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Abstract

A government delegates a build-operate-transfer project to a private firm. At the contracting stage, the operating cost is unknown. The firm can increase the likelihood of facing a low cost (the good state), rather than a high cost (the bad state), by exerting costly effort when building the infrastructure. Once this is in place, the firm learns the true cost and begins to operate. If some partner reneges on the contract thereafter, the court of justice has a limited ability to enforce penalties. Break-up of the partnership occasions a replacement cost for the government that is higher the earlier the contract is terminated. We show that the contract is self-enforcing, entailing no distortions away from efficiency, only if the firm is instructed to invest both own and borrowed funds in the project, and the duration of the contract is set longer in the good state than in the bad state. The firm's investment should not be massive. The debt payment to the lender, which ultimately lies on the government, should be conditioned on the firm not defaulting on the project. The result that the contract should have a longer duration in the good state is at odds with the prescription of the literature on "flexible-term" contracts, which recommends a longer duration when the operating conditions are unfavourable.

Keywords: Public-private partnerships; state-dependent duration; fixed-term contract; limited enforcement; renegotiation; break-up

J.E.L. Classification Numbers: D82; H57; H81

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1 Introduction

MOTIVATION Long-term relationships in principal-agent models have been studied extensively, though mainly with regards to transactions that can be ruled by means of repeated contracts (see Baron and Besanko [5]). Within this literature, awareness has been shown of the importance that limits in the enforcement ability of the courts of justice may have in contractual design (see Levin [34], who focuses on relational contracts). Long-term relationships not structured as a sequence of short-term contracts have been little studied, and never studied with regards to frameworks of limited enforcement. This is the case of public-private partnerships (PPPs), under which a private consortium is delegated the financing of a public infrastructure (whether in total or in part), its construction, and its long-term operation. PPPs are long termed for two reasons. First, they require large investments being made up-front. Second, there are synergies between construction and operation, which are internalized if the firm that builds the infrastructure is also delegated its management for a sufficiently big number of years, before the activity is reverted to the public sector.¹ As PPPs are known to be highly vulnerable to the partners' opportunism,² there is a clear need for a better understanding of the way in which enforceability of PPP contracts is related to their length.

The reason why concerns on the partners' opportunism cannot abstract from considerations on the contractual length is that the costs and benefits, associated with the breach of a longterm contract, are not equally large in all dates in which the breach might occur. To fix ideas, think of a bridge built under a PPP arrangement. If the contract is breached twenty years before the stipulated termination date, then the associated costs and benefits, for the partners, cannot be the same as if the residual duration were of one year only. A simple way to see this is to consider that the termination of a PPP has a negative impact on the reputation and/or credibility of the government. It signals a failure to create an institutional environment, in which the private partner does bring the project to completion, as contractually delegated.³ Because early break-up involves that the government steps in and appropriates the activity for a large number of years, it is natural that the government faces a higher cost, in terms of

¹Hart [25], Bennett and Iossa [6], Martimort and Pouyet [35], and Iossa and Martimort [29] argue in favour of bundling construction and operation and delegating both tasks to a single firm, when synergies are present between the two activities. None of those studies is concerned with the contractual length under limited enforcement.

²See, for instance, the report of Guasch [21] and the cases described in Estache and Wren-Lewis [17].

³Irwin [30] emphasizes that, in government-firm relationships involving large investments, governments are especially concerned with the information that their current behaviour and achievements convey to third parties with whom they might interact in the future, namely firms, investors, customers, and voters. In the same vein, Trebicock and Rosenstock [40] acknowledge that governments face transaction costs, when PPPs are broken up early on. Using a longitudinal dataset of 40 developing economies, over the period 1990 – 2000, Banarjee *et alii* [4] show that economies in which the risk of assets expropriation is known to be high, fail to attract private investors, who do not perceive them as providing a safe haven.

reputation and/or credibility, the more prematurely is the partnership terminated.

The goal of this paper is to investigate how a PPP contract should be designed, particularly, how its duration should be set, in order to make it self-enforcing. This requires structuring the contract in such a way that neither party displays an interest in breaching it in any period through the termination date. Making the contract self-enforcing, without entailing distortions away from efficiency, is not immediate. The need to avoid breaches has to be reconciled, first, with the need to induce the private partner to internalize the synergies between tasks, thus preventing moral hazard. Additionally, it has to be reconciled with the need to tackle the informational gap, which appears as the firm becomes experienced with the operating conditions. We consider these issues in our investigation. We also take into account that, in PPP arrangements, the project can be financed by combining three distinct sources, namely public funds and private funds provided both by the firm and by an external lender, such as a bank. The financial structure of the project will prove to have an important impact on the partners' behaviour during the execution of the contract.

The main result of our study is that, when the judicial system has limited or no enforcement power, the contract cannot be made self-enforcing, unless the contractual length is conditioned on the operating conditions, to be realized after the contract is signed. The idea of linking the contractual length to the realized state of nature is in line with the works on flexible-term contracts developed by Engel et alii [11] - [12]. These authors propose that the contract specify how much revenues the firm should cumulate, in discounted terms, regardless of the realized operating conditions. Consequently, the firm operates for a bigger number of years, when the operating conditions are *unfavourable*, because, under those circumstances, it takes longer to cumulate the revenues established in the contract. However, we find that, in environments in which information problems (moral hazard and adverse selection) and limited enforceability have bite, a different strategy should be adopted. That is, the contract should have a longer duration in *favourable* states. In this study, the result is obtained in a more subtle way, as it is necessary to consider the renegotiation game in which the partners engage, following to a contractual breach.⁴ The payoffs attained by the partners in the renegotiation process depend on the payments the government owes to the firm and, indirectly, to the lender, as time goes by. Those payments depend, in turn, on the residual contractual length, hence, implicitly, on the duration of the contract in each state of nature. These factors will all be determinant for an optimal choice of the contractual length.

⁴In the study of Engel *et alii* [11] - [12], there is no need to consider the renegotiation process because renegotiation could only follow from contractual incompleteness, an issue which is removed by making the firm's payoff independent of the operating conditions.

SETUP AND MAIN RESULTS Our analytical framework is as follows. Both the government and the firm are risk-neutral. When the incentive contract is signed, the operating marginal cost (the state of nature) is unknown. The contract stipulates a state-dependent allocation, which includes the firm's profit and the contractual length. In addition, it stipulates the mix of public funds, firm's own funds and private loan to be invested. The contract is complemented by a set of termination clauses, stating the payments that the firm and the lender will receive from the government, if the partnership is terminated. Feasible penalties are bounded because the enforcement power of the court of justice is limited. Moral hazard arises at the time when the firm builds the infrastructure. Adverse selection appears as soon as the infrastructure is in place and the firm observes the true cost, which can be either low (the good state) or high (the bad state). The partners are unable to commit to the respective obligations. Thus, once the state becomes known, given the incentives to information release provided by the government to the firm, some party may renege on the contract, in which case either a new agreement is reached or the partnership is terminated.⁵ Break-up of the relationship and consequent retrieval of the activity occasion a "replacement cost" to the government, which is higher the bigger the residual contractual length.

Flexibility in adjusting per-period profits Tackling moral hazard requires exposing the firm to a certain amount of risk, below which the firm would shirk when building the infrastructure. This involves inducing a sufficiently large difference between the firm's cumulated profits in the two states. Yet, too large a profit wedge would trigger adverse selection. We find that, when transferring the desirable amount of risk to the firm, adverse selection is prevented, by inducing appropriate adjustments in the contractual terms, to be compensated with variations in the per-period profits of the firm. Specifically, a raise in the contractual length in the good state should be compensated with a reduction in the per-period profit in that state; a reduction in the contractual length in the bad state should be compensated with a raise in the per-period profit in that state. These compensations are feasible, as long as the contractual length is finite in the good state and strictly positive in the bad state. The interest of this finding resides in that conditioning the duration of the contract on the state of nature grants a flexibility gain to the contract designer. A wider range of values of per-period profits is available, in either state, to tackle information problems. This possibility appears to be crucial at mitigating the partners' *ex-post* opportunism, which depends finely on the residual profits (hence, on the per-period

⁵The assumption that moral hazard is followed by adverse selection and, possibly, contractual breach is in line with several studies on public-private contracting, namely Laffont, [32], Guasch *et alii* [22] - [23], Iossa and Martimort [27], Danau and Vinella [9]. As in the latter study, in this work, *both* the firm and the government are unable to commit and their breach payoffs depend on the contractual length. However, unlike in that paper, we here allow for the contractual term to be conditioned on the state and for the court of justice to enforce limited breach penalties.

profits) earned by the firm under the contract.

The lender should lose money when the firm defaults Two widely debated aspects of PPPs concern the reasons why projects should be highly leveraged and the kind of guarantees that governments should provide in order to attract external sponsors.⁶ Our study predicts that, if the firm takes out a loan, then, within the limit of the enforceable payments, that loan can be used strategically to discourage the partners from returning to the contracting table. Because the debt burden lies, ultimately, on the government, it can be employed as a tool to destroy the surplus that the government could share with the firm in a new negotiation. For this tool to be most effective, the government should be called upon to pay the entire debt to the lender, when the partnership is terminated on its own initiative. By contrast, it should be waived any payment to the lender, when the partnership is terminated on the firm's initiative, in which case break-up is assimilated to the default of a private project. Intuitively, if the government owes no debt payment when the partnership is prematurely terminated, then it may want to induce break-up and appropriate the lender's investment. Moreover, if the debt obligation of the government is independent of the opportunistic behaviour of the firm, then the firm may seek a new negotiation to appropriate more surplus.

This all has implications on the design of the government's liabilities $vis-\dot{a}-vis$ the firm's lender. They have to be set such that, should the government renege, it would seek to renegotiate; should the firm renege, it would seek to terminate the partnership.⁷

The firm should invest in the project, but not too much For the firm to be motivated to honour the contract, it should be instructed to invest money in the project up-front. However, if the firm were required to invest massively, then it might be convenient, for the government, to terminate the partnership prematurely and appropriate that investment. Thus, the firm's contribution is essential but it should not be too large. Although this result is very intuitive, it is, yet, very important. Policy makers frequently argue that PPPs are desirable because the private sector finances the projects. Consequently, it is not necessary to raise distortionary taxes to develop projects and more public resources can be devoted to other policies, which are socially

⁶See Irwin *et alii* [31] for a detailed discussion on the provision of governmental guarantees in PPP projects. See also the report prepared by NAO [37] on the London underground project delegated to Metronet, according to which 95% of the latter's debt was guaranteed by the government.

⁷Looking at this result only, one might be led to conclude that break-up of the PPP created for the London underground project was the best epilogue, after Metronet breached the contract. However, the fact that the contract was not honoured signals that the design was poor. Particularly, the problem might have lied in the design of the debt liabilities, provided a large part of the debt was guaranteed by Transport for London (the agency in charge of the daily operation of the public transport network in London) unconditionally. Our findings suggest that, if private default is to occur, rather than renegotiation, conditional on the firm breaching the contract, this is exactly because the debt burden ling on the government can be structured in such a way that the contract does remain in place. Hence, the firm does not renege, in fact, and default does not follow.

desirable but unprofitable. Engel *et alii* [13] - [14] point out that a PPP is just as costly as any other government investment. It has exactly the same impact on the government inter-temporal budget constraint and only alters the timing of government revenues and disbursements. Thus, if there are any savings to private financing of PPP projects, they ensue from that costly intermediation by public agencies can be avoided, as private contributions are tantamount to direct transfers from firms to consumers.⁸ Our result offers a new argument for private financing of PPP projects.⁹

The very contribution of our study is embedded in the following two results, which provide the reasons why the contract should have a longer duration under favourable operating conditions. Both of them are related to the difficulty of mitigating the government's opportunism without, yet, triggering the firm's opportunism.

Under limited enforcement, debt liabilities cannot be optimally structured, unless the contract has a longer duration in the good state of nature Conditioning the contractual length on the state of nature, involves that also the debt burden, ling on the government at each instant during the execution of the contract, is made state-dependent.¹⁰ Differentiating liabilities between states is necessary in that, as we explained, they affect the opportunism of the partners in opposite directions. Absent that possibility, the incentive of the government to renege in the good state, in which it owes a high profit to the firm, could not be eliminated together with the incentive of the firm to renege in the bad state, in which it receives a low profit.¹¹

Under limited enforcement, a "neither-too-high-nor-too-low" investment cannot be recommended from the firm, unless the contract has a longer duration in the good state of nature This result fully reflects the need to circumvent enforcement difficulties, on one side, when also solving information problems, on the other. The former task looks easier, if

⁸In the same vein, Hart [25] is negative with the thinking that the private sector is a cheaper source of financing, as compared to the public sector, as it is difficult to imagine an agent that is more able to borrow than the government. He rather regards contracting costs as being the central issue to explain reliance on the private sector.

⁹This result evidences a reversed effect with respect to the familiar hold-up problem. The latter arises when the agent *under*-invests early on in the relationship, anticipating that the relationship might be terminated before a transaction takes place, in which case the investment would be foregone. In our setting, the firm cares of the relationship being preserved, if it provides a sufficiently *large* contribution up-front.

¹⁰For instance, if the per-period debt obligation is the coupon of a bond, the value of the coupon is lower the longer is the duration of the contract.

¹¹Essentially, this result follows from that the government, not the lender, provides incentives to the firm. Letting the lender act as a second principal would introduce complications, in our model, without yet allowing for any real improvement in contractual performance. We find, indeed, that conditioning the duration on the state and setting a proper financial structure of the project does lead to self-enforceability of the contract.

it considered that, in each period during the execution of the contract, the government is more eager to preserve the relationship, the more costly would a break-up be, hence the bigger is the residual contractual length. Resting on this, it is intuitive that the duration of the contract should be set sufficiently long, in the state in which the government is more prone to take opportunistic actions, *i.e.*, in the good state. However, in so doing, one should take care of not exacerbating the information problems, instead. Actually, this can be avoided, by taking advantage of the enhanced possibility of adjusting the firm's per-period profits, which becomes available when the contractual length is differentiated in the two states, as we said. Ultimately, the strategy is to set a longer duration in the good state than in the bad state.

When the judicial enforcement power is poor, institutions like Export Credit Agencies can be called upon to restructure debt liabilities so as to foster enforceability In a large variety of businesses, including PPP projects, financial institutions such as Export Credit Agencies are called upon to protect lenders against payment default by borrowing corporations. In PPPs, concerns on lenders' protection arise because of lenders' exposure to political and non-commercial risks, including breach of contract and private investment expropriation. Our last result is that, when the judicial system is poor and replacement of the firm is not very costly for the government, involving such institutions may be helpful to make the contract selfenforcing. Their intervention would allow for the debt payment to the lender to be structured in a more flexible manner than the government affords through the PPP contract. Specifically, it would be possible to establish that the government deposit some money with the agency up-front, which the agency should then deliver to the lender, depending on the evolution of the relationship between the government and the firm. The effectiveness of this double transfer rests on that its size does not need to be related to the size of the contractual debt. Thanks to this, the partners' opportunism can be lessened to the most by setting liabilities as follows. Again, the lender foregoes any further payment if, in the bad state, the firm reneges on the contract. Novel to this scenario, the lender receives *more* than the contractual debt if, in the good state, the government reneges on the contract. This novelty has an important implication. Unlike in absence of the agency, not only the firm, but also the government can be led to obtain no more than its break-up payoff in the renegotiation process, thus making contractual breach less attractive. Therefore, reliance on institutions such as Export Credit Agencies reinforces the effectiveness of debt payments as a tool, complementary to the state-dependent duration, against the partners' opportunism.

OUTLINE The reminder of the paper is organized as follows. In section 2, we describe the model. In section 3, we present the payoffs of the partners when the contract is honoured and when it is breached. In section 4, we state the programme which must be solved to characterize

the optimal contract, embodying both information constraints and enforcement constraints. In section 5, results are drawn. Section 6 illustrates the flexibility gain that the contract designer enjoys when the contractual length is state-dependent. In section 7, the analysis is extended to identify the benefit of involving financial institutions, such as Export Credit Agencies, in PPPs. In section 8, results are discussed in relation to the existing literature. Section 9 concludes.

2 The model

A government (G) delegates a public project to a private firm (F). The project includes the construction and the management of an infrastructure, which will be used to provide a good (or service) to society. F is a Special-Purpose-Vehicle (SPV), expressly created by a group of private investors to perform these tasks. The contract is signed and the construction takes place at date 0. F provides the good thereafter, through the termination date T > 0. For simplicity, we assume that the infrastructure has an infinite life, during which it does not depreciate. If T is finite, then the infrastructure is transferred to G at the end of the contract, as is typical of PPP arrangements.

Financing To build the infrastructure, F must bear a sunk cost of I > 0. To finance this cost, it invests own funds of $M \ge 0$ up-front, and not any further amount during the development of the project.¹² Additionally, F borrows funds of $C \ge 0$ from a lender (L) on the credit market and receives an up-front transfer of $t_0 \in \mathbb{R}$ from G, such that $M + C + t_0 = I$. By allowing for t_0 to be negative, we admit the possibility of F paying a fee to be awarded the contract.¹³

Returns from operation At each instant $\tau \in (0, T)$, F delivers q units of the good at a total cost of $\theta q + K$, where K > 0. In return for production, F receives a transfer of t from G and collects revenues p(q)q from the market. Out of those, it makes a payment of d to L. The profit of F is, thus, $\pi = p(q)q + t - (\theta q + K) - d$. Consumption of q units of the good yields a gross surplus of S(q), such that $S'(\cdot) > 0$, $S''(\cdot) < 0$, S(0) = 0, and the Inada's conditions hold. Customers purchase the output produced at each τ at a price of $p(q) \equiv S'(q)$. G attaches to the project a value equal to consumer surplus, net of the transfer made to the firm. The instantaneous value of the project to G is, thus, S(q) - p(q)q - t.¹⁴

¹²See, for instance, Engel *et alii* [13] and the guidelines provided by EPEC [16] on financial arrangements typical of PPPs. Here, the relevant aspect is that the transactions made by the parent firms through the SPV are *off* balance sheet, involving that, during the development of the project, they cannot be required to make further contributions, in addition to those made up-front.

¹³Public funds used in the projects can also be intended as money borrowed by the government itself.

¹⁴To be rigorous, spending one unit of public funds requires collecting more than one unit of money from taxpayers. To capture this circumstance formally, we could introduce some parameter $\lambda > 0$, expressing the

Information problems There are two subsequent information problems, namely, moral hazard followed by adverse selection. When building the infrastructure, F exerts an unobservable effort, denoted $a \in \{0, 1\}$, which occasions a disutility of $\psi(0) = 0 < \psi(1) = \psi$. The value of a affects the distribution of the marginal cost of production. The latter can take values of θ_l and θ_h , such that $0 < \theta_l < \theta_h$. The "good" realization θ_l occurs with probability ν_1 , if a = 1, and with probability ν_0 , if a = 0. As we refer to a PPP project, we can reasonably assume that exerting effort makes it more likely that the marginal cost will be low, *i.e.*, $0 < \nu_0 < \nu_1 < 1$. F observes the realization of θ privately, as soon as the infrastructure is in place and before production begins. θ is an inner characteristic of the infrastructure, thus its value remains the same during the life of the project. Henceforth, we denote $i \in \{l, h\}$ the state of nature.¹⁵

The contracts Applying the Revelation Principle, the incentive contract that G offers to F includes the menu of allocations $(\{q_l, t_l; T_l\}, \{q_h, t_h; T_h\})$. The peculiarity of our approach, relative to the literature on incentive contracts, is that we let the contractual length be conditioned on the state of nature. This, of course, encompasses the standard case of a fixed-term contract, such that $T_l = T_h \equiv T$. Additionally, the contract between G and F specifies the triplet $\{M, C, t_0\}$ of financing sources of the project. In turn, the contract between L and F stipulates that, given the loan of C that F obtains from L, F must reimburse an amount of D_0 , in discounted terms. Assuming that the credit market is perfectly competitive, $D_0 = C$.¹⁶ Although D_0 is independent of *i*, provided L does not screen types, the repayment d_i that F owes to L, in each period of operation, depends on the length of time, namely T_i , during which it must be made. Hence, the value of the debt at date τ is $D_{i,\tau} = \int_{\tau}^{T_i} d_i e^{-r(x-\tau)} d\tau$, where *r* is the discount rate, and it is such that $D_{l,0} = D_{h,0} = D_0$. Once T_i and C are set in the PPP contract, D_0 and, implicitly, d_i are also determined. This further yields the entire sequence of debt values $\{D_{i,\tau}\}_{i=l,h,\tau \in (0,T_i)}$. As the choice of t_0 is implied by that of M and C, and as $D_0 = C$, we will refer to $\{M, D_0\}$ as to the pair of financing decision variables of the project.

Contractual frictions Both partners are unable to commit to contractual obligations. Thus, in any state $i \in \{l, h\}$, at any $\tau \in (0, T_i)$, either F or G may renege on the PPP contract. Then,

shadow cost of public funds. Then, a transfer of t would cost $(1 + \lambda)t$ to G. However, because this would have no qualitative impact on results, we neglect the shadow cost of public funds, for simplicity.

¹⁵While we account for the issue of moral hazard in construction, we neglect that of moral hazard in operation, which is related to the effort that would be necessary to maintain the infrastructure. This choice has no substantial impact on our analysis. To see this, denote b > 0 the maintenance effort and take $\theta = \beta - b$, where β is the value of the marginal cost if the infrastructure is not maintained. By conditioning the compensation on θ , rather than on β , G makes F residual claimant of the benefit from effort b. F is, thus, motivated to maintain the infrastructure (see, for instance, Laffont and Martimort [33], ch.7).

¹⁶The assumption of perfect competition is without loss of generality as what matters, in our model, is that the debt of F is proportional to the loan taken out on the credit market.

either they reach a new agreement (*renegotiation*) or the partnership is terminated (*break-up*). The possibility of renegotiating follows from that break-up does not come for free to G. This is because break-up, and the consequent replacement of the firm, mirrors a failure by the government to fulfil its responsibility for the concerned project being brought to completion in partnership with the delegated firm. It is, thus, natural that G will incur a cost in terms of reputation and/or credibility, which will be higher the earlier is the partnership terminated. In consideration of this, we define this "replacement cost" as a continuously differentiable function R_x , such that $R_x > 0$ and $R'_x \equiv (dR/dx) > 0$, $\forall x > 0$. It means that, for any given $T_i > 0$, the cost of R_{δ_i} , where $\delta_i = (T_i - \tau)$, which G faces at τ , is higher the longer is the time interval (τ, T_i) . To keep the analysis well focused, R_{δ_i} is the only dynamic variable of the model, once T_i is set. At a later stage, it will allow us to represent the circumstance that the surplus, to be shared in a hypothetical renegotiation, depends on the residual contractual length.

Whether or not the PPP contract is executed, the credit contract may be breached. Indeed, as F is unable to commit, it may stop reimbursing L.

Breach penalties and enforceability A court of justice cannot oblige the parties to execute the contract, when at least one of them is unwilling to do so. However, in the same vein as in the recent literature on non-enforceable contracts, we allow for the court to impose penalties (sanctions), following a breach. Penalties do not concern F. Break-up is assimilated to the default of a private project, involving that G relieves the SPV and the private investors cannot be called upon to inject further resources, in addition to those contributed to the project upfront. However, as it stops abiding by the contract, F renounces to recoup a part of its initial investment. This loss will act naturally as a penalty, for the firm, in the event of a breach. By contrast, penalties can be conceived for G. If the partnership is terminated, then G takes the assets, which embody the investments of F and L. In consideration of this, we assume that the court of justice can impose to G a maximum penalty of $\overline{P} \ge 0$ in favour of F, and a maximum reimbursement of $\overline{D} \geq 0$ in favour of L. The magnitude of \overline{P} and \overline{D} captures the court's enforcement ability, which might be limited, say, due to an innate weakness of the judicial system or to legislative restrictions and voids. Thus, large (small) values of \overline{P} and \overline{D} mean that the court has a strong (weak) enforcement power. We also allow for \overline{P} and \overline{D} to take different values, say, mirroring different degrees of legal protection for firms and banks.

Termination clauses Given the contractual frictions that can arise, the PPP contract is complemented with the profile of termination clauses $\{P, D_0^F, D_0^G\}$. The application of the latter is conditioned on the partners' specific actions during the execution of the contracts. G will pay to F a penalty of $P \in [0, \overline{P}]$ if, at some date $\tau \in (0, T_i)$, in some state *i*, the PPP is terminated on its own initiative. Additionally, in that case, G will reimburse to L an amount of $D_{i,\tau}^G$, whereas it will reimburse an amount of $D_{i,\tau}^F$ if the PPP is terminated on the initiative of F. L receives the stipulated amount of $D_{i,\tau}$, instead, if the partners reach a new deal. Therefore, the termination clauses allow for the possibility that L not be repaid, depending on the *ex-post* behaviour of G and F. Similarly to the contractual liabilities $D_{l,0}$ and $D_{h,0}$, the termination reimbursements are independent of the state, involving that $D_{i,0}^F = D_0^F$ and $D_{i,0}^G = D_0^G$, $\forall i$. Lastly, L cannot be reimbursed more than it lent to F; hence, max $\{D_0^F, D_0^G\} \leq D_0$.¹⁷

Timing To sum up, events unfold as follows. At date 0, G offers the PPP contract to F, including the menu $\{q_i, t_i; T_i\}_{i=l,h}$ and the financing pair $\{M, D_0\}$. The contract is complemented with the breach provisions $\{P, D_0^F, D_0^G\}$. If F accepts the offer, then the contract is signed. Accordingly, F contributes own funds of M, it takes out a loan of D_0 , signing the credit contract with L, it receives t_0 from G, and it invests I in the project. When building the infrastructure, F exerts effort a. Once the infrastructure is in place, F observes θ_i and makes a report to G. The corresponding allocation $\{q_i, t_i; T_i\}$ is picked from the contractual menu. Provided the contract is incentive-compatible, the report is truthful and the allocation choice releases information on the realized state publicly. F operates according to that allocation thereafter, unless, at some point, contractual frictions arise. In that case, the termination clauses apply, depending upon the renegotiation outcome.

3 The partners' payoffs

We now present the payoffs the partners obtain when the contract is honoured and when it is breached.

¹⁷Although the orientation of EPEC is that, if contractual frictions arise on the private partner's initiative, then lenders be allowed to rescue the PPP project, by taking remedial actions before the government terminates the relationship, national legislations are rather heterogeneous (see EPEC [15] and [16]). Some countries recognize no right to lenders, others provide for alternative solutions. In this patchy context, clarifying the lenders' rights in PPP projects is essential. While, in the private sector, a firm's default leads to either reorganization of the activity or liquidation of the assets by the creditors, in PPP projects the assets are relieved by the government and the activity is transferred to the public sector. Accordingly, lenders should be given the possibility of stepping in, only if there are remedies, which they can actually put in place. However, this is not the case because the firm's cash-flow is endogenous to the incentive scheme designed by the government. Under these circumstances, more than allowing lenders to step in, in order to avoid private default, it is important to establish how liable they should be made, together with the firm and/or the government, for that default. This is what the pair of debt payments $\{D_0^F, D_0^G\}$ is meant for in our model.

3.1 The contract is executed

At each date $\tau \in (0, T_i)$, in each state $i \in \{l, h\}$, the present value of the stream of future profits of F is given by $\Pi_{i,\tau} = \int_{\tau}^{T_i} \pi_i e^{-r(x-\tau)} dx$. The date-0 payoff of F is thus:

$$\widetilde{\Pi}_{i} = \Pi_{i,0} - \left(M + \psi\left(a\right)\right).$$

The stream of future yields, discounted at date τ , which G obtains from private management, is given by $V_{i,\tau} = \int_{\tau}^{T_i} [S(q_i) - p(q_i) q_i - t_i] e^{-r(x-\tau)} dx$. Replacing $t_i = \pi_i - p(q_i) q_i + (\theta_i q_i + K) + d_i$ and defining $w(q_i) \equiv S(q_i) - (\theta q_i + K)$, this further becomes:

$$V_{i,\tau} = \int_{\tau}^{T_i} w(q_i) e^{-r(x-\tau)} dx - (\Pi_{i,\tau} + D_{i,\tau}) dx$$

Thus, the date-0 discounted return of G from private management is equal to $V_{i,0} - t_0 = \int_0^{T_i} w(q_i) e^{-rx} dx - (\prod_{i,0} + I - M)$, and the payoff from the entire life of the project is given by:

$$W_{i} = \int_{0}^{T_{i}} w(q_{i}) e^{-rx} dx - (\Pi_{i,0} + I - M) + \int_{T_{i}}^{\infty} w(q_{i}^{*}) e^{-ry} dy$$

where q_{i}^{*} is the output level that maximizes $w\left(\cdot\right)$, defined by the marginal-cost pricing rule:

$$p(q_i^*) = \theta_i. \tag{1}$$

3.2 The contract is breached

Suppose that F truthfully reports the observed state $i \in \{l, h\}$ to G so that any subsequent interaction takes place under complete information.¹⁸ Further suppose that, at some $\tau \in (0, T_i)$, contractual frictions arise. That is, either F or G reneges on the contract. They return to the contracting table and negotiate on the pair $\{q_i, \Pi_i\}$.¹⁹ We hereafter describe how the partners' payoffs are determined in that case. To avoid confusion, we will use the superscripts b, F; rn, F;b, G; rn, G to refer, respectively, to break-up and renegotiation when F reneges on the contract and when G reneges on the contract.

 $^{^{18}}$ Under incentive-compatibility of the contract that G designs for F, this is actually the case. On this point, see Section 4 below and, in particular, foornote 4.

¹⁹Guasch [21] provides examples of renegotiations involving changes in user fees, contractual length and/or public transfers. In our model, this translates into a new negotiation on q_i , T_i and t_i . However, as T_i and t_i are both embodied in Π_i , being one a substitute for the other, renegotiation actually occurs on the pair $\{q_i, \Pi_i\}$. Technical details on the renegotiation game are relegated to Appendix A.

F reneges on the contract Suppose that F reneges and that renegotiation fails. Then, the partnership is broken up and F is relieved of the activity. Under the assumption that F faces a zero outside opportunity in operation, it is left with a payoff of zero. G takes control of the infrastructure. Thereafter, either the activity is run by G itself or it is delegated to a new firm. In either situation, an output of q_i^* is produced and G obtains the entire net surplus from the activity, namely $w_i(q_i^*)\frac{1-e^{-r\delta_i}}{r}$. This is because production takes place under complete information and no further investment is required to complete the project. Consequently, there is no reason to move away from the decision rule in (1), and G can content itself with covering the production costs, if it runs the activity, or wash out any operating profits from the new firm, if it delegates the activity. However, G incurs a replacement cost of R_{δ_i} plus a payment of $D_{i,\tau}^F$, which it owes to L in this break-up scenario. The discounted cumulated returns, through date T_i , for F and G, are given by:

$$\Pi_{i,\tau}^{b,F} = 0 \tag{2}$$

$$V_{i,\tau}^{b,F} = w_i(q_i^*) \frac{1 - e^{-r\delta_i}}{r} - R_{\delta_i} - D_{i,\tau}^F.$$
(3)

Next suppose that renegotiation succeeds. The payoffs of F and G are given by:

$$\Pi_{i,\tau}^{rn,F} = (1-\alpha) \left[R_{\delta_i} - \left(D_{i,\tau} - D_{i,\tau}^F \right) \right]$$
(4)

$$V_{i,\tau}^{rn,F} = w_i(q_i^*) \frac{1 - e^{-r\delta_i}}{r} - D_{i,\tau} - (1 - \alpha) \left[R_{\delta_i} - \left(D_{i,\tau} - D_{i,\tau}^F \right) \right].$$
(5)

These expressions are derived under the implicit assumption that the contract is renegotiated at date τ , and not further renegotiated beyond that date. This means that each partner anticipates that the regime (hence, the payoff) following the contractual breach will remain unchanged thereafter.²⁰ To interpret (4) and (5), it is necessary to understand how the renegotiation process unfolds. With probability α , G makes a take-it-or-leave-it offer to F; with probability $1 - \alpha$, F makes a take-it-or-leave-it offer to G. In either case, output is still set to q_i^* and G appropriates the entire net surplus of the activity, namely $w_i(q_i^*)\frac{1-e^{-r\delta_i}}{r}$, for the reasons previously illustrated. On the other hand, G faces the debt liability, which persists in the renegotiated contract, namely $D_{i,\tau}$. Moreover, when F makes the offer, G is extracted the whole benefit from renegotiation. This benefit, captured by the term $R_{\delta_i} - (D_{i,\tau} - D_{i,\tau}^F)$, is given by the replacement cost, net of the additional debt burden ling on G as the partnership is preserved, rather than being terminated on the initiative of F. When G makes the offer, F

²⁰The assumption that renegotiation takes place only once, following information release, enables us to keep the model tractable to analyse the relation between break-up and renegotiation. Allowing for repeated renegotiations would introduce complications in the attainment of contractual incentive-compatibility. Although this issue would deserve investigation, it is beyond the scope of the present work.

obtains a payoff of zero, that is, the same payoff it would get if the partnership were terminated.

G reneges on the contract Suppose that G reneges on the contract and that the partnership is terminated. Then, the partners' payoffs are given by:

$$\Pi^{b,G}_{i,\tau} = P \tag{6}$$

$$V_{i,\tau}^{b,G} = w_i(q_i^*) \frac{1 - e^{-r\delta_i}}{r} - R_{\delta_i} - \left(P + D_{i,\tau}^G\right).$$
(7)

As responsible for break-up, G owes both a penalty of P to F and a reimbursement of $D_{i,\tau}^G$ to L. If a new agreement is reached, instead, then F and G, respectively, obtain:

$$\Pi_{i,\tau}^{rn,G} = (1-\alpha) \left[R_{\delta_i} - \left(D_{i,\tau} - D_{i,\tau}^G - P \right) \right]$$
(8)

$$V_{i,\tau}^{rn,G} = w_i(q_i^*) \frac{1 - e^{-r\delta_i}}{r} - D_{i,\tau} - (1 - \alpha) \left[R_{\delta_i} - \left(D_{i,\tau} - D_{i,\tau}^G - P \right) \right].$$
(9)

Mutatis mutandis, these expressions replicate those in (4) and (5), up to the presence of P, which enters the payoff of F positively and that of G negatively. This is easily explained. By reaching a new deal, G escapes the termination payment to F, which it would need to make in case of break-up. This saving is extracted by the firm, when it makes the offer to G.

4 The programme

Assume that, at the contracting stage, F faces a zero outside opportunity and that effort is desirable.²¹ Let $\Delta \theta = \theta_h - \theta_l$, $\Delta \nu = \nu_1 - \nu_0$ and $\Delta \Pi = \Pi_{l,0} - \Pi_{h,0}$. Referring to $\Pi_{i,0}$, rather than to t_i , for all *i*, with a standard change of variable, the programme of G is written as follows:

$$\underset{\{q_{i},\Pi_{i,0};T_{i}\}_{i=l,h};\{M,D_{0}\};\{P,D_{0}^{F},D_{0}^{G}\}}{Max} \mathbb{E}\left[W_{i}\right],$$

subject to the participation constraint

$$\mathbb{E}\left[\Pi_{i,0}\right] \ge M + \psi,\tag{10}$$

²¹Effort is desirable when $\mathbb{E}[w(q_i^*)] - \widetilde{\mathbb{E}}[w(q_i^*)] > r\psi$, where \mathbb{E} and $\widetilde{\mathbb{E}}$ are the expectation operators over the two states l and h corresponding, respectively, to a = 1 and a = 0.

the information constraints

$$\Delta \Pi \ge \frac{\psi}{\Delta \nu} \tag{11}$$

$$\Delta \Pi \le \int_0^{r_l} \Delta \theta q_l e^{-rx} dx \tag{12}$$

$$\Delta \Pi \ge \int_0^{T_h} \Delta \theta q_h e^{-rx} dx, \tag{13}$$

the self-enforcing constraints

$$\Pi_{i,\tau} \ge \max\left\{\Pi_{i,\tau}^{b,F}, \Pi_{i,\tau}^{rn,F}\right\}, \ \forall i,\tau$$
(14)

$$V_{i,\tau} \ge \max\left\{V_{i,\tau}^{b,G}, V_{i,\tau}^{rn,G}\right\}, \ \forall i,\tau,$$
(15)

the limited-enforcement constraints

$$P \le \overline{P}, \ D_0 \le \overline{D}, \tag{16}$$

and the break-up liabilities constraints

$$\max\left\{D_0^F, D_0^G\right\} \le D_0.$$
(17)

The solution to this programme is the optimal self-enforcing contract.

Before turning to the characterization of the solution, we briefly describe the constraints. (10) prevents F from incurring losses in expectation. (11) is the moral-hazard constraint whereby F is prevented from shirking at the construction stage, provided a = 1 is desirable to G. This constraint requires that the amount of risk of $\Delta \Pi$, to which F is exposed if it operates in state h, rather than in state l, be large enough to motivate F to make state l more likely by exerting costly effort. (12) and (13) are the adverse-selection constraints whereby F is not tempted to announce, respectively, l in state h and h in state l. As from (12), a lie is prevented in state h, if the benefit of $\Delta \Pi$, induced by that lie, does not exceed the penalty that F would incur by understating the cost, which is given by the difference between the true high cost and the announced low cost, in each production period through date T_l . As from (13), information is released in state l, if the benefit of $\Delta \Pi$, which F appropriates by reporting l rather than h, is at least as large as the gain that F would obtain by exaggerating the cost, which is given by the difference between the fake high cost and the true low cost, in each production period through date T_h .²² (14) and (15) warrant that both break-up and renegotiation yield a lower

 $^{^{22}}$ The formulation of the adverse-selection constraints in (12) and (13) is reminiscent of the one that is found in repeated adverse-selection problems à *la* Baron and Besanko [5]. In those problems, private information is

payoff, respectively, to F and G, than the PPP contract. (16) states that, whether the contract is honoured or it is breached, payments must be set within the enforcement limits. Lastly, (17) ensures that, if the partnership is terminated prematurely, then the payment owed to L does not exceed the contractual debt burden.²³

4.1 Social efficiency

In a hypothetical situation with complete information and full commitment, constraints (11) - (16) do not appear in the programme. G only needs to properly select the triplets $\{q_l, t_l; T_l\}$ and $\{q_h, t_h; T_h\}$. In each state i = l, h, social efficiency is attained by setting the output at the level q_i^* as well as the profit and the termination date at the levels $\Pi_{i,0}^*$ and T_i^* , such that (10) is satisfied as an equality, involving that F breaks even in expectation.

The goal of our analysis is to identify conditions under which there exist pairs $\{\Pi_{l,0}^*, T_l^*\}$ and $\{\Pi_{h,0}^*, T_h^*\}$, together with financial variables $\{M, D_0\}$ and termination payments $\{P, D_0^F, D_0^G\}$, such that the optimal contract attains efficiency under asymmetric information and limited enforcement. Results are stated and discussed below.

5 Results

Information constraints (11) to (13) are formulated in terms of $\Delta \Pi$, T_l and T_h . Hence, it is convenient to refer to the triplet { $\Delta \Pi$, T_l , T_h }, rather than to the pairs { $\Pi_{l,0}$, T_l } and { $\Pi_{h,0}$, T_h }. This is possible because the profits of F, such that (10) is saturated, can be expressed as a function of $\Delta \Pi$:

$$\Pi_{l,0}^{*} = M + \psi + (1 - \nu_{1}) \Delta \Pi$$
(18a)

$$\Pi_{h,0}^* = M + \psi - \nu_1 \Delta \Pi.$$
(18b)

These expressions show that, apart from recovering the initial contribution of $M + \psi$, both monetary and non-monetary, F receives a "reward" of $(1 - \nu_1) \Delta \Pi$ in state l, and faces a "punishment" of $\nu_1 \Delta \Pi$ in state h. Once $\Delta \Pi$ is chosen in compliance with (11), (12) and (13)

not persistent and the agent makes a new report to the principal in each subsequent period. In our model, the marginal cost of operation is drawn once for all and the firm reports to the government only at date 0. However, a lie at that date can be assimilated to a repetition of the same lie, yielding the same output obligation and the same compensation right, in each subsequent period through the termination date. Essentially, what differs, in our setting, is that the number of periods, during which the firm could benefit from that lie, is endogenous to the contract.

 $^{^{23}}$ Together with (12) and (13), other constraints should be considered, whereby F does not misrepresent information, in view of a contractual breach. These constraints are here omitted because, as we show in Appendix B, they are implied by constraints (12) to (15). For settings where the additional adverse-selection constraints might be binding, see Bester and Strausz [7], Laffont [32], Guasch *et alii* [22] - [23].

are met as well, provided that the contractual terms are set as follows:

$$T_l \geq \underline{T}(\Delta \Pi) \equiv \frac{1}{r} \ln \frac{\Delta \theta q_l}{\Delta \theta q_l - r \Delta \Pi} \quad \text{if} \quad \Delta \Pi < \Delta \theta \frac{q_l}{r}$$
 (19a)

$$T_h \leq \overline{T}(\Delta \Pi) \equiv \frac{1}{r} \ln \frac{\Delta \theta q_h}{\Delta \theta q_h - r \Delta \Pi} \text{ if } \Delta \Pi < \Delta \theta \frac{q_h}{r}.$$
 (19b)

The condition $\Delta \Pi < \Delta \theta \frac{q_l}{r}$, which appears in (19a), is necessary for (12) to hold. If $\Delta \Pi \ge \Delta \theta \frac{q_h}{r}$, then (13) is satisfied, regardless of how T_h is set. Moreover, for (12) to hold together with (11), exerting effort must be not too costly to F, *i.e.*, we must have $\psi \le \Delta \nu \Delta \theta \frac{q_l}{r}$. As we are interested in identifying the conditions under which the contract decentralizes an efficient allocation, we assume that this is the case, indeed. Then, $\Delta \Pi$ must be chosen within the interval $\left[\frac{\psi}{\Delta \nu}, \Delta \theta \frac{q_l}{r}\right]$.

Lemma 1 The contract that stipulates an efficient allocation satisfies (11) to (13), if and only if the triplet $\{\Delta\Pi, T_l^*, T_h^*\}$ is such that $\Delta\Pi \in \left[\frac{\psi}{\Delta\nu}, \Delta\theta \frac{q_l^*}{r}\right], T_l^* \in [\underline{T}(\Delta\Pi), \infty)$, and either $T_h^* \in (0, \overline{T}(\Delta\Pi)]$, if $\Delta\Pi < \Delta\theta \frac{q_h^*}{r}$, or $T_h^* \in (0, \infty)$, otherwise. In particular, when $T_l^* = T_h^* \equiv T^*$, $T^* \in [\underline{T}(\Delta\Pi), \overline{T}(\Delta\Pi)]$.

The core insight conveyed by the lemma is that, in the presence of information problems, the range of feasible termination dates might be narrower, if the contract is bound to have a fixed term of T, than is if the contract has a state-dependent duration. Specifically, if moral hazard is not too strong, such that:

$$\psi < \Delta \nu \Delta \theta \frac{q_h^*}{r},\tag{20}$$

and if $\Delta \Pi \in \left[\frac{\psi}{\Delta \nu}, \Delta \theta \frac{q_h^*}{r}\right)$, then a fixed-term contract cannot have a duration longer than $\overline{T}(\Delta \Pi)$. By contrast, this is possible, in the good state, for a contract with a state-dependent term. Allowing for a time extension of $T_l - T$ does not make F more prone to exaggerate the cost, because the benefit induced by that lie depends on T_h , rather than on T_l . Moreover, while a fixed-term contract cannot have a duration shorter than $\underline{T}(\Delta \Pi)$, this is possible, in the bad state, for a contract with a state-dependent term. Imposing a time cut of $T - T_h$ does not make F more eager to understate the cost, because the penalty associated with that lie depends on T_l , rather than on T_h . From now on, for our study to be meaningful, we restrict attention to situations in which (20) holds, hence it is possible to set $T_l > \overline{T}(\Delta \Pi)$.

To understand what the best strategy is to incentivize the two partners to honour the contract, first recall that, following to a contractual renege, the negotiation between G and F concerns the per-period quantity q_i and the residual profit $\prod_{i,\tau}$, the state *i* being commonly known, at that stage. We saw that, if the contract is breached, then, for either partner, the optimal level of production is q_i^* , which maximizes the operating return $w_i(q_i) \frac{1-e^{-r\delta_i}}{r}$ to be shared in the negotiation process. In consideration of this, we can only concentrate on profits.

Provided a profit of $\Pi_{i,0}^*$ is stipulated in the contract, its residual value (*i.e.*, the contractual operating profit of F) and, accordingly, the contractual payoff of G, at date $\tau \in (0, T_i)$, amount to:

$$\Pi_{i,\tau}^* = \Pi_{i,0}^* \frac{1 - e^{-r\delta_i}}{1 - e^{-rT_i}}$$
(21)

$$V_{i,\tau}^{*} = w_{i}(q_{i}^{*}) \frac{1 - e^{-r\delta_{i}}}{r} - \left(\Pi_{i,\tau}^{*} + D_{i,\tau}\right), \qquad (22)$$

where $D_{i,\tau} = D_0 \frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}$. We need to check whether $\Pi_{i,0}^*$ can be set such that $\Pi_{i,\tau}^*$, together with $V_{i,\tau}^*$, satisfies the self-enforcing constraints in all periods through the termination date. If it is so in either state, then we can conclude that the optimal contract does effect an efficient allocation, just as in the seminal work of Harris and Raviv [24], in which the contract is signed ex ante and the parties fully commit to the contract. The core issue, here, is that the lack of enforceability introduces a limit on the liabilities to which the firm can be exposed. If, in some state of nature, the contractual operating profit falls too low, at some date τ , then F will be unwilling to continue to honour the contract. On the other hand, G may be unsatisfied with the contractual return, which is lower the higher are profit and debt burden.

Resting on (21) and (22) and recalling (18a) and (18b), one can identify two crucial effects, which the private contributions to the project have on the partners' contractual payoffs, hence on their *ex-post* decisions. On the one hand, debt reduces the contractual payoff of G, whereas it does not affect the profit of F. This is because F acts just as an intermediary between G and L, transferring money from the former to the latter. Hence, although the loan is taken out by F, the debt burden ultimately lies on G. On the other hand, as $\Pi_{i,0}^*$ increases with M, the own investment raises the payoff of F. By contrast, just as debt, it reduces the payoff of G. Given these contrasting effects, it is not evident that appropriate values of the private contributions can be found, such that (14) and (15) are both satisfied in either state.

Debt liabilities Let us begin with the debt. We shall assess how large it should be chosen, for the self-enforcing constraints to be met. It should be considered that, for either partner, the debt determines the benefit to be obtained, when reneging on the contract. The following equivalences hold:

$$\Pi_{i,\tau}^{b,F} \geq \Pi_{i,\tau}^{rn,F} \Leftrightarrow D_0 - D_0^F \geq R_{\delta_i} \frac{1 - e^{-rT_i}}{1 - e^{-r\delta_i}}$$

$$\tag{23}$$

$$V_{i,\tau}^{b,G} \geq V_{i,\tau}^{rn,G} \Leftrightarrow D_0 - D_0^G \geq R_{\delta_i} \frac{1 - e^{-rI_i}}{1 - e^{-r\delta_i}}.$$
(24)

They involve that, for the renegotiation game to result in a break-up, L must be exposed to the risk associated with F or G behaving opportunistically. That is, termination payments should be sufficiently little, relative to debt. This is explained as follows. As compared to the case of successful renegotiation, in the event of break-up, G saves an amount of $D_{i,\tau} - D_{i,\tau}^F =$ $(D_0 - D_0^F) \frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}$, if F reneges; an amount of $D_{i,\tau} - D_{i,\tau}^G = (D_0 - D_0^G) \frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}$, if G itself reneges. When these savings are large enough, G is better off by terminating the partnership, despite that it will incur a cost of R_{δ_i} . In turn, under these circumstances, there is no benefit that F can extract from G, by proposing renegotiation. Hence, the partnership is terminated, indeed. Noticeably, it is because L is a *third* party to the relationship between G and F, that conditioning the payments to L on the *ex-post* behaviour of F and G affects the outcome of the renegotiation game.

We now need to understand whether it is actually convenient to expose L to risk and, if so, whether that is the case regardless of which partner reneges. At a first glance, as breakup represents the status-quo point in the renegotiation game, one might deduce that, for the self-enforcing constraints to be relaxed, the best is to have (23) and (24) satisfied jointly, in which case no partner can extract more than the break-up payoff from the other. While this is true with regards to F, it is not with regards to G. The reason is that, if G reneges and break-up follows, then G saves an amount of $(D_0 - D_0^G) \frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}$, in terms of debt payment. If the contract is renegotiated, instead, then this benefit is to be shared with F. Hence, for G, the saving only amounts to $(1 - \alpha) (D_0 - D_0^G) \frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}$. Therefore, in order to make the contract self-enforceable, it is better to create conditions under which G would prefer renegotiation to break-up.

Knowing that the best is to have (23) met and (24) violated, the debt stipulated in the contract affects (14) and (15) in opposite ways. It is possible to reconcile these two effects, provided F and G do not display incentives to renege in the same state of nature.

Lemma 2 (14) holds $\forall i$, only if it holds for i = h. (15) holds $\forall i$, only if it holds for i = l.

It is intuitive that F is tempted to renege in the bad state, in which it receives a low profit, whereas G is tempted to renege in the good state, in which it owes a high profit to F. Under these circumstances, it is possible to have (23) satisfied and (24) violated, only if the debt profile is differentiated between states, *i.e.*, $D_{h,\tau} \neq D_{l,\tau}$. This is the case, if and only if the contractual term is not fixed. **Lemma 3** (14) and (15) are weakest when $\{D_0, D_0^F, D_0^G\}$ and $\{T_l, T_h\}$ are such that:

$$D_0^F = 0, \ D_0^G = D_0 \tag{25}$$

$$D_0 \geq R_{\delta_h} \frac{1 - e^{-r\delta_h}}{1 - e^{-r\delta_h}}, \ \forall \delta_h \in (0, T_h)$$
(26)

$$D_0 \leq R_{\delta_l} \frac{1 - e^{-rT_l}}{1 - e^{-r\delta_l}}, \ \forall \delta_l \in (0, T_l).$$

$$(27)$$

(26) and (27) are jointly satisfied, only if $T_l > T_h$.

The lemma suggests that ideal is to expose L to risk, conditional on the *ex-post* actions of F only. Specifically, if, at a certain point during the operation phase, F reneges and the partnership is terminated, then L will forego the debt payment it was supposed to receive. Notice that this result provides a justification for relying on private debt in PPP projects. It is precisely because the loan is taken out by the firm, rather than by the government, that the debt can remain unpaid, when the firm defaults on the activity. Instead, there is no point to expose L to risk related to the *ex-post* actions of G. If the PPP is terminated on the initiative of G, then the best strategy is to reimburse L entirely, in which case the incentive of G to renege is weakest. As this requires setting $T_l > T_h$, it is already suggestive of the main result of the study, which we will state in the proposition below.

Investment of the firm Before turning to the main result, we need to consider the firm's contribution to the project. Specifically, we need to establish under which conditions values of M can be found, such that (14) holds in state h, together with (15) in state l.

Lemma 4 $\exists M \geq 0$ such that (14) and (15) are satisfied jointly, only if:

$$P \ge \Delta \Pi \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} - (1 - \alpha) R_{\delta_l}, \ \forall \delta_l \in (0, T_l), \ \forall T_l.$$

$$(28)$$

Lemma 4 identifies a lower bound to the breach penalty to be set in favour of F. At any given date τ , at which the contract could be reneged upon, the exact level of the bound depends on the contractual length in the good state only. This looks intuitive, if it considered that G owes money to F, only if renege occurs on its own initiative, which is a relevant possibility when F operates at a low cost. Observe that, when the contract has a fixed term of T, it is $\Delta \Pi \frac{1-e^{-r\delta}}{1-e^{-rT}} = \Pi_{l,\tau}^* - \Pi_{h,\tau}^*$. Accordingly, when $T_l \neq T_h$, the expression $\Delta \Pi \frac{1-e^{-r\delta_l}}{1-e^{-rT_l}}$ represents a measure of the discounted wedge between the residual profits of F, "normalized" as if the termination date were T_l in the two states. This normalization is required because, while P is a lump-sum transfer, to be possibly made in state l, profits are actually paid on a per-period basis, through the respective termination dates. Resting on this, (28) can be interpreted by recalling that too high a value of $\Pi_{l,\tau}^*$ encourages G to renege, whereas too low a value of $\Pi_{h,\tau}^*$ encourages F to renege. All else equal, when the residual profit wedge is large, it is difficult to lessen the opportunism of G, without exacerbating that of F. Then, the breach penalty is to be raised, to that end. This requirement is, nonetheless, weakened by the presence of the replacement cost. If renegotiation succeeds, then G is to concede an amount of $(1 - \alpha) R_{\delta_l}$ to F. By not breaching the contract, G saves that amount. Therefore, R_{δ_l} and P are substitutes in motivating G to abide by the contract. The higher is R_{δ_l} , the more does G prefer the initial contract to a new deal, the less useful is the penalty, the lower can it be set.

Lemma 5 For $T_l = T_h \equiv T$, Lemma 3 does not apply. Then, $\exists M \ge 0$ such that (14) and (15) are satisfied jointly, only if:

$$P \ge \Delta \Pi \frac{1 - e^{-r\delta}}{1 - e^{-rT}}, \ \forall \delta \in (0, T), \forall T.$$

$$(29)$$

As from Lemma 3, there is no way to make renegotiation preferable to break-up for G, when the contractual term is fixed. Knowing that this is a situation in which it is more difficult to incentivize G to behave in a virtuous manner, it is not surprising that (29) is tighter than (28).

Resting on Lemma 4 and 5, it is easy to see that, when the penalty P cannot be set high, it is necessary to differentiate the termination date between states, in order to have Lemma 3 satisfied. As the conditions in Lemma 3 must hold for any value of the replacement cost, the analysis cannot be finalized, without considering the properties of the function R_x . Henceforth, to ease exposition, we assume that either $\frac{R'_x}{R_x} \leq \frac{re^{-rx}}{1-e^{-rx}}$ or $\frac{R'_x}{R_x} \geq \frac{re^{-rx}}{1-e^{-rx}}$, for all x > 0, *i.e.*, the value of the replacement cost, discounted at date 0, is either non-increasing or non-decreasing in x. It involves that the difference $(D_{i,\tau} - R_{\delta_i})$ is monotonic for all $\delta_i \in (0, T_i)$ and all $T_i > 0$. Under this assumption, each of the two conditions in Lemma 3 is tightest either for $\delta_i = 0$ or for $\delta_i = T_i$. Once it is checked that each such condition holds for the value of δ_i for which it is tightest, we are assured that it holds for all values of δ_i .

Further denoting $\Upsilon(R_T)$ the inverse function of R_T , which is increasing in T, and $\Psi(D_0) = \frac{1}{r} \ln \frac{R'_0}{R'_0 - r(1+\lambda)D_0}$, we are now ready to state the main result of our study.

Proposition 1 (I) Suppose that $\overline{P} \geq \frac{\psi}{\Delta \nu}$. Then, an efficient allocation is implementable, with a fixed-term contractual policy $(T_l^* = T_h^* \equiv T^*)$, if and only if:

$$\underline{T}\left(\frac{\psi}{\Delta\nu}\right) \le T^* \le \overline{T}\left(\frac{\psi}{\Delta\nu}\right).$$

(II) Suppose that $\overline{P} < \frac{\psi}{\Delta \nu}$. Then, an efficient allocation is implementable, if and only if

 $\Upsilon\left(\frac{1}{1-\alpha}\left(\frac{\psi}{\Delta\nu}-\overline{P}\right)\right)$ is finite and $T_l^* > T_h^*$, such that:

$$T_{h}^{*} \leq \min\left\{\Psi\left(D_{0}\right); \Upsilon\left(D_{0}\right); \overline{T}\left(\frac{\psi}{\Delta\nu}\right)\right\}$$

$$(30)$$

$$T_{l}^{*} \geq \max\left\{\Psi\left(D_{0}\right); \Upsilon\left(D_{0}\right); \underline{T}\left(\frac{\psi}{\Delta\nu}\right); \Upsilon\left(\frac{1}{1-\alpha}\left(\frac{\psi}{\Delta\nu}-\overline{P}\right)\right)\right\}.$$
(31)

The main prediction of the proposition is that, in environments in which only small penalties are enforceable, there are *two reasons* for which the contractual length should be differentiated between states. First, to ensure that risk transfer to the lender is conditioned on the *ex-post* behaviour of the firm (Lemma 3), the two termination dates must be drawn from two ranges of values, separated by the thresholds $\Upsilon(D_0)$ and $\Psi(D_0)$, in such a way that T_h^* does not exceed the smaller of the two and T_l^* does not lie below the larger of the two. Formally, $\Upsilon(D_0) \leq \Psi(D_0)$ if and only if $\frac{R'_x}{R_x} \geq \frac{re^{-rx}}{1-e^{-rx}}$. Second, while setting T_l^* large makes G more likely to behave in a virtuous manner in the good state, it does not trigger cheating in that same state (as could be with a fixed term), provided that T_h^* is set small, instead. Formally, this second benefit of a state-dependent duration appears when $\Upsilon\left(\frac{1}{1-\alpha}\left(\frac{\psi}{\Delta\nu}-\overline{P}\right)\right)$ is larger than $\overline{T}\left(\frac{\psi}{\Delta\nu}\right)$. Therefore, the double benefit, which a state-dependent duration delivers, follows from that, first, the lender is a third party to the relationship between the government and the firm and, second, the replacement cost depends on the residual contractual length.

The necessity of $\Upsilon\left(\frac{1}{1-\alpha}\left(\frac{\psi}{\Delta\nu}-\overline{P}\right)\right)$ being finite shows the limits of this contractual policy. Provided the breach penalty and the replacement cost are substitutes, in mitigating the temptation of G to renege, the lower the enforceable penalty \overline{P} is, the higher the replacement cost should be, for the contract that stipulates an efficient allocation to be honoured. By contrast, thanks to the possibility of adjusting contractual terms, it does not matter how small \overline{D} exactly is, as long as some (even negligible) debt payment can be enforced so that, as is essential, external financiers are indeed available to concede a loan. The only consequence to \overline{D} being small is that the ranges of feasible values of T_l and T_h are narrower. Self-enforceability is, nonetheless, at hand. The following corollary formalizes this point.

Corollary 1 $\exists D_0, T_l^*, T_h^*$ such that Proposition 1 holds for $\overline{P} = 0$, if and only if $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$ is finite. If $\overline{P} = 0$, $\overline{D} \to 0$, and $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$ is finite, then $\exists T_l^*, T_h^*$ such that Proposition 1 holds.

This result has an important practical implication. When replacing the firm is sufficiently costly to the government (so that $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$ is finite), the contract can be made self-enforcing, even without any termination penalty in favour of the firm, and even if the court of justice can only provide a very limited protection to the lender.

6 From fixed to state-dependent duration: a flexibility gain

The results in Proposition 1 and Corollary 1 depend essentially on whether or not Lemma 4 holds. For this to be the case, F should be faced with as little risk as possible. Formally, this involves binding (11). Thus, the optimal risk transfer to F amounts to $\Delta \Pi = \frac{\psi}{\Delta \nu}$. Now recall from Lemma 1 that, given a certain value of $\Delta \Pi$, G enjoys more flexibility at adjusting the two termination dates of T_l and T_h than a fixed term of T. In this section, we show that, to keep the risk transfer equal to $\Delta \Pi = \frac{\psi}{\Delta \nu}$, adjustments in the contractual terms must be compensated with adjustments in the per-period profits of F, and that the flexibility gain aforementioned consists precisely in the enhanced possibility of compensating those adjustments, when the duration of the contract is state-contingent. By formalizing the "substitutability" between contractual terms and per-period profits, we will also be able to provide a graphical illustration of the second benefit of a state-dependent duration, which we emphasized in the comment to Proposition 1.

For the profit wedge to be such that (12) and (13) are satisfied, it must take the following expression:

$$\Delta \Pi = \Delta \theta \int_0^{T_j} z_j e^{-rx} dx, \qquad (32)$$

where $j \in \{l, h\}$, not necessarily coinciding with the true state *i*, and with the additional requirement that $\int_0^{T_l} z_l e^{-rx} dx = \int_0^{T_h} z_h e^{-rx} dx$. Expressed in this way, $\Delta \Pi$ denotes the cumulated discounted wedge between the per-period rewards and punishments to be faced by F through the termination date. This per-period measure is "normalized" as if that date were either T_l in both states or T_h in both states. Specifically, for j = l, (32) means that a per-period reward of $(1 - \nu_1) \Delta \theta z_l$ is granted to F for T_l periods, where $z_l \in \left(\frac{r\psi}{\Delta\nu\Delta\theta}, q_l^*\right]$; for j = h, it means that a per-period punishment of $\nu_1 \Delta \theta z_h$ is inflicted to F for T_h periods, where $z_h \in [q_h^*, \infty)$. When deciding about $\Delta \Pi$ and T_l , G is basically choosing z_l , hence the per-period reward in a contract with duration T_l . Symmetrically, when deciding about $\Delta \Pi$ and T_h , G is choosing z_h , hence the per-period punishment in a contract with duration T_h . By extending the duration T_l beyond $\overline{T}\left(\frac{\psi}{\Delta \nu}\right)$, G is able to decrease the per-period reward below the minimum value of $(1 - \nu_1) \Delta \theta q_h^*$, which is feasible with a fixed term. Analogously, by shortening the duration T_h below $\underline{T}\left(\frac{\psi}{\Delta\nu}\right)$, G can raise the per-period punishment above the maximum value of $\nu_1 \Delta \theta q_l^*$, which is feasible with a fixed term. In either case, the flexibility gain is related to the adverseselection constraints, which can be relaxed by adjusting the contractual term in the exact state in which a lie would be convenient for F.



Figure 1: Ranges of values of T_l^* and T_h^* when $R_x > \frac{1}{1-\alpha} \frac{\psi}{\Delta \nu} (1-e^{-rx}) > \Delta \theta q_h^* \frac{1-e^{-rx}}{r}$.

Using (32) and taking, for simplicity, $\overline{P} = 0$, we can rewrite (28) as:

$$R_{T_l} \ge \frac{1}{1-\alpha} \Delta \theta z_l \frac{1-e^{-rT_l}}{r}, \ \forall T_l > 0.$$

$$(33)$$

This says that the replacement cost, faced by G when the residual contractual length is T_l , must not fall below the profit wedge of $\Delta \theta z_l \frac{1-e^{-rT_l}}{r}$, as inflated by the ratio $\frac{1}{1-\alpha} > 1$. Considering that $1 - \alpha$ captures the bargaining power of F in the renegotiation game, it involves that, all else equal, self-enforceability of the contract is easier to attain the stronger is that power. Intuitively, the more solid is the position of F, when the partners negotiate, the more surplus can F extract from G, hence the less attractive is renegotiation for G and the less eager will G be to renege on the contract seeking a new deal. The formulation in (33) evidences that, in the end, given a per-period reward of $(1 - \nu_1) \Delta \theta z_l$ granted to F for T_l periods, set to tackle information problems, the possibility of making the contract self-enforcing only depends on the contractual period T_l .

Referring to $x \in (0, \infty)$ as to the relevant time interval, Figure 1 illustrates the situation, presented in Proposition 1, in which $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$ is finite and larger than $\overline{T}\left(\frac{\psi}{\Delta\nu}\right)$. By specifying the necessary condition (28) as (33), we see that there exists some finite value of T_l , such that $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$ is finite, when, for all x, the replacement cost of R_x is larger than $\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\left(1-e^{-rx}\right)$. Up to the ratio $\frac{1}{1-\alpha}$, this is the minimum value of the profit wedge consistent with the information constraints. Recall that the minimum profit wedge is attained when the per-period reward of F tends to a value of $(1-\nu_1)\frac{r\psi}{\Delta\nu}$. A per-period reward, set larger than this value, can be reduced to allow for a raise in T_l . The area between the curve $\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\left(1-e^{-rx}\right)$ and the horizontal line $\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}$, to the right of $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$, is basically the area within which G has the flexibility to adjust the per-period reward by inducing changes in the contractual length, given the amount of risk of $\Delta\Pi = \frac{\psi}{\Delta\nu}$ to be imposed to F. In the graph, $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right) > \overline{T}\left(\frac{\psi}{\Delta\nu}\right)$ because, for all values of x, the replacement cost is larger than $\Delta\theta q_h^* \frac{1-e^{-rx}}{r}$. This is the value that the profit wedge takes, when the per-period punishment in the bad state is set to $\nu_1 \Delta \theta q_h^*$, which is the lowest value such that F truth-tells in that state. The graph thus highlights the necessity of setting $T_l > T_h$, in order to address enforcement issues together with information issues.

7 Contract enforceability and financial institutions

Proposition 1 and Corollary 1 suggest that, in environments in which the judicial power is very poor, it is essential that early replacement of the firm be so costly to the government that the value of $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$ is finite. The contract could not be made self-enforcing otherwise.

In consideration of this, one may wonder whether financial institutions could be attached a role in promoting efficient contractual outcomes, when the judicial power is poor and early replacement of the firm is not costly enough to prevent opportunism. Such institutions could be, for instance, Export Credit Agencies (ECAs). These private or quasi-governmental entities, providing cover to lenders against borrowers' payment defaults, are now deeply involved in PFI/PPP projects worldwide, particularly in large infrastructure and industrial projects, which are risky and highly capital-intensive, and have long gestation periods. Being diversified in country risks, ECAs are so well placed to cover political and non-commercial risks, including expropriation and breach of contract, that they often afford to back projects which the World Bank Group and other multilateral banks find too risky to support.²⁴

The purpose of this section is to show that, indeed, institutions like ECAs can help make

²⁴Originally created to facilitate exports of goods and services, ECAs have began to operate in project financing starting from the Nineties. In recent years, they have supported between US \$50 and 70 billion annually, in medium and long-term transactions in developed and, above all, developing countries. Many EU member countries have increased the financial capacity of their official ECAs, in order to remedy the serious lack of provision for financing trade transactions, created by the global financial crisis. See Sader [39] for the role that both bilateral and multilateral ECAs play to provide political risk insurance by pledging guarantees on the debt package in the realization of BOT-type infrastructure projects realized in developing countries. For detailed information about ECAs activities, visit: http://www.eca-watch.org/. The OECD maintains a list of all official ECA websites, where data can be found about ECAs in different countries, at: http://www.oecd.org/trade/exportcredits/eca.htm.

PPP contracts successful. From our previous findings, this is not apparent as the value of $\Upsilon\left(\frac{1}{1-\alpha}\frac{\psi}{\Delta\nu}\right)$ is unrelated to the debt value.

First recall that the debt has opposite effects on the incentives of G and F to renege on the contract. However, this is only due to the fact that, when renegotiation succeeds, the liability ling on G is still D_0 , as in the initial contract. We now assume that, when drafting the contract, G and F stipulate that, should a new agreement be reached in some state i at some date τ , the liability would be equal to some amount of $D_{i,\tau}^{rn}$, such that $D_{i,\tau}^{rn} = D_0^{rn} \frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}$, rather than to $D_{i,\tau} = D_0 \frac{1 - e^{-r\delta_i}}{1 - e^{-rT_i}}$. To fix ideas, imagine that, at the time when the contract is signed, G deposits an amount of D_0^{rn} up-front with the ECA, which will then releases an amount of $D_{i,\tau}^{rn}$ to L, when the new agreement is reached. We further assume that it is possible to set $D_0^{rn} > D_0$. Under this arrangement, the date-0 value of the payment to L is D_0 , if the PPP contract is executed; it is D_0^{rn} , if a new deal is reached; it is equal to zero, if the PPP is terminated prematurely, regardless of who reneges on the contract. This means that, just as before, L is remunerated if the partnership remains in place, whereas it is not if the partnership is broken up. However, because L is exposed to the risk of not being paid, provided the partnership may be terminated if some partner reneges, it is now assigned a risk premium, with a discounted value of $D_0^{rn} - D_0$, in the event of a successful renegotiation. The conditions under which the two partners are discouraged from reneging, namely (26) and (27), are replaced by:

$$D_0^{rn} \geq R_{\delta_h} \frac{1 - e^{-rT_h}}{1 - e^{-r\delta_h}}, \ \forall \delta_h \in (0, T_h)$$

$$(34)$$

$$D_0^{rn} \geq R_{\delta_l} \frac{1 - e^{-rT_l}}{1 - e^{-r\delta_l}}, \ \forall \delta_l \in (0, T_l).$$
(35)

The core difference, with respect to the previous setting, is that, by letting D_0^{rn} be independent of D_0 , the best is now to make G better off if the partnership is terminated, than if a new agreement is reached, when it reneges on the contract. Anticipating that the best it could obtain is the break-up payoff, G is less eager to renege. Therefore, the need to lessen the opportunism of G is more handily reconciled with that to lessen the opportunism of F. Formally, (28) is replaced by the weaker condition:

$$P \ge \Delta \Pi \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} - R_{\delta_l}, \ \forall \delta_l \in (0, T_l), \ \forall T_l.$$

$$(36)$$

This leads us to draw the result below. In order to emphasize the important role that financial institutions like ECAs can play, despite that they can only intervene to enforce debt payments, the result is stated for $\overline{P} = 0$, thus ruling out any further enforcing mechanism.

Proposition 2 Suppose that (34) and (35) are satisfied, and that $\overline{P} = 0$. Then, $\exists T_l^*, T_h^*$ such that the efficient allocation is implementable, if and only if $\Upsilon\left(\frac{\psi}{\Delta\nu}\right)$ is finite. If $\Upsilon\left(\frac{\psi}{\Delta\nu}\right) > \overline{T}\left(\frac{\psi}{\Delta\nu}\right)$,

then the efficient allocation is implemented, only if $T_l^* > T_h^*$.

When relying on an agency, the lower bound on T_l , namely $\Upsilon\left(\frac{\psi}{\Delta\nu}\right)$, is smaller than in situations where no agency is involved. Consequently, if there exists no finite value of T_l satisfying Proposition 1 and Corollary 1, such a value might be found when an ECA is called upon to intervene in the PPP.

The finding that the efficient allocation cannot be implemented by means of a fixed-term contract, if $\Upsilon\left(\frac{\psi}{\Delta\nu}\right) > \overline{T}\left(\frac{\psi}{\Delta\nu}\right)$, is reminiscent of the content of Corollary 1 in Danau and Vinella [9]. Here, we allow for $T_l > \overline{T}\left(\frac{\psi}{\Delta\nu}\right)$ and it comes out that it is desirable to differentiate the contractual length in the two states also when an agency is involved. As above, the result is illustrated by rewriting (36), for P = 0, as follows:

$$R_{T_l} \ge \Delta \theta z_l \frac{1 - e^{-rT_l}}{r}, \ \forall T_l > 0.$$
(37)

This condition is the same as (33), except that the ratio $\frac{1}{1-\alpha}$ no longer appears in the right-hand side. This is because the bargaining power of F in the renegotiation process does not matter, in this framework, as G would not seek a new agreement, should it renege on the contract. A graphical illustration is provided in Figure 2, referring again to $x \in (0, \infty)$ as to the relevant time interval. Like in the previous setting, if the contractual term were fixed, then the smallest feasible value of the residual profit wedge would be $\Delta \theta q_h^* \frac{1-e^{-rx}}{r}$. By allowing for $T_l > \overline{T} \left(\frac{\psi}{\Delta \nu}\right)$, as we explained, the wedge can be decreased below that value, for any x. Then, it is possible to find a finite value of T_l , greater than $\overline{T} \left(\frac{\psi}{\Delta \nu}\right)$, such that the contract is self-enforcing, if and only if the replacement cost R_x exceeds the smallest feasible value the residual profit wedge can take with a state-dependent duration, namely $\frac{\psi}{\Delta \nu} \left(1 - e^{-rx}\right)$. When this is the case, we can identify the shortest term the contract can have in state l as being $\Upsilon \left(\frac{\psi}{\Delta \nu}\right)$, *i.e.*, the value of xsuch that $R_x = \frac{\psi}{\Delta \nu}$.

To conclude the analysis, one last point is worth making. Conditions (28) and (36) cannot be relaxed by allowing the per-period profits to follow a dynamic pattern. To see this, take (36), for instance, and rewrite it for P = 0 and $T_l = T_h \equiv T$:

$$R_{\delta} \ge \Pi_{l,\tau}^* - \Pi_{h,\tau}^*, \ \forall \delta \in (0,T) \,. \tag{38}$$

Inspection of (38) emphasizes that what really matters, for self-enforceability of the contract, is the relationship between replacement cost and residual profits, both referred to the entire residual contractual length, rather than to each single period. Suppose that the per-period profit is let vary over time. This involves allowing for a per-period reward of $(1 - \nu_1) \Delta \theta z_{x-\tau}$ and a per-period punishment of $\nu_1 \Delta \theta z_{x-\tau}$, where $z_{x-\tau}$ depends on the residual time length of



Figure 2: Ranges of values of T_l^* and T_h^* when $\Delta \theta q_h^* \frac{1-e^{-rx}}{r} > R_x > \frac{\psi}{\Delta \nu} (1-e^{-rx})$ and (34) and (35) hold.

 $x - \tau$, for all possible $x \in (\tau, T)$. Accordingly, $\Pi_{l,\tau}^* - \Pi_{h,\tau}^* = \int_0^{\delta} z_x e^{-rx} dx$. Taking $\delta \to T$, (13) is written, in this framework, as $\int_0^T z_x e^{-rx} dx \ge \int_0^T q_h^* e^{-rx} dx$, and (38) as $R_{\delta} \ge \int_0^T z_x e^{-rx} dx$. These two conditions hold jointly, only if $\Upsilon\left(\frac{\psi}{\Delta\nu}\right) \le \overline{T}\left(\frac{\psi}{\Delta\nu}\right)$, as from Proposition 2.

8 Relation to the literature

Contracts with a state-dependent duration Engel *et alii* [11] show that conditioning the contractual length on the state of nature represents an effective tool to ensure that there is no discrepancy in the streams of profits the firm will obtain in the different states, as the project is developed. They argue that, the firm being fully insured, *ex-post* renegotiations will be avoided. In line with these authors, we find that the firm should be exposed to a limited risk. In order to lessen the partners' opportunism, the wedge between the streams of profits the firm will cumulate in the two states should be set as little as possible. Specifically, our analysis predicts that the best is to let the firm bear just enough risk to prevent shirking in construction. However, unlike in Engel *et alii* [11], the risk transfer to the firm does not

need to change as the contractual length is adjusted in the different states. This is because, in our framework, per-period profits are *endogenous*. We showed that, given a certain risk transfer, per-period profits and contractual length are substitutes. Thus, once the risk transfer is downsized to the minimum that amount of risk can be maintained, when inducing changes in the contractual terms, by compensating those changes with *variations in the wedge between per-period* (*rather than cumulated*) *profits*. This proves dramatically useful to incentivize the partners, in environments in which enforcement problems coexist with information problems. To fix ideas, suppose that the risk transfer were minimized by adjusting the contractual length, while keeping per-period profits fixed, as in flexible-term contracts. Then, in each period, the firm would earn so much in the good state and so little in the bad state that it would not be possible to prevent *ex-post* opportunistic actions. In particular, regardless of the insurance received in the contract, the firm would be tempted to renege in the bad state, pursuing a more convenient deal.

Renegotiation of incentive contracts Most studies on long-term contractual relationships focus on repeated interactions between buyers and sellers. Dewatripont [10], Hart and Tirole [26], Rey and Salanié [38] show that the parties to an incentive contract, signed at *interim*, may want to renegotiate the allocation initially stipulated, once private information is revealed. This desire arises because, under complete information, a Pareto-improving allocation is available to the parties. To prevent this outcome, the contract must be made robust to renegotiation. In our setting, the *status-quo* point of the renegotiation process is the break-up of the relationship. Thus, to prevent renege, the contract must be made robust to both renegotiation and breakup. Furthermore, as the contract is signed *ex ante*, it stipulates an efficient allocation (a result which traces back to Harris and Raviv [24]), unless self-enforcing constraints are binding. Thus, if a new deal is ever reached *ex post*, then the allocation to which it leads is Pareto-improving on the break-up allocation, not on the contractual allocation.

Off equilibrium payoffs in long-term relationships Abreu [1] shows that, in repeated games, it is possible to induce some equilibrium path, by setting penal codes for off equilibrium strategies, which lead to the worst outcome for each player. In line with Abreu [1], Levin [34], with regards to relational contracts, and Martimort et alii [36], in a context of dynamic buyer-seller relationships, specify that the worst equilibrium is played, following a contractual breach. In the same vein, in our setting, for the partners to be motivated to honour the contract, the surplus to be shared in a hypothetical ex-post negotiation should be minimized (if not destroyed). To that end, the strategic tool to be used is the debt payment to be made to the lender, conditional on the partners' ex-post actions. However, as the lender is a third party to the government-firm relationship, the worst possible outcome should be induced, only if the firm

reneges on the contract. It should not be induced, instead, if the government reneges. This is what leads to the first benefit of a state-dependent duration that we identified. Referring to situations where no court of justice can warrant contractual enforcement, Fuchs [18] shows that the introduction of a third party is helpful to keep a long-term relationship in place. In line with that study, in our framework, the key point is that, following a contractual breach, some player is to make a payment to the third party, thus eliminating the stake potentially at hand off the contractual equilibrium.

Efficiency of self-enforcing long-term contracts Levin [34] shows that the result of Harris and Raviv [24] that efficiency is attained with *ex-ante* contracting, can be obtained under limited enforcement as well. We found a similar result in a framework in which, unlike in Levin [34], adverse selection follows moral hazard. In this framework, to achieve the goal, it is necessary to condition the contractual length on the state of nature. This facilitates the task of addressing the two information problems, while also keeping the partners' opportunism under control.

9 Concluding remarks

There are two essential lessons, on PPPs in infrastructure projects, to be drawn from our analysis. First, what causes contracts with a state-dependent duration to perform better than fixed-term contracts, is the concomitant inability of *both* the firm and the government to commit to contractual obligations, in frameworks where, due to moral-hazard concerns, the profit scheme must be designed such that some risk is transferred to the firm. This conclusion would not carry over, if only one partner were unable to commit, as is often assumed in the literature on delegation of public projects. While, in practice, we observe that either the firm or the government reneges on the contract, *a priori*, it is reasonable that, if one partner can take that initiative, then also the other partner has the same possibility. Second, when each of the two partners is to be incentivized to abide by the contract, it is necessary to move away from the flexible-term approach previously proposed. Rather than letting the firm manage the activity for a longer period when it faces unfavourable conditions, the contract should be lengthened when conditions are favourable.

Our study evidenced that the conflict between opportunistic interests of the partners is stronger at early stages in the operation phase. This might explain why, in PPPs, it is often the case that contractual frictions arise early on during the execution of the contract. Our results suggest that conditioning the contractual length on the state of nature can be a useful tool to mitigate early vulnerability of PPP contracts.

The essential link between the financial structure of the infrastructure project and the

duration of the PPP contract, detected in our analysis, justifies both the reliance on private investments, in the double form of firm's capital and bank loans, and the adoption of a statedependent contractual term. The duration of the contract must be conditioned on the state precisely because this contractual policy allows to fine-tune the firm's contribution and the debt liabilities vis-a-vis the lender, in such a way that the partners' opportunism is lessened.

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A The renegotiation game

A.1 Break-up

It is straightforward to compute $\Pi_{i,\tau}^{b,F}$, $\Pi_{i,\tau}^{b,G}$, $V_{i,\tau}^{b,F}$ and $V_{i,\tau}^{b,G}$ and to see that $V_{i,\tau}^{b,F}$ and $V_{i,\tau}^{b,G}$ are maximized, respectively, by setting output to q_i^* .

A.2 Renegotiation

Suppose that, after some party reneges at date τ in state *i*, renegotiation succeeds. The variables newly determined, through the renegotiation process, are $\{q_i^{ar}, \Pi_{i,\tau}^{ar}\}$. This pair of variables will replace the contractual pair $\{q_i^*, \Pi_{i,\tau}^*\}$.

A.2.1 G makes an offer to F

G requires F to produce a quantity of q_i^{ar} , at each $x \in (\tau, T_i)$, in return for a discounted profit of $\prod_{i,\tau}^{ar}$. G obtains:

$$V_{i,\tau}^{ar} = w_i \left(q_i^{ar} \right) \frac{1 - e^{-r\delta_i}}{r} - \left(\Pi_{i,\tau}^{ar} + D_{i,\tau} \right).$$

To maximize this return, G chooses a value of $\Pi_{i,\tau}^{ar}$, such that F obtains exactly its break-up payoff, *i.e.*, $\Pi_{i,\tau}^{ar} = \Pi_{i,\tau}^{b,G}$, if renege occurred on the initiative of G, $\Pi_{i,\tau}^{ar} = \Pi_{i,\tau}^{b,F}$, if it occurred on

the initiative of F. Then:

$$\Pi_{i,\tau}^{ar} = \Pi_{i,\tau}^{b,G} \Leftrightarrow V_{i,\tau}^{ar} = w_i \left(q_i^{ar} \right) \frac{1 - e^{-r\delta_i}}{r} - \left(P + D_{i,\tau} \right) \equiv V_{i,\tau}^G$$
$$\Pi_{i,\tau}^{ar} = \Pi_{i,\tau}^{b,F} \Leftrightarrow V_{i,\tau}^{ar} = w_i \left(q_i^{ar} \right) \frac{1 - e^{-r\delta_i}}{r} - D_{i,\tau} \equiv V_{i,\tau}^F$$

The optimal quantity for G is $q_i^{ar} = q_i^*$.

A.2.2 F makes an offer to G

F proposes to produce an output of q_i^{ar} , at each $x \in (\tau, T_i)$, in return for a discounted profit of $\Pi_{i,\tau}^{ar}$. F leaves G with a payoff of $V_{i,\tau}^{ar} = V_{i,\tau}^{b,G}$, in the event that G reneged, and with a payoff of $V_{i,\tau}^{ar} = V_{i,\tau}^{b,F}$, if the event that F itself reneged. Then:

$$V_{i,\tau}^{ar} = V_{i,\tau}^{b,G} \Leftrightarrow \Pi_{i,\tau}^{ar} = R_{\delta_i} - (D_{i,\tau} - D_{i,\tau}^F) - (w_i(q_i^{ar}) - w_i(q_i^*)) \equiv \Pi_{i,\tau}^F$$

$$V_{i,\tau}^{ar} = V_{i,\tau}^{b,F} \Leftrightarrow \Pi_{i,\tau}^{ar} = R_{\delta_i} + P - (D_{i,\tau} - D_{i,\tau}^G) - (w_i(q_i^{ar}) - w_i(q_i^*)) \equiv \Pi_{i,\tau}^G.$$

The optimal quantity for F is $q_i^{ar} = q_i^*$.

A.2.3 Expected renegotiation payoffs

G makes the offer with probability α , F with probability $1-\alpha$. When F reneges, the expected payoffs from renegotiation are given by:

$$\Pi_{i,\tau}^{rn,F} = \alpha \Pi_{i,\tau}^{b,F} + (1-\alpha) \Pi_{i,\tau}^{F}$$
$$V_{i,\tau}^{rn,F} = \alpha V_{i,\tau}^{F} + (1-\alpha) V_{i,\tau}^{b,F}.$$

When G reneges, they are given by:

$$\begin{aligned} \Pi^{rn,G}_{i,\tau} &= \alpha \Pi^{b,G}_{i,\tau} + (1-\alpha) \Pi^G_{i,\tau} \\ V^{rn,G}_{i,\tau} &= \alpha V^G_{i,\tau} + (1-\alpha) V^{b,G}_{i,\tau}. \end{aligned}$$

Replacing the expressions of the profits of F and the returns of G previously found, we obtain (4), (5), (8) and (9).

B Removing the incentives of F to cheat anticipating renege

We identify conditions under which F has no incentive to lie on θ_i , anticipating that some party will renege at some date $\tau \in (0, T_i)$.

Let $\Pi_{i,\tau}^{RN}$ denote the payoff that F would obtain in state *i*, discounted at time τ , if it were to cheat at the outset of the operation phase and renegotiation were to occur at τ . Also let $\pi_{i,x}^*$

the instantaneous profit in state $i \in \{l, h\}$ at instant $x \in (\tau, T_i)$. F has no incentive to lie if and only if

$$\Pi_{l,0}^{*} \geq \int_{0}^{\tau} \left(\pi_{h,x}^{*} + \Delta \theta q_{h}^{*} \right) e^{-rx} dx + \max \left\{ 0; \Pi_{l,\tau}^{RN} \right\}$$
(39a)

$$\Pi_{h,0}^{*} \geq \int_{0}^{\tau} \left(\pi_{l,x}^{*} - \Delta \theta q_{l}^{*} \right) e^{-rx} dx + \max \left\{ 0; \Pi_{h,\tau}^{RN} \right\}.$$
(39b)

We hereafter show that (39a) is satisfied. If F reports h at date 0, in state l, and the contract is renegotiated at some $\tau \in (0, T_h)$, the residual profit of F corresponding to the period (τ, T_h) is:

$$\Pi_{l,\tau}^{RN} = \Pi_{h,\tau}^{rn} + \int_{\tau}^{T_h} \Delta \theta q_h^* e^{-r(x-\tau)} d\tau,$$

where $\Pi_{h,\tau}^{rn}$ is the profit that a type h can extract at renegotiation, this being either $\Pi_{h,\tau}^{rn,F}$ or $\Pi^{rn,G}_{h,\tau}.$ Hence:

$$\Pi_{l,\tau}^{RN} = \int_{\tau}^{T_h} \pi_{l,\tau}^{RN} e^{-r(x-\tau)} d\tau = \Pi_{h,\tau}^{rn} + \int_{\tau}^{T_h} \Delta \theta q_h^* e^{-r(x-\tau)} d\tau.$$

(39a) becomes

$$\Pi_{l,0}^{*} \geq \Pi_{h,0}^{*} + \int_{0}^{T_{h}} \Delta \theta q_{h}^{*} e^{-rx} dx$$

$$+ e^{-r\tau} \left[\max \left\{ 0; \Pi_{h,\tau}^{rn} + \int_{\tau}^{T_{h}} \Delta \theta q_{h}^{*} e^{-r(x-\tau)} dx \right\} - \left(\Pi_{h,0}^{*} + \int_{\tau}^{T_{h}} \Delta \theta q_{h}^{*} e^{-r(x-\tau)} dx \right) \right].$$
(40)

Using (14), the expression added to $\Pi_{h,0}^* + \int_0^{T_h} \Delta \theta q_h^* e^{-rx} dx$ is non-positive. Hence (39a) is implied by (13) and (14).

Symmetrically, (39b) is implied by (12) and (14).

Proof of lemmas \mathbf{C}

C.1 Proof of Lemma 2 and 3

If (23) holds, then (14) is written as $\Pi_{i,\tau}^* \ge 0$. As $\Pi_{l,\tau}^* > \Pi_{h,\tau}^*$, (14) reduces to $\Pi_{h,\tau}^* \ge 0$. If (23) is violated, then (14) is written as $\Pi_{i,\tau}^* \ge \Pi_{i,\tau}^{rn,F}$, where $\Pi_{i,\tau}^{rn,F} > 0 \quad \forall i$. Hence, it is tighter than $\Pi_{h,\tau}^* \ge 0$. Hence (14) is most relaxed when (23) holds in state h, which means when $D_{h,\tau}^F = 0, \, \forall \tau.$ (23) is rewritten as (26) and (14) as $\Pi_{h,\tau}^* \ge 0.$

If (24) holds, then (15) is written as $\Pi_{i,\tau}^* \leq P$. If (24) is violated, then (15) is written as:

$$\Pi_{i,\tau}^* \le (1-\alpha) R_{\delta_i} - D_{i,\tau} - \left[P + \alpha D_{i,\tau} + (1-\alpha) D_{i,\tau}^G \right].$$

The easiest way to have this condition satisfied is to set $D_{i,\tau}^G = D_{i,\tau}$, in which case it is rewritten as:

$$\Pi_{i,\tau}^* \le (1-\alpha) R_{\delta_i} + P.$$

This is weaker than $\Pi_{i,\tau}^* \leq P$, so that to have (15) satisfied, it is necessary that it holds when (24) is violated. First take i = h so that (15) is rewritten $\Pi_{h,\tau}^* \leq (1 - \alpha) R_{\delta_h} + P$. This condition holds, together with $\Pi_{h,\tau}^* \geq 0$, $\forall \Pi_{h,\tau}^* \in [0, (1 - \alpha) R_{\delta_h} + P]$. Hence, to have (15) satisfied jointly with $\Pi_{h,\tau}^* \geq 0$, it is necessary and sufficient that it holds for i = l. The condition under which (24) is violated is rewritten as (27).

C.2 Proof of Lemma 4

Recall from the proof of Lemma 2 and 3 that (14) is rewritten as $\Pi_{h,\tau}^* \ge 0$ and (15) as $\Pi_{l,\tau}^* \le (1-\alpha) R_{\delta_l} + P$. Using (18b) and (18a) in (21), we can write:

$$\Pi_{h,\tau}^* \geq 0 \Leftrightarrow M \geq \nu_1 \Delta \Pi - \psi$$

$$\Pi_{l,\tau}^* \leq (1-\alpha) R_{\delta_l} + P \Leftrightarrow M \leq [(1-\alpha) R_{\delta_l} + P] \frac{1 - e^{-rT_l}}{1 - e^{-r\delta_l}} - (1-\nu_1) \Delta \Pi - \psi.$$

These conditions are jointly satisfied, only if (28) holds.

C.3 Proof of Lemma 5

As $T_l = T_h$, it is impossible to have (23) satisfied in state h, while having (24) violated in state l.

Take (23) and (24) to be both satisfied. From the proof of Lemma 2 and 4, (14) and (15) reduce to $\Pi_{h,\tau}^* \ge 0$ and $\Pi_{i,\tau}^* \le P \ \forall i$, the latter being tighter for i = l. Using (18b) and (18a) in (21), we have:

$$\begin{aligned} \Pi_{h,\tau}^* &\geq & 0 \Leftrightarrow M \geq \nu_1 \Delta \Pi - \psi \\ \Pi_{l,\tau}^* &\leq & P \Leftrightarrow M \leq P \frac{1 - e^{-rT_l}}{1 - e^{-r\delta_l}} - (1 - \nu_1) \Delta \Pi - \psi. \end{aligned}$$

These conditions are jointly satisfied, only if (29) holds.

Take (23) and (24) to be both violated. (14) is rewritten as $\Pi_{i,\tau}^* \ge \Pi_{i,\tau}^{rn,F} \forall i$. Using $T_l = T_h$ in (4), it is $\Pi_{l,\tau}^{rn,F} = \Pi_{h,\tau}^{rn,F}$. As $\Pi_{l,\tau}^* > \Pi_{h,\tau}^*$, (14) reduces to $\Pi_{h,\tau}^* \ge \Pi_{h,\tau}^{rn,F}$. (15) is rewritten as:

$$\Pi_{i,\tau}^* \le (1-\alpha) R_{\delta_i} - D_{i,\tau} - \left[P + \alpha D_{i,\tau} + (1-\alpha) D_{i,\tau}^G\right].$$

As above, the easiest way to satisfy this condition is to set $D_{i,\tau}^G = D_{i,\tau}$, in which case it is rewritten as:

$$\Pi_{i,\tau}^* \le (1-\alpha) R_{\delta_i} + P.$$

As $T_l = T_h$ and $\Pi_{l,\tau}^* > \Pi_{h,\tau}^*$, this is tighter for i = l. Using (18b) and (18a) in (21), (14) and

(15) are rewritten as:

$$\Pi_{h,\tau}^* \geq \Pi_{h,\tau}^{rn,F} \Leftrightarrow M \geq (1-\alpha) R_{\delta_h} \frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} - \psi + \nu_1 \Delta \Pi$$

$$\Pi_{l,\tau}^* \leq (1-\alpha) R_{\delta_l} + P \Leftrightarrow M \leq [(1-\alpha) R_{\delta_l} + P] \frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} - (1-\nu_1) \Delta \Pi - \psi$$

These conditions are jointly satisfied, only if (29) holds.

When $T_l = T_h$, there does not exist a case in which (23) is violated and (24) is satisfied.

Proof of Proposition 1 D

From the proof of Lemma 4, recall that $\exists M \geq 0$ such that (28) is satisfied, if and only if (26) and (27) hold. First suppose that $\frac{R'_x}{R_x} \geq r \frac{e^{-rx}}{1-e^{-rx}} \quad \forall x \geq 0$. Then, (26) holds for all δ_h , if and only if:

$$D_0 \ge R_{T_h} \Leftrightarrow T_h \le \Upsilon(D_0).$$

(27) holds $\forall \delta_l$, if and only if:

$$D_0 \le R'_0 \frac{1 - e^{-rT_l}}{r} \Leftrightarrow T_l \ge \Psi(D_0).$$

Next suppose that $\frac{R'_x}{R_x} \leq r \frac{e^{-rx}}{1-e^{-rx}} \quad \forall x \geq 0$. Then, (26) holds $\forall \delta_h$ if and only if:

$$D_0 \ge R'_0 \frac{1 - e^{-rT_h}}{r} \Leftrightarrow T_h \le \Psi(D_0)$$

(27) holds $\forall \delta_l$ if and only if:

$$D_0 \leq R_{T_l} \Leftrightarrow T_l \geq \Upsilon(D_0)$$

Take $T_l = T_h$. According to Lemma 5, (29) is necessary. As $\frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}$ increases with δ_i , (28)

is satisfied for all δ_i , if and only if $P \ge \Delta \Pi$. Knowing that $\Delta \Pi \ge \frac{\psi}{\Delta \nu}$ (Lemma 1), $\exists T_l$ such that (28) holds if and only if $T_l \ge \Upsilon\left(\frac{\psi}{\Delta \nu}\right)$. For $\Delta \Pi = \frac{\psi}{\Delta \nu}$, the remaining conditions in Lemma 1 become $T_l \ge \underline{T}\left(\frac{\psi}{\Delta \nu}\right)$ and $T_h \le \overline{T}\left(\frac{\psi}{\Delta \nu}\right)$.