stops as soon as the number of trait- $a$  agents reaches the threshold  $1 + \tilde{q}_2$ . When  $Q_t \in ]1 + \tilde{q}_2, 2]$ , the city is ‘stationary mobile’ as the cultural substitution effect is strong and there is no incentive to exert any socialization effort in either urban area.

Multiple history-dependent steady states are consistent with the divergence of human-capital levels among cities being explained by initial education (see, among others, Glaeser and Saiz, 2005, Berry and Glaeser, 2005, Glaeser, Resseger and Tobio, 2008). For instance, in US metropolitan areas, Glaeser, Resseger and Tobio (2008) emphasize a:

“[...] 73 percent correlation between the share of adults with college degrees in the year 2000 and the share of adults with college degrees in the year 1940. The college share of the population in 1940 is able to explain more than 50 percent of the variation in the college share today, which suggests

the enormous power of historical forces in shaping the skill composition of cities today.” (p. 18)<sup>17</sup>

We can also carry out the following comparative-static analysis

**Proposition 3** (i) *A rise in  $\Delta w$ ,  $a$ ,  $\tau$  reduces the number of ‘socially-immobile’ cities:  $\tilde{q}_1$  falls and  $\tilde{q}_2$  rises, so that the set of values  $Q_t$  for which the dynamics are stationary falls.*

(ii) *A rise in  $c$ ,  $\theta$  increases the number of ‘socially-immobile’ cities:  $\tilde{q}_1$  rises and  $\tilde{q}_2$  falls, so that the set of values  $Q_t$  for which the dynamics are stationary rises.*

Proposition 3 underlines that the characteristics of the population dynamics depend on the incentives to socialize.<sup>18</sup> When either the returns to education,  $\Delta w$ , the extra gain from education  $a$  or the efficiency of socialization effort,  $\tau$ , rise then there are greater incentives to exert socialization effort. As a result, the set of values of  $Q_t$  for which the population dynamics are stationary shrinks. Any population starting in  $[\tilde{q}_1, \tilde{q}_2]$  or  $[1 + \tilde{q}_1, 1 + \tilde{q}_2]$  will attain higher steady-state education. On the contrary, a rise in  $c$  or  $\theta$  reduces the incentives for socialization effort: cities are more likely to attain a lower level of steady-state education.

## 6 Is Social Segregation Optimal?

We now turn to the issue whether it is efficient to let people sort themselves into urban areas, or, in other words, should we implement particular urban policies? Our efficiency criterion is the long-run rate of education.

**Proposition 4** *For  $Q_t \in [\tilde{q}_1, \min\{2\tilde{q}_1, \tilde{q}_2\}] \cup [\max\{2\tilde{q}_2, 1 + \tilde{q}_1\}, 1 + \tilde{q}_2]$  segregation is efficient; for  $Q_t \in ]\tilde{q}_2, 1 + \tilde{q}_1[$  segregation is inefficient.*

The above proposition stresses that segregation has ambiguous implications for city education performance. The intuition is that segregation does not always provide incentives favoring the transmission of education preferences. When the fraction of cultural trait  $a$  is relatively low, i.e.  $Q_t \in [\tilde{q}_1, \min\{2\tilde{q}_1, \tilde{q}_2\}]$ , and the urban equilibrium is segregated, the concentration of the trait- $a$  population in urban area 1 is sufficiently high that peer effects create an incentive to transmit education preferences in urban area 1. Here the city is ‘socially mobile’ and the long-run rate of education rises. By way of contrast, were the city to be mixed, it would be ‘socially immobile’. Spreading the trait- $a$  population out over both urban areas yields peer effects that are too low

<sup>17</sup>Although the research cited here invokes other mechanisms than ours to explain this correlation.

<sup>18</sup>These results rely on expressions of  $\bar{q}_1$  and  $\bar{q}_2$  given in Appendix 8.2.



everywhere to create incentives to transmit cultural trait  $a$ . As such, education remains constant over time.

In the case where the fraction of the trait- $a$  population is larger, such that  $Q_t \in ]\max\{2\tilde{q}_2, 1 + \tilde{q}_1\}, 1 + \tilde{q}_2]$ , segregation leads to a fraction of  $a$  agents in area 2 that dilutes the cultural substitution effect. There are hence incentives to transmit trait  $a$  in urban area 2. The segregated city experiences rising education. By contrast, in a mixed city, cultural substitution turns out to be large in both urban areas, so that type- $a$  parents transmit their trait in neither urban area and the education rate remains constant.

A policy promoting integration is optimal for intermediate values of  $Q_t$ , i.e.  $Q_t \in ]\tilde{q}_2, 1 + \tilde{q}_1[$ .<sup>19</sup> When the city is segregated, the fraction of type- $a$  individuals is high in area 1 and low in area 2. In area 2, the peer effects are then too low to create incentives to transmit education preferences. In addition, in area 1, as the fraction of  $a$  agents is high, cultural substitution is too strong, so that these agents have no incentive to exert socialization effort. Since there is no transmission of education preferences in either area, the rate of education remains constant. On the one hand, integration reduces the fraction of type- $a$  agents in area 1, so that peer effects may overcome the cultural substitution effect and type- $a$  agents have an incentive to transmit their trait in this area. On the other hand, integration increases the fraction of type- $a$  agents in area 2, so that peer effects become sufficiently large to encourage these agents to exert socialization effort. As such, the fraction of cultural trait  $a$  increases and education spreads out in the city.<sup>20</sup>

Whereas Bénabou (1996a) stresses that the degree of complementarity between individuals' levels of human capital at the community and the society levels is key to assess the efficiency of a segregated equilibrium, we emphasize that segregation may be costly when it generates a strong cultural substitution effect and impedes the cultural transmission of preferences for education.

These findings echo the empirical evidence that ethnic groups are not affected similarly by ethnic concentration (see Borjas, 1995, Edin *et al.*, 2003, Cutler *et al.* 2005, 2008). Our result that the efficiency of segregation depends non-monotonically on the distribution of traits in the whole population has important implications. It suggests that poverty deconcentration and integration policies must circumvent the difficulty to identify the degree of neighborhood social mix most favorable to education. According to Galster (2002)'s meta-analysis of the empirical evidence on the impact of

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<sup>19</sup>We call integration a urban configuration where both types of agents inhabit both urban areas.

<sup>20</sup>We should mention that there also exist intervals of  $Q_t$  such that we cannot rank the mixed and segregated equilibria with respect to efficiency. This is the case when, whether the city be integrated or segregated, type- $a$  parents have no incentives to exert socialization effort.

poverty concentration on socio-economic success, if behavioral problems are related to neighborhood poverty rates within a range of approximately 15-40% of poverty rate,

“This implies that net social benefits will be larger if neighborhoods with greater than roughly 15% poverty rates are replaced with (an appropriately larger number) of neighborhoods having less than 15% poverty rates. However, net social benefits will be smaller if neighborhoods with greater than about 40% poverty rates are replaced with (an appropriately larger number) of neighborhoods having between about 15-40% poverty rates. Put more bluntly in policy terms, unless very low-poverty neighborhoods can be opened up for occupation by the poor, deconcentration efforts should halt, because merely transferring the poor from high- to moderate-poverty neighborhoods is likely to be socially inefficient.” (p. 322, Galster, 2002)

Our framework highlights two types of policy that may help restore efficiency. First, urban policies could affect the socio-economic composition of urban areas. Enforcing quotas of inhabitants from a given social category is one way of promoting social mixing in a given urban area.<sup>21</sup> Housing-subsidy programs, by affecting the rent differential, could make the integrated city stable. Housing subsidies that are targeted at the poor (type-*b* families in our model), such as the Moving to Opportunity Program or Low-Income Tax Credits in the US, could make type-*b* parents more willing than their type-*a* counterparts to live in urban area 1.<sup>22</sup> Second, our framework allows us to highlight other kinds of policies that would impact both integration and segregation forces.

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<sup>21</sup>One example of a quota policy is the SRU law (loi relative à la Solidarité et au Renouvellement Urbains) in force in France since 2000. In French municipalities with at least 3,500 inhabitants (1,500 inhabitants in the Paris administrative region, Ile-de-France), 20% of the available housing stock must be public housing. Municipalities with figures below this ratio have to pay fines (see Gobillon and Vignolles, 2014, for an evaluation of this policy).

<sup>22</sup>Let us define by  $\beta$  the fraction of trait-*b* parents' rent expenditures paid by the government (see Brueckner, 2011, for a presentation and an analysis of various housing policies). Assume that  $Q_t/2 \in [\tilde{q}^1, \tilde{q}^2]$  so that trait-*a* families exert socialization effort at the symmetric equilibrium. The difference in the bid-rent slopes can be expressed as follows:

$$\left. \frac{\partial \bar{p}^{b,1}}{\partial q_t^1} \right|_{q_t^1=Q_t/2} - \left. \frac{\partial \bar{p}^{a,1}}{\partial q_t^1} \right|_{q_t^1=Q_t/2} = 2 \left[ \frac{\Delta w - b - c(1 - Q_t)}{1 - \beta} - c\left(\tau + \frac{Q_t}{2}(1 - \tau)\right) \right].$$

When  $\beta = 0$ , we have  $\left. \frac{\partial \bar{p}^{b,1}}{\partial q_t^1} \right|_{q_t^1=Q_t/2} < \left. \frac{\partial \bar{p}^{a,1}}{\partial q_t^1} \right|_{q_t^1=Q_t/2}$  for any  $Q_t$ . As the bid-rent slope differential is continuously increasing with respect to  $\beta$ , we can find values of  $Q_t$ ,  $\beta$  and  $\Delta w$  such that there exists a unique  $\beta^*$  above which  $\left. \frac{\partial \bar{p}^{b,1}}{\partial q_t^1} \right|_{q_t^1=Q_t/2} > \left. \frac{\partial \bar{p}^{a,1}}{\partial q_t^1} \right|_{q_t^1=Q_t/2}$ .

**Proposition 5** *When Assumption 4 is no more satisfied, that is parameters are such that*

$$4(a + b)^2 < \tau (\tau(a + \Delta w)^2 - 4c\theta),$$

*if  $Q_t \in ]2\tilde{q}_2, Q^*[$ ,  $Q^* \leq 1 + \tilde{q}_1$ , integration and segregation are stable equilibria and integration is efficient.*

This Proposition stresses that the ability of individuals to substitute between location decision and socialization effort may allow integrated equilibria to emerge and to restore efficiency. When cultural intolerance is low and/or the gain from the socialization effort is high, i.e.  $4(a + b)^2 < \tau(a + \Delta w)^2 - 4c\theta$ , instruments become sufficiently substitutable so that at equilibrium some type- $a$  parents have an incentive to live in area 2 where they save the rent but exert a high socialization effort which offsets low oblique transmission while, some others prefer to pay the rent for living in area 1 to enjoy high oblique transmission. An equilibrium configuration where both traits  $a$  and  $b$  individuals have the same willingness to pay to live in urban area 1, i.e. the bid-rent differential (13) is equal to 0, can arise. Compared to the segregated equilibrium where oblique transmission is the only way to transmit preferences, the integrated equilibrium is efficient. In this latter case, the socialization effort exerted in area 2 positively affects the dynamics of education which increases the long run level of education.

A number of policies could reverse the inequality in Assumption 4 by increasing the substitutability between instruments and give rise to an integrated and efficient equilibrium. One would be to increase the returns to education, that is increase  $\Delta w$  or reduce  $c$ . Another would be to affect the preference parameters  $a$  and  $b$ . Considering that  $a$  and  $b$  may capture some identity benefits, we can emphasize the role of a different kind of policy which aims to change social norms, that is group-based policies (Liu *et al.*, 2014). An example is the charter school policy which aims to change behaviours that are detrimental to education by, for instance, implementing strict discipline or introducing long school days (see Angrist *et. al.*, 2013, Curto and Fryer, 2014).

## 7 Conclusion

How segregation impacts the transmission of traits which are critical for economic success? Does the existence of opposite cultures leads to residential segregation? This paper provides some answer by developing a model which interacts neighbourhood formation and cultural transmission.

Our framework involves a trade-off for parents who decide where to live and how much to spend in

the socialization of their children to their cultural trait. We study urban equilibria and highlight new forces driving location decisions. All else equal, cultural intolerance makes parents more willing to live in areas which favours the transmission of their own trait (i.e where oblique transmission is high) and so pushes to segregation. However, the degree of substitutability between the two instruments of socialization, i.e, the socialization effort and the residential choice, which is endogenous also matters. The two instruments are complements if peer effects are high since it create incentives for parents to live and exert a high socialization effort in the same neighbourhood. In this case there is further incentives to segregate. They are substitute if the cultural substitution effect is high as parents are then encouraged to make a higher socialization effort in the area where they are in a minority. In this case, there exists an incentive to live in mixed urban area.

Studying the joint dynamics of segregation and the distribution of cultural traits favouring economic success we emphasize the existence of multiple history-dependent steady states. We deduce that segregation has non linear effects on the transmission of traits which favour socio-economic success. We study efficiency and show that segregation may or may not be optimal depending on the composition of the population which creates different combination of peer effects/cultural substitution effect. Interestingly, we show that when socialization instruments are sufficiently substitutable an integrated equilibrium does emerge and increases the incentive to actively transmit the more successful trait which promotes education in the long run.

The model could be extended along several lines. First, there is a consensus that housing market dynamics impact segregation (see the review of Rosenthal and Ross, 2014). The model is flexible enough to introduce some housing market features such as tenure choice, housing depreciation and maintenance, development and redevelopment of housing stock. These features would influence the ability of individuals to substitute between socialization instruments and allow us to explore their implications on the pattern of segregation and cultural dynamics. Second, the model assumes that the process of oblique transmission is unbiased as children conform role models they encounter. Another interesting extension would be to examine the consequences of different biases on the incentives to segregate as it would affect the efficiency of oblique transmission (see Saez-Marti and Sjögren, 2008, for a non-urban model of cultural transmission with positive bias, negative bias and conformism). Third, an interesting extension would be to consider that parents would decide collectively a socialization effort provided by institutions such as schools. This collective socialization mechanism could affect incentives to segregate and have interesting implications for the dynamics of cultural traits. Finally, relaxing the assumption that children attend the school of their urban area would allow

to differentiate the social arenas where peer effects and oblique transmission are determined. For instance, considering that peer effects are circumscribed within schools whereas oblique transmission is produced in the urban area would affect the trade-off faced by parents when deciding the place of residence. This extension could shed new light on the consequences of school choice systems on segregation and inequality dynamics.

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## 8 Appendix

### 8.1 A General Version of the Model

We construct a general framework with exogenous spatial socio-economic segregation to show that our main results regarding the dynamics of cultural traits in segregated versus integrated cities continue to hold.

#### 8.1.1 Children's Educational Choice

**Preferences.** We keep the same notation as before and assume that preferences satisfy the following properties.

**Assumption 5** For all  $i \in \{a, b\}$ ,

$$(i) U^i(w_r, e, \lambda_t^j) > U^i(w_p, e, \lambda_t^j),$$

$$(ii) U^i(w, \bar{e}, \lambda_t^j) < U^i(w, \underline{e}, \lambda_t^j),$$

$$(iii) \partial U^i(w_r, \bar{e}, \lambda_t^j) / \partial \lambda_t^j = Cst. > 0, \partial U^i(w_r, \underline{e}, \lambda_t^j) / \partial \lambda_t^j = 0.$$

For all  $\lambda_t^j$ ,

$$(iv) U^a(w_r, \bar{e}, \lambda_t^j) > U^b(w_r, \bar{e}, \lambda_t^j),$$

$$(v) U^a(w_p, \underline{e}, \lambda_t^j) < U^b(w_p, \underline{e}, \lambda_t^j).$$

We assume that utility is increasing in income (item i), education is costly (item ii), there exist local spillovers which take the form of positive peer effects in education (item iii), and type- $a$  agents have higher returns to education than do type- $b$  agents (items iv and v). We retain the assumption that exerting effort of  $\bar{e}$  ( $\underline{e}$ ) guarantees turning out rich (poor).

A type- $a$  child chooses education if and only if

$$U^a(w_r, \bar{e}, \lambda_t^j) > U^a(w_p, \underline{e}, \lambda_t^j).$$

A type- $b$  child chooses education if and only if

$$U^b(w_r, \bar{e}, \lambda_t^j) > U^b(w_p, \underline{e}, \lambda_t^j).$$

We also assume the following.

**Assumption 6** (i)  $U^a(w_r, \bar{e}, 0) > U^a(w_p, \underline{e}, 0)$ , (ii)  $U^b(w_r, \bar{e}, 0) < U^b(w_p, \underline{e}, 0)$ , (iii)  $U^b(w_r, \bar{e}, 1) > U^b(w_p, \underline{e}, 1)$ .

According to item (iii), we consider a case where education is profitable enough so that when peer effects are high, type- $b$  agents have an incentive to educate. We can then deduce the fraction of educated agents in neighbourhood  $j$  at time  $t$ . In particular, it is straightforward to show that there exists  $\lambda^*$  such that<sup>23</sup>

$$\begin{aligned}\lambda_t^j \leq \lambda^* &\Leftrightarrow \lambda_t^j = q_t^j \\ \lambda_t^j > \lambda^* &\Leftrightarrow \lambda_t^j = 1.\end{aligned}$$

### 8.1.2 Parents' socialization decisions

The dynamics of preferences follows the same lines as before. However, we assume that type- $i$  parents may exert a continuous effort  $d^i$  to transmit their preferences. We consider parents' optimal socialization choice. In what follows, we omit the time index. We denote by  $V_{ii'}^j$  the gain to a type- $i$  parent of having a type- $i'$  child in neighbourhood  $j$ . Given children's education choices, for type- $a$  parents we have

$$\begin{aligned}V_{aa}^j &= U^a(w_r, \bar{e}, \lambda^j) \\ V_{ab}^j &= U^a(w_p, \underline{e}, \lambda^j) \quad \text{if } \lambda^j \leq \lambda^*, \\ V_{ab}^j &= U^a(w_r, \bar{e}, \lambda^j) \quad \text{if } \lambda^j > \lambda^*,\end{aligned}$$

which implies that

$$\begin{aligned}\Delta V_a^j(\lambda^j) &= U^a(w_r, \bar{e}, \lambda^j) - U^a(w_p, \underline{e}, \lambda^j) > 0 \quad \text{if } \lambda^j \leq \lambda^*, \\ \Delta V_a^j(\lambda^j) &= 0 \quad \text{if } \lambda^j > \lambda^*.\end{aligned}$$

For type- $b$  parents,

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<sup>23</sup>This non-strict inequality is adopted by convention. It does not affect the result.

$$V_{bb}^j = U^b(w_p, \underline{e}, \lambda^j) \quad \text{if } \lambda^j \leq \lambda^*,$$

$$V_{bb}^j = U^b(w_r, \bar{e}, \lambda^j) \quad \text{if } \lambda^j > \lambda^*,$$

$$V_{ba}^j = U^b(w_r, \bar{e}, \lambda^j),$$

which implies

$$\Delta V_b^j(\lambda^j) = U^b(w_p, \underline{e}, \lambda^j) - U^b(w_r, \bar{e}, \lambda^j) \geq 0 \quad \text{if } \lambda^j \leq \lambda^*,$$

$$\Delta V_b^j(\lambda^j) = 0 \quad \text{if } \lambda^j > \lambda^*.$$

Exerting a socialisation effort implies a cost of  $C(d) = d^2/2$ . Parents then maximise the expected gain associated with the type of the child given the cost of effort. The optimal effort of type- $a$  parents, in neighbourhood  $j$  is given by

$$d^{j,a} = (1 - q^j) \Delta V_a^j(\lambda^j) = (1 - \lambda^j) \Delta V_a^j(\lambda^j) \equiv d^a(\lambda^j), \quad \text{if } q^j \leq \lambda^*,$$

$$d^{j,a} = 0 \quad \text{otherwise.}$$

The optimal effort of type- $b$  parents in neighbourhood  $j$  is given by

$$d^{j,b} = q^j \Delta V_b^j(\lambda^j) = \lambda^j \Delta V_b^j(\lambda^j) \equiv d^b(\lambda^j), \quad \text{if } q^j \leq \lambda^*,$$

$$d^{j,b} = 0 \quad \text{otherwise.}$$

The dynamics of the fraction of type- $a$  agents in neighbourhood  $j$  is then

$$q_{t+1}^j = q_t^j + q_t^j(1 - q_t^j)(d^a(q_t^j) - d^b(q_t^j)), \quad \text{if } q^j \leq \lambda^*,$$

$$q_{t+1}^j = q_t^j \quad \text{otherwise.}$$

The dynamics of the fraction of educated agents is given by the dynamics of  $q^j$  if  $q^j \leq \lambda^*$  and by  $\lambda_t^j = 1, \forall t$  otherwise. We call  $\Lambda_t = \lambda_t^1 + \lambda_t^2$  the fraction of educated agents in the whole population at time  $t$ .

### 8.1.3 Dynamics of cultural traits

**Lemma 1** *Suppose that  $\Delta V^b / \Delta V^a > \frac{d\Delta V^b}{d\lambda} / \Delta V^b - 1$ , then the equation  $\lambda_{t+1} - \lambda_t = 0$  has four fixed points  $\lambda = 0$ ,  $\lambda = \tilde{\lambda}_1 \in ]0, 1[$ ,  $\lambda = \tilde{\lambda}_2 \in ]0, 1[$ , and  $\lambda = 1$ , with  $\tilde{\lambda}_1 < \tilde{\lambda}_2$ . For all  $\lambda_0 \in [0, \tilde{\lambda}_2[$ ,  $\lambda_t \rightarrow \tilde{\lambda}_1$  and for all  $\lambda_0 > \tilde{\lambda}_2$ ,  $\lambda_t \rightarrow 1$ .*

**Comment.** This Lemma proposes an important result, that the fraction of educated agents may converge to an intermediate steady state  $\tilde{\lambda}_1$ . The intuition is the following. When  $\lambda_0$  is low  $\lambda_0 < \tilde{\lambda}_1$ , oblique transmission is low (resp. high) for parents of type  $a$  (resp.  $b$ ). Due to cultural substitutability, type- $a$  parents have a greater incentive to transmit their trait than do type- $b$  parents, so that the fraction of type- $a$  agents, which is also the fraction of educated agents, increases. For intermediate values of  $\lambda_0$ ,  $\tilde{\lambda}_1 < \lambda_0 < \tilde{\lambda}_2$ , oblique transmission falls (increases) for parents of type  $b$  ( $a$ ) which increases (reduces) the socialization effort of  $b$  ( $a$ ) parents. If, furthermore, the peer effects in education are sufficiently low, for such intermediate values of the fraction of educated agents, type- $b$  parents find it more profitable to transmit their trait than do type- $a$  parents, so that the fraction of type- $a$  agents falls to reach an intermediate steady state level ( $\lambda_t$  converges to  $\tilde{\lambda}_1$ ). For  $\lambda_0 > \tilde{\lambda}_2$ , the peer effects in education become sufficiently high that, in spite of high oblique transmission, type- $a$  agents exert high socialization effort which overcomes the effort of type- $b$  agents. The fraction of agents  $a$  reaches the threshold  $\lambda^*$  and  $\lambda_t = 1 \forall t$ .

#### Proof. (1) Existence

We omit the indexation by  $j$ . We denote by  $f_1$  the map  $[0, \lambda^*] \rightarrow [0, \lambda^*]$ , which is such that  $f_1(\lambda_t) = \lambda_t + \lambda_t(1 - \lambda_t)(d^a(\lambda_t) - d^b(\lambda_t))$ , and  $f_2$  the map  $[\lambda^*, 1]$  such that  $f_2(\lambda_t) = 1$ . First,  $\lambda = 1$  is a steady state of the map  $f_2$ . Also, the equation  $f_1(\lambda_t) = \lambda_t$  has one obvious fixed point which is 0. The other fixed points must solve  $d^a(\lambda) - d^b(\lambda)$ . From item (iii) of Assumption 5, both functions  $d^a(\lambda)$  and  $d^b(\lambda)$  are concave. Also, given items (i), (ii), (iii) of Assumption 6,  $d^a(0) > 0$ ,  $d^a(\lambda^*) > 0$ ,  $d^b(0) = 0$ , and  $d^b(\lambda^*) = 0$ . The two functions intersect twice if and only if  $d^b(\lambda_{max}) > d^a(\lambda_{max})$ , where  $\lambda_{max} = \operatorname{argmax}_{\lambda} d^b(\lambda)$ . This is equivalent to the assumption made in Lemma 1. We deduce that the map  $\lambda$  has two other fixed points  $\tilde{\lambda}_1$  and  $\tilde{\lambda}_2$ , which both belong to the interval  $]0, 1[$ .

#### (2) Stability

We now calculate the derivative of the map  $f_1$ .

$$\frac{df_1}{d\lambda} = 1 + (1 - 2\lambda)(d^a(\lambda) - d^b(\lambda)) + \lambda(1 - \lambda)\left(\frac{\partial d^a(\lambda)}{\partial \lambda} - \frac{\partial d^b(\lambda)}{\partial \lambda}\right).$$

We have

- (i)  $\frac{df}{d\lambda}|_0 = 1 + e^a(0) > 1$
- (ii)  $\frac{df}{d\lambda}|\tilde{\lambda}_1 = 1 + \lambda(1 - \lambda)\left(\frac{\partial d^a}{\partial \lambda}|\tilde{\lambda}_1 - \frac{\partial d^b}{\partial \lambda}|\tilde{\lambda}_1\right) < 1$ , since  $\frac{\partial d^a}{\partial \lambda}|\tilde{\lambda}_1 - \frac{\partial d^b}{\partial \lambda}|\tilde{\lambda}_1 < 0$ .
- (iii)  $\frac{df}{d\lambda}|\tilde{\lambda}_2 = 1 + \lambda(1 - \lambda)\left(\frac{\partial d^a}{\partial \lambda}|\tilde{\lambda}_2 - \frac{\partial d^b}{\partial \lambda}|\tilde{\lambda}_2\right) > 1$ , since  $\frac{\partial d^a}{\partial \lambda}|\tilde{\lambda}_2 - \frac{\partial d^b}{\partial \lambda}|\tilde{\lambda}_2 > 0$ .

These conditions imply that 0 and  $\tilde{\lambda}_2$  are repelling fixed points and  $\tilde{\lambda}_1$  is attracting. Suppose that  $\lambda_0 \in [0, \tilde{\lambda}_1]$ . Then, for  $\lambda_0 \in [0, \tilde{\lambda}_1]$ , the sequence  $\lambda_t$  is monotonically increasing and bounded from above by  $\tilde{\lambda}_1$  (if  $\lambda_t = \tilde{\lambda}_1$ , then  $\lambda_{t+1} = f_1(\lambda_t) = \tilde{\lambda}_1$ ). This therefore has a limit  $L$ , so that

$$\begin{aligned} \lim_{t \rightarrow \infty} |\lambda_t - L| &= 0 \\ \Rightarrow \lim_{t \rightarrow \infty} |f_1(\lambda_t) - f(L)| &= 0 \quad \text{due to the continuity of } f_1 \\ \Rightarrow \lim_{t \rightarrow \infty} |\lambda_{t+1} - f_1(L)| &= 0. \end{aligned}$$

Therefore  $L$  is a fixed point of  $f_1$ . Since  $f_1$  is in  $[0, \tilde{\lambda}_1]$  it can be 0 or  $\tilde{\lambda}_1$ . Since 0 is repelling and  $\tilde{\lambda}_1$  attracting, we deduce that  $L = \tilde{\lambda}_1$ , or for any  $\lambda_0 \in [0, \tilde{\lambda}_1]$  the sequence  $\lambda_t$  converges to  $\tilde{\lambda}_1$ .

Similar reasoning allows us to show that the sequence  $\lambda_t$  converges to  $\tilde{\lambda}_1$ ,  $\forall \lambda_0 \in [\tilde{\lambda}_1, \tilde{\lambda}_2]$ .

For  $\lambda_0 \in ]\tilde{\lambda}_2, \lambda^*]$ , the sequence  $\lambda_t$  is monotonically increasing and  $f_1(\lambda^*) > \lambda^*$ , so that there exists some  $\tilde{t}$  such that  $\lambda_{\tilde{t}} > \lambda^*$ , which implies that  $\lambda_t = 1$  for all  $t > \tilde{t}$ , and the sequence  $\lambda_t$  converges to 1. ■

#### 8.1.4 Long-Run Equilibria under Segregation versus Integration

In what follows we assume that when peer effects are low enough, the gain from directly transmitting trait  $a$  is relatively low, i.e. the parameters are such that  $d^a(0)$  is close to zero:<sup>24</sup>

##### Assumption 7

$$d^a(0) < -\tilde{\lambda}_1(1 - \tilde{\lambda}_1)\left(\frac{\partial d^a}{\partial \lambda}|\tilde{\lambda}_1 - \frac{\partial d^b}{\partial \lambda}|\tilde{\lambda}_1\right)$$

<sup>24</sup>This assumption allows us to simplify the proofs and the presentation, but is not crucial for the results.



**Proposition 6** *Suppose that the city is segregated. For all  $\Lambda_0 \in [0, \tilde{\lambda}_2]$ , the fraction of type- $a$  agents  $\Lambda_t$  converges to  $\Lambda = \tilde{\lambda}_1$ . For all  $\Lambda_0 \in ]\tilde{\lambda}_2, 1 + \tilde{\lambda}_2]$ ,  $\Lambda_t$  converges to  $1 + \tilde{\lambda}_1$ . For all  $\Lambda_0 > 1 + \tilde{\lambda}_2$ ,  $\Lambda_t$  converges to 2.*

**Comment.** The dynamics of the fraction of trait- $a$  agents exhibits multiple steady states. The intuition is as follows. Segregation provides different incentives to transmit one's own cultural traits in each neighbourhood. When the total fraction of type- $a$  agents is low (i.e.  $\Lambda_0 < \tilde{\lambda}_1$ ), the fraction of type- $a$  agents in both neighbourhoods is small. Since oblique transmission is low (resp. high) for parents of type  $a$  (resp.  $b$ ), the latter have a greater incentive to transmit their trait than do type- $b$  parents. This is true in each urban area, so that the fraction of  $a$  agents increases. When  $\tilde{\lambda}_1 < \Lambda_0 < \tilde{\lambda}_2$ , there is a small fraction of  $a$  agents in neighbourhood 2 but an intermediate fraction of type- $a$  agents in neighbourhood 1. In this urban area, oblique transmission is higher (lower) for type- $a$  ( $b$ ) parents and the peer effects (which increase the incentives to transmit trait  $a$ ) are still low, so that type- $b$  parents exert a higher socialization effort than do type- $a$  parents. This negatively affects the dynamics of trait  $a$  in neighbourhood 1 and in the whole city (since in neighbourhood 2, the increase in the fraction of  $a$  agents is low). The fraction of type- $a$  agents thus reaches an intermediate steady state level  $\Lambda = \tilde{\lambda}_1$ .

For some higher values of the fraction of type- $a$  agents (i.e.  $\Lambda_0 \in ]\tilde{\lambda}_2, 1 + \tilde{\lambda}_2]$ ), due to segregation, peer effects increase in neighbourhood 1 so that in this neighbourhood the fraction of type- $a$  agents increases (until it reaches  $\lambda^*$  and becomes constant). If, furthermore, the fraction of type- $a$  agents in neighbourhood 2 is lower than  $\tilde{\lambda}_1$  then, in this area as well, the fraction of  $a$  agents increases. However, it cannot reach a high steady state. If  $\lambda_2 > \tilde{\lambda}_1$ , then the fraction of  $a$  agents in neighbourhood 2 is intermediate, in which case type- $b$  parents have a greater incentive to transmit their trait. Since the increase in the fraction of  $a$  agents in area 1 is close to or equal to zero, the fraction of  $a$  agents in the whole population falls to an intermediate steady state.

Finally, when the initial fraction of educated agents is sufficiently high (i.e.  $\Lambda_0 > 1 + \tilde{\lambda}_2$ ) in both neighbourhoods, the peer effects are high, so that the benefit of transmitting their trait is higher for type- $a$  than for type- $b$  parents. The fraction of type- $a$  agents increases up to its high steady-state level.

**Proof.**

Under segregation it is straightforward to show that the equation  $\Lambda_{t+1} = \Lambda_t$  has seven solutions  $\Lambda = 0$ ,  $\Lambda = \tilde{\lambda}_1$ ,  $\Lambda = \tilde{\lambda}_2$ ,  $\Lambda = 1$ ,  $\Lambda = 1 + \tilde{\lambda}_1$ ,  $\Lambda = 1 + \tilde{\lambda}_2$ ,  $\Lambda = 2$ .

**Stability**

A fixed point  $\Lambda$  is attracting if  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_{\Lambda} < 0$  (since  $\frac{d\Lambda_{t+1}}{d\Lambda_t} = \frac{d\lambda_{t+1}^1}{d\lambda_t^1} + \frac{d\lambda_{t+1}^2}{d\lambda_t^2} > 0$ ). We below calculate this derivative.

$$\begin{aligned} \frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t} &= \frac{d(\lambda_{t+1}^1 - \lambda_t^1) + (\lambda_{t+1}^2 - \lambda_t^2)}{d(\lambda_t^1 + \lambda_t^2)} \\ &= \frac{d(\lambda_{t+1}^1 - \lambda_t^1)}{d(\lambda_t^1 + \lambda_t^2)} + \frac{d(\lambda_{t+1}^2 - \lambda_t^2)}{d(\lambda_t^1 + \lambda_t^2)} \\ &= \frac{d(\lambda_{t+1}^1 - \lambda_t^1)}{d\lambda_t^1} + \frac{d(\lambda_{t+1}^2 - \lambda_t^2)}{d\lambda_t^2}. \end{aligned}$$

From the proof of Lemma 1, we can then deduce the values of this derivative at any fixed point as

- (i)  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_0 = d^a(0) + d^a(0) > 0$ .
- (ii)  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_{\tilde{\lambda}_1} = \tilde{\lambda}_1(1 - \tilde{\lambda}_1)(\frac{\partial d^a}{\partial \lambda}|_{\tilde{\lambda}_1} - \frac{\partial d^b}{\partial \lambda}|_{\tilde{\lambda}_1}) + d^a(0) < 0$  from Assumption 7.
- (iii)  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_{\tilde{\lambda}_2} = \tilde{\lambda}_2(1 - \tilde{\lambda}_2)(\frac{\partial d^a}{\partial \lambda}|_{\tilde{\lambda}_2} - \frac{\partial d^b}{\partial \lambda}|_{\tilde{\lambda}_2}) + d^a(0) > 0$ .
- (iv)  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_1 = d^a(0) > 0$ .
- (v)  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_{1+\tilde{\lambda}_1} = \tilde{\lambda}_1(1 - \tilde{\lambda}_1)(\frac{\partial d^a}{\partial \lambda}|_{\tilde{\lambda}_1} - \frac{\partial d^b}{\partial \lambda}|_{\tilde{\lambda}_1}) < 0$ .
- (vi)  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_{1+\tilde{\lambda}_2} = \tilde{\lambda}_2(1 - \tilde{\lambda}_2)(\frac{\partial d^a}{\partial \lambda}|_{\tilde{\lambda}_2} - \frac{\partial d^b}{\partial \lambda}|_{\tilde{\lambda}_2}) > 0$ .
- (v)  $\frac{d(\Lambda_{t+1} - \Lambda_t)}{d\Lambda_t}|_2 = 0$ .

The sequence  $\Lambda_t$  is monotonic (as  $\frac{d\Lambda_{t+1}}{d\Lambda_t} \geq 0$ ).

-Suppose that  $\Lambda_0 \in [0, \tilde{\lambda}_1]$ . The sequence  $\Lambda_t$  is monotonically increasing as both of the sequences  $\lambda_t^1$  and  $\lambda_t^2$  are monotonically increasing. Arguments analogous to those in the proof of Lemma 1 then allow us to conclude that the sequence  $\Lambda_t$  converges to  $\tilde{\lambda}_1$ .

-Suppose that  $\Lambda_0 \in [\tilde{\lambda}_1, \tilde{\lambda}_2]$ . The sequence  $\Lambda_t$  is monotonically decreasing ( $\Lambda_{t+1} - \Lambda_t = \lambda^1(1 - \lambda^1)(d^a(\lambda^1) - d^b(\lambda^1)) + \lambda^2(1 - \lambda^2)(d^a(\lambda^2) - d^b(\lambda^2)) = \lambda^1(1 - \lambda^1)(d^a(\lambda^1) - d^b(\lambda^1)) < 0$ ), which allows us to conclude that the sequence  $\Lambda_t$  converges to  $\tilde{\lambda}_1$ .

-Suppose that  $\Lambda_0 \in [\tilde{\lambda}_2, 1 + \tilde{\lambda}_1]$ . The sequence  $\Lambda_t$  is monotonically increasing since both sequences

$\lambda_t^1$  and  $\lambda_t^2$  are monotonically increasing.<sup>25</sup>  $\Lambda_t$  converges to  $1 + \tilde{\lambda}_1$  for any  $Q_0 \in [0, 1 + \tilde{\lambda}_1]$ .

-Suppose that  $\Lambda_0 \in [1 + \tilde{\lambda}_1, 1 + \tilde{\lambda}_2]$ . The sequence  $\Lambda_t$  is monotonically decreasing ( $\Lambda_{t+1} - \Lambda_t = \lambda^2(1 - \lambda^2)(d^a(\lambda^2) - d^b(\lambda^2)) < 0$ ).  $\Lambda_t$  converges to  $1 + \tilde{\lambda}_1$  for any  $\Lambda_0 \in [1 + \tilde{\lambda}_1, 1 + \tilde{\lambda}_2]$ .

-Suppose that  $\Lambda_0 \in [1 + \tilde{\lambda}_2, 2]$ . The sequence  $\Lambda_t$  is monotonically increasing. Hence  $\Lambda_t$  converges to 2 for any  $\Lambda_0 \in [1 + \tilde{\lambda}_2, 2]$ . ■

**Proposition 7** *Suppose that the city is perfectly integrated, i.e.  $\lambda_t^1 = \lambda_t^2 = \Lambda_t/2$  at each time  $t$ . The system has two stable steady states  $\Lambda = 2\tilde{q}_1$  and  $\Lambda = 2$ . For all  $\Lambda_0 \in [0, 2\tilde{\lambda}_2]$  the system converges to  $2\tilde{\lambda}_1$ , and for all  $\Lambda_0 \in [2\tilde{\lambda}_2, 2]$  the system converges to 2.*

**Comment.** When the city is perfectly integrated both neighbourhoods provide the same incentives for parents to transmit their trait. As long as the initial fraction of  $a$  agents is low ( $\Lambda_0 \in [0, 2\tilde{\lambda}_2]$ ), the fraction of  $a$  agents cannot attain a high steady-state value. Whenever  $\Lambda_t > 2\tilde{\lambda}_1$ , peer effects in both neighbourhoods are low and the cultural substitution effect not high (low) enough for type- $b$  ( $a$ ) parents, so that the benefit of having a child of their own type is higher for type- $b$  than for type- $a$  parents. The fraction of  $a$  agents thus falls to  $2\tilde{\lambda}_1$ . If, however, the initial fraction of  $a$  agents is high enough ( $\Lambda_0 \in [2\tilde{\lambda}_2, 2]$ ), the peer effects are high in both urban areas and the fraction of  $a$  agents increases up to a high steady-state value.

**Proof.**

We can straightforwardly show that, if we impose  $\lambda_t^1 = \lambda_t^2 = \Lambda_t/2$ , the equation  $\Lambda_{t+1} = \Lambda_t$  has four solutions:  $\Lambda = 0$ ,  $\Lambda = 2\tilde{\lambda}_1$ ,  $\Lambda = 2\tilde{\lambda}_2$  and  $\Lambda = 2$ .

**Stability**

We have already calculated the derivatives  $\frac{d(\Lambda_{t+1}-\Lambda_t)}{d\Lambda_t}|_0 > 0$  and  $\frac{d(\Lambda_{t+1}-\Lambda_t)}{d\Lambda_t}|_2 = 0$ . In addition,

- (i)  $\frac{d(\Lambda_{t+1}-\Lambda_t)}{d\Lambda_t}|_{2\tilde{\lambda}_1} = 2\tilde{\lambda}_1(1 - \tilde{\lambda}_1)(\frac{\partial d^a}{\partial \lambda}|_{\tilde{\lambda}_1} - \frac{\partial d^b}{\partial \lambda}|_{\tilde{\lambda}_1}) < 0$ .
- (ii)  $\frac{d(\Lambda_{t+1}-\Lambda_t)}{d\Lambda_t}|_{2\tilde{\lambda}_2} = 2\tilde{\lambda}_2(1 - \tilde{\lambda}_2)(\frac{\partial d^a}{\partial \lambda}|_{\tilde{\lambda}_2} - \frac{\partial d^b}{\partial \lambda}|_{\tilde{\lambda}_2}) > 0$ .

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<sup>25</sup>Note that the sequence  $\lambda_t^1$  is not strictly increasing, since it is constant whenever  $\lambda_t^1 > \lambda^*$ .

Using similar arguments to those above, we show that the sequence  $\Lambda_t$  converges to  $2\tilde{\lambda}_1$  for all  $\Lambda_0 \in [0, 2\tilde{\lambda}_2]$ , and to 2 for all  $\Lambda_0 \in [2\tilde{\lambda}_2, 2]$ . ■

**Proposition 8** *There are ranges of  $\Lambda_0$  such that segregation leads to a higher long-run rate of education, and other ranges of  $\Lambda_0$  such that integration leads to a higher long-run rate of education.*

**Comment.** The urban configuration affects the incentives to transmit each trait, so that it has an impact on the long-run equilibrium. For some values of  $\Lambda$ , the integrated city provides too few incentives to transmit trait  $a$  in both neighbourhoods. Segregation may (i) increase the peer effects in neighbourhood 1, and (ii) increase the oblique transmission of trait  $b$  in area 2 (i.e. the cultural substitution effect for type- $b$  agents). Both of these changes increase the incentives for type- $a$  parents to transmit their trait relative to type- $b$  parents, which positively affects the dynamics of trait  $a$  and so the long-run equilibrium. Conversely, for some values of  $\Lambda$ , the integrated city provides high incentives to transmit trait  $a$  in both neighbourhoods. Segregation negatively affects the fraction of  $a$  agents in neighbourhood 2, leading to a fall in peer effects in this area. This produces greater incentives for type- $b$  parents than for type- $a$  parents to transmit their trait in area 2. If, furthermore, the fraction of  $a$  agents in area 1 is such that nobody exerts socialization effort ( $\lambda_t > \lambda^*$ ), the fraction of  $a$  agents in the whole population falls to the lower long-run equilibrium value of education.

**Proof.** Suppose that  $\Lambda_0 \in [\tilde{\lambda}_2, 2\tilde{\lambda}_2]$  and the city is perfectly integrated. Given Proposition 7, the long-run rate of education is  $\Lambda = 2\tilde{\lambda}_1$ . Suppose that some shock leads the city to become segregated. Given Proposition 6,  $\Lambda_t$  converges to  $\Lambda = 1 + \tilde{\lambda}_1 > 2\tilde{\lambda}_1$ , and the long-run rate of education rises.

Suppose that  $\Lambda_0 \in [2\tilde{\lambda}_2, 1 + \tilde{\lambda}_2]$  and the city is segregated. Given Proposition 6, the long-run rate of education is  $\Lambda = 1 + \tilde{\lambda}_1$ . Suppose that some shock leads the city to become perfectly integrated. Given Proposition 7,  $\Lambda_t$  converges to  $\Lambda = 2 > 1 + \tilde{\lambda}_1$ , and the long-run rate of education rises. ■

## 8.2 Assumptions 2 and 3

**Assumption 2** Let us denote the LHS of (9) by  $P^a(q)$ . This is a polynomial of order two, going from  $[0, 1]$  into  $\mathbb{R}$ . It is concave with  $P^a(1) = -\theta < 0$ . Furthermore,  $q_{\max}^a$ , which is such that  $P^{a'}(q_{\max}^a) = 0$ , is given by

$$q_{\max}^a = 1 - \frac{\Delta w}{2c}.$$

In particular, the parameters  $c, w_r, w_p, \tau, \theta$  are such that  $P^a(q) = (1 - q)\tau(\Delta w - (1 - q)c + a) - \theta$  satisfies

- (i)  $P^a(0) = \tau(\Delta w - c + a) - \theta < 0$ ,
- (ii)  $q_{\max}^a = 1 - \frac{\Delta w}{2c} > 0 \Leftrightarrow 2c > \Delta w$ ,
- (iii)  $P^a(q_{\max}^a) > 0 \Leftrightarrow \left(\frac{\Delta w}{2c}\right)\tau\left(\frac{\Delta w}{2} + a\right) - \theta \geq 0$
- (iv)  $P^a\left(\frac{1}{2}\right) > 0 \Leftrightarrow \frac{\tau}{2}\left(\Delta w - \frac{c}{2} + a\right) > \theta$ .

From items (i)-(iii) and  $P^a(1) < 0$ , we can deduce that  $P^a(q)$  has two positive roots,  $\tilde{q}_1$  and  $\tilde{q}_2$ , given by

$$\tilde{q}_1 = 1 - \frac{\tau(a + \Delta w) + \sqrt{D}}{2\tau c}, \quad \tilde{q}_2 = 1 - \frac{\tau(a + \Delta w) - \sqrt{D}}{2\tau c}$$

with

$$D = \tau(\tau(a + \Delta w)^2 - 4c\theta).$$

As  $\tau < 1$ , it is easy to see that  $\tilde{q}_2 < 1$ . It is also easy to show that  $\tilde{q}_1 + \tilde{q}_2 < 1$ . Finally, item (iv) implies that  $\tilde{q}_1 < 1/2 < \tilde{q}_2$ .

**Assumption 3** Let us denote the left-hand side of (11) by  $P^b(q^j)$ . The function  $P^b : [0, 1] \rightarrow \mathbb{R}$  is concave with  $P^b(0) = -\theta < 0$  and  $P^b(1) = -\tau(\Delta w - b) - \theta$ . We calculate  $q_{\max}^b$ , which is such that  $P^b(q_{\max}^b) = 0$ , to find

$$q_{\max}^b = \frac{1}{2} - \frac{(\Delta w - b)}{2c}.$$

We assume that the parameters  $\tau, \Delta w, c, b$  and  $\theta$  are such that

$$P^b(q_{\max}^j) < 0 \Leftrightarrow \frac{\tau}{c}\left(\frac{\Delta w - b}{2} - \frac{c}{2}\right)^2 - \theta < 0.$$

### 8.3 Proof of Proposition 1

For any  $Q_t \in [0, 2]$ , and any  $q_t^1 \in [Q_t/2, Q_t]$  and  $q_t^2 = Q_t - q_t^1 \in [0, Q_t/2]$ , we consider the bid-rent differential and characterize the implied urban equilibrium.

**Case 1** When  $Q_t \in [0, \tilde{q}_1]$ , for any  $q_t^1 \in [Q_t/2, Q_t]$  and  $q_t^2 \in [0, Q_t/2]$ , we have  $q_t^2 \leq q_t^1 \leq \tilde{q}_1$ . We know from equation (10) that type- $a$  agents exert no effort in both urban areas. From (13), the

rent differential then equals

$$\bar{\rho}^{b,1} - \bar{\rho}^{a,1} = -(q_t^1 - q_t^2)[b + a] \leq 0 \text{ as } q_t^1 \geq q_t^2. \quad (14)$$

**Result** For any  $Q_t \in [0, \tilde{q}_1]$ , type- $a$  parents outbid type- $b$  parents and both  $[\rho_t^* = \bar{\rho}^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = 0, d^{*2,a} = 0]$  and  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = 0, d^{*2,a} = 0]$  are urban equilibria. However,  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = 0, d^{*2,a} = 0]$  is unstable. To show this, consider a small perturbation  $\epsilon > 0$  that increases the number of type- $a$  inhabitants, respectively type- $b$  inhabitants, in urban area 1, respectively 2. We easily find that  $\bar{\rho}^{b,1} - \bar{\rho}^{a,1} = -(2\epsilon)[b + a] < 0$ , meaning that type- $a$  individuals have greater incentives to outbid their type- $b$  counterparts to live in area 1, preventing the symmetric equilibrium to emerge.

In all of the cases that follow, suppose that  $2\tilde{q}_1 < \tilde{q}_2$ .<sup>26</sup>

**Case 2** When  $Q_t \in [\tilde{q}_1, \tilde{q}_2]$ , we have various possible situations.

**Subcase 2.1** Suppose that  $2\tilde{q}_1 < \tilde{q}_2$  and  $Q_t \in [\tilde{q}_1, 2\tilde{q}_1]$ .

*i/* Suppose that  $q_t^1 \in [Q_t/2, \tilde{q}_1]$ ,  $q_t^2 \in [Q_t - \tilde{q}_1, Q_t/2]$ , which implies that  $q_t^2 \leq q_t^1 \leq \tilde{q}_1$ : type- $a$  agents exert no effort in either urban area. The rent differential is (14).

*ii/* Suppose that  $q_t^1 \in [\tilde{q}_1, Q_t]$  and  $q_t^2 \in [0, Q_t - \tilde{q}_1]$ , then  $q_t^2 \leq \tilde{q}_1 \leq q_t^1 < \tilde{q}_2$ : type- $a$  agents exert a socialization effort  $\tau$  in area 1 and no effort in area 2. From (13), the rent differential is

$$\bar{\rho}^{b,1} - \bar{\rho}^{a,1} = -(q_t^1 - q_t^2)[b + a] - \tau(1 - q_t^1)(\Delta V_a^1) + \theta. \quad (15)$$

Since  $\tilde{q}_1 \leq q_t^1 < \tilde{q}_2$ , we have  $-\tau(1 - q_t^1)(\Delta V_a^1) + \theta < 0$ , leading to

$$\bar{\rho}^{b,1} - \bar{\rho}^{a,1} < 0.$$

Type- $a$  parents outbid type- $b$ .

**Subcase 2.2** Suppose that  $Q_t \in [2\tilde{q}_1, \tilde{q}_2]$ .

<sup>26</sup>The proof for the case with  $2\tilde{q}_1 > \tilde{q}_2$  is very similar, so that we omit the details to restrict the number of cases.

*i/* When  $q^1 \in [Q_t/2, Q_t - \tilde{q}_1]$  and  $q^2 \in [\tilde{q}_1, Q_t/2]$ , then  $\tilde{q}_1 \leq q_t^2 \leq q_t^1 < \tilde{q}_2$ : type-*a* agents exert effort in both urban areas. From (13), (6) and (8), the rent differential then equals

$$\bar{\rho}_t^{b,1} - \bar{\rho}_t^{a,1} = (q_t^1 - q_t^2) [-b - a - \tau(c - \Delta w - a + c(1 - Q_t))]. \quad (16)$$

The second factor in brackets rises in  $Q_t$ . For  $Q_t = 2$ , it equals  $[-b + \tau\Delta w - (1 - \tau)a]$ , which is negative by Assumption 1. Since we assumed  $(q_t^1 - q_t^2) > 0$ , the rent differential (16) is therefore negative for any  $Q_t \in [0, 2]$  (provided that  $q_t^1 \geq q_t^2$ ). Type-*a* individuals outbid type-*b* individuals.

*ii/* When  $q_t^1 \in [Q_t - \tilde{q}_1, Q_t]$  and  $q_t^2 \in [0, \tilde{q}_1]$ , then  $q_t^2 \leq \tilde{q}_1 < q_t^1 < \tilde{q}_2$ : type-*a* agents exert effort in urban area 1 and no effort in urban area 2, so that the rent differential is given by (15) which is negative meaning that type-*a* individuals outbid type-*b*.

To sum up, when  $Q_t \in [\tilde{q}_1, \tilde{q}_2]$ ,

$$\text{If } Q_t \in [\tilde{q}_1, 2\tilde{q}_1], \quad \text{the rent differential is } \begin{cases} (14) \text{ when } q_t^1 \in [\frac{Q_t}{2}, \tilde{q}_1], q_t^2 \in [Q_t - \tilde{q}_1, \frac{Q_t}{2}], \\ (15) \text{ when } q_t^1 \in [\tilde{q}_1, Q_t], q_t^2 \in [0, Q_t - \tilde{q}_1]. \end{cases}$$

$$\text{If } Q_t \in [2\tilde{q}_1, \tilde{q}_2], \quad \text{the rent differential is } \begin{cases} (16) \text{ when } q_t^1 \in [\frac{Q_t}{2}, Q_t - \tilde{q}_1], q_t^2 \in [\tilde{q}_1, \frac{Q_t}{2}]. \\ (15) \text{ when } q_t^1 \in [Q_t - \tilde{q}_1, Q_t], q_t^2 \in [0, \tilde{q}_1]. \end{cases}$$

**Result** For any  $Q_t \in [\tilde{q}_1, \tilde{q}_2]$ ,

- when  $Q_t \in [\tilde{q}_1, 2\tilde{q}_1]$ ,  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = 0, d^{*2,a} = 0]$  and  $[\rho_t^* = \bar{\rho}_t^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = \tau, d^{*2,a} = 0]$  are urban equilibria. As the first is unstable, the second is the unique urban equilibrium;
- when  $Q_t \in [2\tilde{q}_1, \tilde{q}_2]$ ,  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = \tau, d^{*2,a} = \tau]$  and  $[\rho_t^* = \bar{\rho}_t^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = \tau, d^{*2,a} = 0]$  are urban equilibria. As the first is unstable, the second is the unique urban equilibrium.

**Case 3** When  $Q_t \in [\tilde{q}_2, 1]$ .

**Subcase 3.1**  $Q_t \in [\tilde{q}_2, \tilde{q}_1 + \tilde{q}_2]$ .

*i/* If  $q_t^1 \in [\frac{Q_t}{2}, Q_t - \tilde{q}_1]$  and  $q_t^2 \in [\tilde{q}_1, \frac{Q_t}{2}]$  then  $\tilde{q}_1 \leq q_t^2 \leq q_t^1 \leq \tilde{q}_2$ : type-*a* agents exert  $\tau$  in both urban areas. The rent differential is given by (16).

*ii/* If  $q_t^1 \in [Q_t - \tilde{q}_1, \tilde{q}_2]$  and  $q_t^2 \in [Q_t - \tilde{q}_2, \tilde{q}_1]$  then  $q_t^2 \leq \tilde{q}_1 \leq q_t^1 \leq \tilde{q}_2$ : type-*a* agents exert effort in

urban area 1 and no effort in urban area 2. The rent differential is given by (15).

iii/ If  $q_t^1 \in [\tilde{q}_2, Q_t]$  and  $q_t^2 \in [0, Q_t - \tilde{q}_2]$  then  $q_t^2 \leq \tilde{q}_1 < \tilde{q}_2 \leq q_t^1$ : type- $a$  agents exert no effort in either urban area. The rent differential is given by (14).

**Subcase 3.2**  $Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1]$

i/ If  $q_t^1 \in [\frac{Q_t}{2}, \tilde{q}_2]$  and  $q_t^2 \in [Q_t - \tilde{q}_2, \frac{Q_t}{2}]$  then  $\tilde{q}_1 \leq q_t^2 \leq q_t^1 \leq \tilde{q}_2$ : type- $a$  agents exert effort  $\tau$  in both urban areas. The rent differential is given by (16).

ii/ If  $q_t^1 \in [\tilde{q}_2, Q_t - \tilde{q}_1]$  and  $q_t^2 \in [\tilde{q}_1, Q_t - \tilde{q}_2]$  then  $\tilde{q}_1 \leq q_t^2 \leq \tilde{q}_2 \leq q_t^1$ : type- $a$  agents exert no effort in urban area 1 and effort  $\tau$  in urban area 2. From (13), the rent differential is

$$\bar{\rho}_t^{b,1} - \bar{\rho}_t^{a,1} = -(q^1 - q^2)[b + a] + \tau(1 - q_t^2)(\Delta V_a^2) - \theta. \quad (17)$$

So that

$$\bar{\rho}_t^{b,1} - \bar{\rho}_t^{a,1} \leq 0 \Leftrightarrow (Q_t - 2q_t^2)[b + a] \geq \tau(1 - q_t^2)(\Delta V_a^2) - \theta.$$

Let us define the functions  $\Psi$  and  $\Phi$  going from  $[0, 1]$  into  $\mathbb{R}$  which are such that

$$\begin{aligned} \Psi(q_t^2; Q_t) &= (Q_t - 2q_t^2)[b + a], \\ \Phi(q_t^2) &= \tau(1 - q_t^2)(\Delta V_a^2) - \theta. \end{aligned}$$

Given equation (10), as  $q_t^2 \in [\tilde{q}_1^1, \tilde{q}_2^2]$ , we know that  $\tau(1 - q_t^2)(\Delta V_a^2) - \theta \geq 0$ .

The graph of the function  $\Psi$  is a line with a negative slope which equals  $-2(b + a)$ . The graph of the function  $\Phi$  is a curve which equals zero at  $q_t^2 = \tilde{q}_1^1$  and at  $q_t^2 = \tilde{q}_2^2$  and reaches a positive unique maximum between this two points.

On the one hand, we have  $Q_t < 1$  which implies that  $\Psi(\tilde{q}_2^2; Q_t) < 0$  since  $2\tilde{q}_2^2 > 1$ . On the other hand,  $Q_t > \tilde{q}_1^1 + \tilde{q}_2^2$  implies that  $\Psi(\tilde{q}_1^1; Q_t) = \tilde{q}_2^2 - \tilde{q}_1^1 > 0$ . Hence there exists a unique  $q^{**} \in [\tilde{q}_1^1, \tilde{q}_2^2]$  such that  $(Q_t - 2q^{**})[b + a] = \tau(1 - q^{**})(\Delta V_a^2) - \theta$ . This is an unstable equilibrium. To show this, consider a small perturbation  $\varepsilon > 0$ . We easily find  $-(Q_t - 2(q^{**} - \varepsilon))[b + a] + \tau(1 - (q^{**} - \varepsilon))(\Delta V_a^2) - \theta < 0$ . This means that a small perturbation consisting of an infinitesimal rise in the number of  $a$  agents in area 1 creates incentives for these agents to migrate into area 1, so that this equilibrium is unstable.

iii/  $q_t^1 \in [Q_t - \tilde{q}_1, Q_t]$  and  $q_t^2 \in [0, \tilde{q}_1]$  then  $q_t^2 \leq \tilde{q}_1 \leq \tilde{q}_2 \leq q_t^1$ : type- $a$  agents exert no effort in either urban area. The rent differential is given by (14).



To sum up, when  $Q_t \in [\tilde{q}_2, 1]$ ,

$$\begin{aligned} \text{if } Q_t \in [\tilde{q}_2, \tilde{q}_1 + \tilde{q}_2], \quad \text{the rent differential is} & \left\{ \begin{array}{l} (16) \text{ when } q_t^1 \in [\frac{Q_t}{2}, Q_t - \tilde{q}_1], q_t^2 \in [\tilde{q}_1, \frac{Q_t}{2}], \\ (15) \text{ when } q_t^1 \in [Q_t - \tilde{q}_1, \tilde{q}_2], q_t^2 \in [Q_t - \tilde{q}_2, \tilde{q}_1], \\ (14) \text{ when } q_t^1 \in [\tilde{q}_2, Q_t], q_t^2 \in [0, Q_t - \tilde{q}_2]. \end{array} \right. \\ \text{if } Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1], \quad \text{the rent differential is} & \left\{ \begin{array}{l} (16) \text{ when } q_t^1 \in [\frac{Q_t}{2}, \tilde{q}_2], q_t^2 \in [Q_t - \tilde{q}_2, \frac{Q_t}{2}], \\ (17) \text{ when } q_t^1 \in [\tilde{q}_2, Q_t - \tilde{q}_1], q_t^2 \in [\tilde{q}_1, Q_t - \tilde{q}_2], \\ (14) \text{ when } q_t^1 \in [Q_t - \tilde{q}_1, Q_t], q_t^2 \in [0, \tilde{q}_1]. \end{array} \right. \end{aligned}$$

**Results** For any  $Q_t \in [\tilde{q}_2, 1]$ ,

- when  $Q_t \in [\tilde{q}_2, \tilde{q}_1 + \tilde{q}_2]$ ,  $[\rho_t^* = \bar{\rho}_t^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = 0, d^{*2,a} = 0]$  and  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = \tau, d^{*2,a} = \tau]$  are the urban equilibria. As the first is unstable, the second is the unique equilibrium which arises;

- when  $Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1]$ ,  $[\rho_t^* = \bar{\rho}_t^{b,1}, q_t^{1*} = Q_t, d^{*1,a} = 0, d^{*2,a} = 0]$  and  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = \tau, d^{*2,a} = \tau]$  are urban equilibria. Only the first arises, as the second is unstable.

In what follows, we assume that  $2\tilde{q}_2 < 1 + \tilde{q}_1$ .<sup>27</sup>

**Case 4** Suppose that  $Q_t \in [1, 1 + \tilde{q}_1]$ .

**Subcase 4.1**  $Q_t \in [1, 2\tilde{q}_2]$ .

*i/* If  $q_t^1 \in [\frac{Q_t}{2}, \tilde{q}_2]$  and  $q_t^2 \in [Q_t - \tilde{q}_2, \frac{Q_t}{2}]$ , then  $\tilde{q}_1 \leq q_t^2 \leq q_t^1 \leq \tilde{q}_2$ : type-*a* agents exert effort in both urban areas. The rent differential is given by (16).

*ii/* If  $q_t^1 \in [\tilde{q}_2, Q_t - \tilde{q}_1]$  and  $q_t^2 \in [\tilde{q}_1, Q_t - \tilde{q}_2]$ , then  $\tilde{q}_1 \leq q_t^2 \leq \tilde{q}_2 \leq q_t^1$ : type-*a* agents exert no effort in urban area 1 and effort in urban area 2. The rent differential is given by (17).

*iii/* If  $q_t^1 \in [Q_t - \tilde{q}_1, 1]$  and  $q_t^2 \in [Q_t - 1, \tilde{q}_1]$ , then  $q_t^2 \leq \tilde{q}_1 \leq \tilde{q}_2 \leq q_t^1$ : type-*a* agents exert no effort in either urban area. The rent differential is given by (14).

<sup>27</sup>The proof for the case  $2\tilde{q}_2 > 1 + \tilde{q}_1$  is very similar. We omit the details here to simplify the presentation.

**Subcase 4.2**  $Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1]$ .

i/ If  $q_t^1 \in [Q_t/2, Q_t - \tilde{q}_2]$  and  $q_t^2 \in [\tilde{q}_2, Q_t/2]$ , then  $\tilde{q}_2 \leq q_t^2 \leq q_t^1$ : type-*a* agents exert no effort in either urban area. The rent differential is given by (14).

ii/ If  $q_t^1 \in [Q_t - \tilde{q}_2, Q_t - \tilde{q}_1]$  and  $q_t^2 \in [\tilde{q}_1, \tilde{q}_2]$ , then  $\tilde{q}_1 \leq q_t^2 \leq \tilde{q}_2 \leq q_t^1$ : type-*a* agents exert no effort in urban area 1 and effort in urban area 2. The rent differential is given by (17).

Suppose that

$$\left| \Psi'(\tilde{q}_2; 2\tilde{q}_2) \right| \geq \left| \Phi'(\tilde{q}_2) \right| \Leftrightarrow 2(b+a) \geq -2\tau c(1 - \tilde{q}^2) + \tau(\Delta w + a).$$

Plugging in the above inequality the expression of  $\tilde{q}^2$  leads to Assumption 4. Since  $\Psi(\tilde{q}_2; 2\tilde{q}_2) = \Phi(\tilde{q}_2)$  and due to the continuity of the two functions in  $q_t^2$  one deduces that  $\Psi(q_t^2; 2\tilde{q}_2) > \Phi(q_t^2)$  for  $q_t^2 > 2\tilde{q}_2$ . Since the lines that are described by  $\Psi(q_t^2; Q_t)$  when  $Q_t$  varies from  $2\tilde{q}_2$  to  $1 + \tilde{q}_1$  are upward translation of the line described by  $\Psi(q_t^2; 2\tilde{q}_2)$ , we deduce that  $\Psi(q_t^2; Q_t) \geq \Phi(\tilde{q}_2)$ ,  $\forall Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1]$ .

iii/ If  $q_t^1 \in [Q_t - \tilde{q}_1, 1]$  and  $q_t^2 \in [Q_t - 1, \tilde{q}_1]$ , then  $q_t^2 \leq \tilde{q}_1 \leq \tilde{q}_2 \leq q_t^1$ : type-*a* agents exert no effort in either urban area. The rent differential is given by (14).

To sum up, when  $Q_t \in [1, 1 + \tilde{q}_1]$ ,

$$\text{if } Q_t \in [1, 2\tilde{q}_2], \quad \text{the rent differential is } \begin{cases} (16) \text{ when } q_t^1 \in [\frac{Q_t}{2}, \tilde{q}_2], q_t^2 \in [Q_t - \tilde{q}_2, \frac{Q_t}{2}], \\ (17) \text{ when } q_t^1 \in [\tilde{q}_2, Q_t - \tilde{q}_1], q_t^2 \in [\tilde{q}_1, Q_t - \tilde{q}_2], \\ (14) \text{ when } q_t^1 \in [Q_t - \tilde{q}_1, 1], q_t^2 \in [Q_t - 1, \tilde{q}_1]. \end{cases}$$

$$\text{if } Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1], \quad \text{the rent differential is } \begin{cases} (14) \text{ when } q_t^1 \in [\frac{Q_t}{2}, Q_t - \tilde{q}_2], q_t^2 \in [\tilde{q}_2, \frac{Q_t}{2}], \\ (17) \text{ when } q_t^1 \in [Q_t - \tilde{q}_2, Q_t - \tilde{q}_1], q_t^2 \in [\tilde{q}_1, \tilde{q}_2], \\ (14) \text{ when } q_t^1 \in [Q_t - \tilde{q}_1, 1], q_t^2 \in [Q_t - 1, \tilde{q}_1]. \end{cases}$$

**Result** For any  $Q_t \in [1, 1 + \tilde{q}_1]$ ,

- when  $Q_t \in [1, 2\tilde{q}_2]$ ,  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = \tau, d^{*2,a} = \tau]$  is an urban equilibrium. It is unstable.

Hence the equilibrium that arises is  $[\rho_t^* = \bar{\rho}_t^{a,1}, q_t^{1*} = 1, d^{*1,a} = 0, d^{*2,a} = 0]$ ,

- when  $Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1]$ ,  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = 0, d^{*2,a} = 0]$  is an urban equilibrium. It is

unstable. The only equilibrium that arises is  $[\rho_t^* = \bar{\rho}_t^{a,1}, q_t^{1*} = 1, d^{*1,a} = 0, d^{*2,a} = 0]$ .

**Case 5** Suppose that  $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$ .

*i/* If  $q_t^1 \in [Q_t/2, Q_t - \tilde{q}_2]$  and  $q_t^2 \in [\tilde{q}_2, Q_t/2]$ , then  $\tilde{q}_2 \leq q_t^2 \leq q_t^1$ : type-*a* agents exert no effort in either urban area. The rent differential is given by (14).

*ii/* If  $q_t^1 \in [Q_t - \tilde{q}_2, 1]$  and  $q_t^2 \in [Q_t - 1, \tilde{q}_2]$ , then  $\tilde{q}_1 \leq q_t^2 \leq \tilde{q}_2 \leq q_t^1$ : type-*a* agents exert no effort in urban area 1 and effort in urban area 2. The rent differential is given by (17).

To sum up when  $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$ ,

$$\text{the rent differential is } \begin{cases} (14) \text{ when } q_t^1 \in [\frac{Q_t}{2}, Q_t - \tilde{q}_2], q_t^2 \in [\tilde{q}_2, \frac{Q_t}{2}], \\ (17) \text{ when } q_t^1 \in [Q_t - \tilde{q}_2, 1], q_t^2 \in [Q_t - 1, \tilde{q}_2]. \end{cases}$$

**Result** For any  $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$ ,

$[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = 0, d^{*2,a} = 0]$  is an urban equilibrium. It is unstable. The equilibrium such that  $[\rho_t^* = \bar{\rho}_t^{a,1}, q_t^{1*} = 1, d^{*1,a} = 0, d^{*2,a} = \tau]$  is the only equilibrium that arises.

**Case 6** When  $Q_t \in [1 + \tilde{q}_2, 2]$ , we have, for  $q_t^1 \in [\frac{Q_t}{2}, 1]$ ,  $q_t^2 \in [Q_t - 1, \frac{Q_t}{2}]$ , which implies  $\tilde{q}_2 \leq q_t^2 \leq q_t^1$ . Type-*a* agents exert no effort in either urban area. The rent differential is given by (14).

**Result**  $[\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*1,a} = 0, d^{*2,a} = 0]$  is an urban equilibrium. It is unstable. The unique equilibrium that arises is  $[\rho_t^* = \bar{\rho}_t^{a,1}, q_t^{1*} = 1, d^{*1,a} = 0, d^{*2,a} = 0]$ .

## 8.4 Proof of Proposition 4

Suppose that  $Q_t \in ]\tilde{q}_2, 1[$  then  $Q_{t+1} = Q_t$  so that  $\bar{Q} = Q_t$  is the long-run rate of education. At time  $t$ , there is an integration policy which consists in transferring a fraction  $\Delta_t$  of agents of type *a* from area 1 to area 2, and which is such that  $\tilde{q}_1 < q_t^1 - \Delta_t < \tilde{q}_2$ . Given equation (10), type-*a* agents now exert socialization effort in area 1 and either  $\tau$  or 0 in area 2, depending on  $Q_t$  and  $\Delta_t$ . In both cases, the dynamics are such that  $Q_{t+1} > Q_t$ . The long-run equilibrium is  $\bar{Q} = 1 > Q_t$ .

Suppose that  $Q_t \in [1, 1 + \tilde{q}_1[$ , then  $Q_{t+1} = Q_t$  so that  $\bar{Q} = Q_t$  is the long-run rate of education. At time  $t$ , there is an integration policy which consists in transferring a fraction  $\Delta_t$  of type-*a* agents from area 1 to area 2, and which is such that  $\tilde{q}_1 < q_t^2 + \Delta_t < \tilde{q}_2$ . Given equation (10), type-*a*

agents now exert a socialization effort in area 2 and either  $\tau$  or 0 in area 2, depending on  $Q_t$  and  $\Delta_t$ . The dynamics is such that  $Q_{t+1} > Q_t$ . The long-run equilibrium is  $\bar{Q} = 1 + \tilde{q}_1 > Q_t$ .

Suppose that  $Q_t \in [\tilde{q}_1, \min\{2\tilde{q}_1, \tilde{q}_2\}]$ , then  $Q_{t+1} = Q_t + (1 - Q_t)Q_t\tau$  and, given Proposition 2,  $\bar{Q} = \tilde{q}_2$ . Suppose, for instance, that at time  $t$  there is an integration policy such that  $q_t^1 = Q_t/2$ , then  $q_t^1 = q_t^2 < \tilde{q}_1$ . Given equation (10), type- $a$  agents exert no socialization effort in either area 1 or area 2. The dynamics is given by  $Q_{t+1} = Q_t$  so that  $\bar{Q} = Q_t/2 < \tilde{q}_2$ .

Suppose that  $Q_t \in ]\max\{2\tilde{q}_2, 1 + \tilde{q}_1\}, 1 + \tilde{q}_2]$ . Given Proposition 2, the population dynamics is such that  $Q_{t+1} = Q_t + (Q_t - 1)(2 - Q_t)\tau$ , so that the long-run rate of education is  $\bar{Q} = 1 + \tilde{q}_2$ . Suppose that at time  $t$  there is an integration policy such that  $q_t^1 = Q_t/2$ , then  $q_t^1 = q_t^2 > \tilde{q}_2$ . Given equation (10), type- $a$  agents exert no socialization effort in either area 1 or area 2. The dynamics is given by  $Q_{t+1} = Q_t$  so that  $\bar{Q} = Q_t/2 < 1 + \tilde{q}_2$ .

Suppose that  $Q_t \in ]0, \tilde{q}_1[$ , the dynamics is given by  $Q_{t+1} = Q_t$ , so that the long-run equilibrium is  $\bar{Q} = Q_t$ . For any integration policy  $q_t^1 < \tilde{q}_1$ , the dynamics do not change. The long-run equilibrium is  $\bar{Q} = Q_t$ . The proof for other cases is very similar, so that we skip the details here.

## 8.5 Proof of Proposition 5

Suppose that

$$4(a + b)^2 < \tau(\tau(a + \Delta w)^2 - 4c\theta).$$

Consider  $Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1[$ . If  $q_t^1 \in [\tilde{q}_2, Q_t - \tilde{q}_1]$  and  $q_t^2 \in [\tilde{q}_1, Q_t - \tilde{q}_2]$ , then  $\tilde{q}_1 \leq q_t^2 \leq \tilde{q}_2 \leq q_t^1$  and the rent differential is given by (17).

First one has,

$$2(b + a) < -2\tau c(1 - \tilde{q}_2^2) + \tau(\Delta w + a) \Leftrightarrow \left| \Psi'(\tilde{q}_2; 2\tilde{q}_2) \right| < \left| \Phi'(\tilde{q}_2) \right|.$$

Since  $\Psi(\tilde{q}_2; 2\tilde{q}_2) = \Phi(\tilde{q}_2)$  and due to the continuity of the two functions in  $q_t^2$  one deduces that  $\Psi(q_t^2; 2\tilde{q}_2) < \Phi(q_t^2)$  for  $q_t^2 \in [\alpha, \tilde{q}_2[$ , with  $0 < \alpha < \tilde{q}_2$ . Moreover, we already show (in subcase 3.2) that  $\Psi(\tilde{q}_1; 2\tilde{q}_2) > \Phi(\tilde{q}_1) = 0$ . We deduce that  $\Psi(q_t^2; 2\tilde{q}_2) = \Phi(q_t^2)$  admits two solutions on  $[0, 1]$ :  $q_t^2 = \tilde{q}_2$  and  $q^{**} < \tilde{q}_2$ . This is equivalent to saying that the line whose equation is given by  $f(q_t^2; 2\tilde{q}_2)$ , say

$L1$ , is a chord of the function  $\Phi$ . By the mean value theorem, we deduce that for some  $q \in ]q^{**}, \tilde{q}_2[$ , there exists a line  $L2$ , parallel to  $L1$ , which is tangent to the curve described by  $\Phi$ . Since (i) all lines describes by  $\Psi(q_t^2; Q_t)$  are upward translation of  $L1$  when  $Q_t$  increases from  $2\tilde{q}_2$  to  $1 + \tilde{q}_1$  and (ii)  $\Phi$  is concave, we deduce that the equation of  $L2$  is given by  $\Psi(q_t^2; Q^*)$ , with  $Q^* > 2\tilde{q}_2$ . Due to the concavity of  $\Phi$ , all of the lines between  $L1$  and  $L2$  are chords of  $\Phi$ . This is equivalent to saying that for  $Q_t \in [2\tilde{q}_2, Q^*[$ , the equation  $\Psi(q_t^2; Q_t) - \Phi(q_t^2) = 0$  has two solutions inferior to  $\tilde{q}_2$ .

The two solutions correspond to interior equilibria: one stable and one unstable. The stability arguments are similar to those used in subcase 3.2, so that we omit the details.

For  $Q_t \in [2\tilde{q}_2, \min\{Q^*, 1 + \tilde{q}_1\}[$ , suppose that the interior stable equilibrium is selected. Given Proposition 4, the population dynamics are such that  $Q_{t+1} > Q_t$ , so that the long-run rate of education is  $\bar{Q} = \min\{Q^*, 1 + \tilde{q}_1\}$ . Now suppose that the segregated equilibrium is selected. Given Proposition 4, the dynamics are given by  $Q_{t+1} = Q_t$ , so that the long-run rate of education is  $\bar{Q} = Q_t < \min\{Q^*, 1 + \tilde{q}_1\}$ .