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How Can Proprietary Software Firms Take Advantage Over Open Source Communities? Another Story of Profitable Piracy

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How Can Proprietary Software Firms Take Advantage Over Open Source Communities? Another Story of Profitable Piracy.

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Abstract

This paper analyzes the impact on a proprietary software (PS) firm's profit of the activities of an open source software (OSS) community and a piracy channel, as well as on welfare. We develop a model in which the PS firm competes by price with both producers and also selects its compatibility strategy towards the OSS solution and its protection strategy towards the software copy (PPS). We show that the existence of the piracy channel incumbent enables the PS firm to reach out higher profit than when piracy is prevented. A key mechanism at stake is that the PS monopolist can define its compatibility strategy so as to level price competition down while extending its market share at the same time. Although it has to provide some protection efforts towards the piracy channel to do so, the extra revenues it generates always overcome such latter costs. From a regulatory point of view, our results stress that welfare is higher when piracy is prevented while the PS firm sets compatibility towards the OSS solution.

Keywords: Anti-copy Protection; Compatibility; Externalities; Open Source Software; Piracy; Proprietary Software.

JEL classification: L11; L82; L86

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"It's easier for our software to compete with Linux when there's piracy than when there's not." Bill Gates, Fortune Magazine, July 17 2007

1 Introduction

For now more than a decade, the development of the software industry has evidenced new patterns of competition. Although some traditional proprietary closed source software (PS, henceforth) firms had been able to crowd other ones from their markets, they today have to deal with atypical software competitors such as open source software (OSS) communities and piracy channels. OSS solutions have gained market shares and often represent a relevant threat to PS products. For instance, Apache leads the market for web servers, as its market share was around 61% of the total in 2012 whereas Microsoft was only 15%. In the market for mobile operating systems, the market share of Android represents 52% whereas that of Apple iOS is 35%. Dealing with piracy, recent surveys estimate the rate for software piracy adoption to 42%. Putting OSS development and software piracy together, one could therefore think that competition intensity on the market for software is likely to increase so that PS firms would eventually lose again market shares. This would apparently all the more hold than both OSS products and illegal software copies are most of times provided at zero price whereas PS firms charge positive license fees to their users. We explain in our paper why this is however not the case, by analyzing the strategic incentives of PS firms to manipulate software network externalities by providing compatible standards with OSS products and to invest in anti-copy protection features.

Theoretical literatures on OSS and digital piracy dealing with competition are already widely available.¹ On the OSS literature's side, many efforts have been carried out to identify the conditions under which OSS and PS solutions may co-exist or one deters the other one. It is now commonly acknowledged that network effects within OSS users (Dalle and Jullien, 2003; Schmidt and Schnitzer, 2003; Casadesus-Masanell and Ghemawat, 2006; Comino and Manenti, 2011), adoption costs for OSS solutions (Mustonen, 2003; Lin, 2004; Lanzi, 2009) and compatibility between PS and OSS products (Economides and Katsamakos, 2006; Llanes and de Elejalde, 2013; Niedermayer, 2013) are key factors explaining why OSS development activities can be

¹See Peitz and Waelbroeck (2006) and Belleflamme and Peitz (2012) for excellent surveys dealing with digital piracy. An extensive survey on the literature of OSS development is Rossi (2006).

detrimental to PS commercial ones. Dealing with software piracy, network effects have also been shown to be of prior importance when measuring the effect of piracy on software firms. One major result is that piracy may have an overall positive impact on a software monopolist, provided that network effects are sufficiently high (Conner and Rumelt, 1991; Takeyama, 1994; Gayer and Shy, 2003). Here, the negative so-called 'competition' effect of piracy on the profit of the software firm is downplayed by the positive demand-enhancing one in such a way that piracy becomes profitable. Such a result does not entirely hold in a duopoly framework because software piracy not only leads to strategic abilities to raise prices but also to competition for market shares (Shy and Thisse, 1999; Peitz, 2004). From this, it results that dropping software protection may be profit-maximizing when digital piracy is considered from a strategic viewpoint.

Extensive research has been conducted to understand the impact of either OSS activities or piracy on PS firms. However, to best of our knowledge, no attempt has to been carried out to understand the twin effect of OSS and piracy activities on PS firms' profitability. We suggest that the competition mechanisms at stake are different when users can choose between three types of software solutions. Focusing on the classical price competition framework, PS firms have to lower their level for license fees if piracy cannot be deterred, which nevertheless allows them to better compete with OSS solutions. Conversely, PS firms have to lower their price when competing with OSS communities, then enabling them to better deal with piracy threat. Introducing network effects, our intuition is that PS firms may benefit from an accommodation strategy towards the diffusion of OSS solutions or software copies, thus being able to charge a higher price compared to a framework without network externality. Moreover, the general question we address here is whether a PS firm should accommodate with OSS communities or with piracy channels. We consider that PS firms can design suitable strategies regarding (i) compatibility between PS and OSS solutions and (ii) protection devices vis-à-vis software copies, then allowing them to control externality levels so as to keep on charging high price and/or extending their market shares. We suggest that PS firms may decide to provide compatible standards with OSS solutions in order to increase their externality effect. By investing in the provision of anti-copy protection devices, PS firms are also able to make the use of software copies inconvenient. Through these two strategies, PS firms are able to increase the software users' willingness to pay for PS solutions and may paradoxically benefit from their both types

of competitors.

We develop a model in which a PS firm competes by price with both an OSS community and a piracy channel. The PS firm also selects its compatibility strategy towards the OSS solution and its protection strategy towards a software - here, PS - copy (PPS, henceforth). Our model rests on the assumption that users do not simultaneously trade off between the three types of software (i.e., PS, OSS and PPS). We introduce heterogeneous software users in our analysis, namely ethical users and unethical users. Ethical users establish their adoption decisions through the comparison of PS and OSS legal solutions. In contrast, unethical users adopt decide to adopt PS or PPS product. In our four-stage model, the PS firm first decides to make its software standard compatible or not compatible with the OSS solution. It secondly selects its protection strategy against the PPS solution and thirdly sets its level for license fee. Software users eventually adopt at the fourth stage. To analyze to what extent the existence of PPS copies allows the PS firm to take advantage over the OSS provider, we compare the outcomes we obtain when piracy is deterred to that we have when piracy is introduced. We show that the compatibility decision of the PS firm towards OSS solution depends on the presence of piracy channels. When piracy does not apply, the PS firm is always better off making its PS solution compatible with the OSS one. Such a result does not hold when an illegal copy is largely diffused over the software market. Another result is that the PS firm systematically sets a positive level for software protection when piracy applies. This underlines that, when piracy is an extended phenomenon, the PS firm actively fights against both OSS and PPS providers by setting out suitable strategies to degrade the PPS solution while extending the network externality for its product through compatibility strategy. Welfare results are also discussed. We find that the introduction of piracy negatively impacts on social welfare. We therefore identify a conflict of interest according to which piracy enhances the profit of the PS firm whereas it is detrimental from the social planner's viewpoint. Our explanation is that the PS firm needs to define a high level for software protection to enjoy from demand-boosting effects from piracy, which is eventually harmful for software users regarding both the increase of PS price increase and the degradation of the users who adopt PPS.

The paper is organized as follows. Section 2 presents the model. Section 3 (resp. section 4) solves for equilibrium PS price, as well as anti-protection and compatibility strategy when piracy does not apply (resp. when piracy applies). A comparative analysis dealing with profits

and welfare is held in section 5. Section 6 concludes and discusses issues for further research.

2 The model

We develop a model in which a proprietary software monopolist firm may face competition from both an open source software community and a piracy channel. We successively consider two cases. In the first case (benchmark case), piracy does not apply and the PS monopolist firm competes with the OSS community. In the second case (piracy case), piracy applies and the PS monopolist competes with both the OSS community and the piracy channel.

2.1 Software products and software providers

In the benchmark case, software products are provided by the PS firm (proprietary software, hereafter PS) or by the OSS community (open source software, hereafter OSS). PS is supplied at price p ($p > 0$) whereas OSS is provided free of charge. Turning to the piracy case, we introduce the piracy channel which also supplies an illegal version of the proprietary software product (pirate proprietary software, hereafter PPS) free of charge. With no loss of generality, we assume that software production is costless for all three software providers. The PS monopolist acts as a profit-maximizing firm, its profit being given by $\pi = D_{PS}p - i(s)$, where D_{PS} denotes the demand of software users for PS and $i(s)$ denotes the cost function the PS monopolist invests in to protect its PS solution towards the diffusion of PPS copies. s ($s > 0$) is the level of protection which is set by the PS firm. The higher the value of s , the less convenient it is for users to use the PPS solution. We further assume that investment cost $i(s)$ in software protection is given by the general quadratic cost specification, here $i(s) = (1/2)s^2$.

2.2 Software users

The population of software users has unit mass. Software users are dispatched over two sub-markets according to their type, namely ethical (e) and unethical (u) users. Ethical users are displayed on a first sub-market (namely, high market) on which they only adopt PS or OSS. They exhibit high valuation for legal software products and they thus do not pay attention to the illegal supply. They can be regarded as institutional users (e.g., public administrations, universities and schools), as well as users who do not wish to adopt PPS or are not aware

of them. Ethical users are of mass α ($\alpha \in [0, 1]$) on the high market. They are uniformly distributed on a Hotelling line of unit length that describes product differentiation between PS and OSS, on which PS is located at 0 whereas OSS is located at 1. Unethical users are displayed on a second sub-market (namely, low market) on which they only adopt PS or PPS. They represent the users who adopt from trading-off between software products from markets on which illegal software products are available (e.g., file-sharing networks). Being displayed on the low market, unethical users pay very little attention to OSS or they are not even aware of its existence. They however know about the existence of PS, as PPS copies directly derive from it. Note that a part of the unethical users may eventually adopt legal software product PS. Indeed, depending on the level of protection the PS monopolist sets, some unethical users may be likely to pay to get access to more convenient PS product. Unethical users are of mass $1 - \alpha$ on the low market. They are uniformly distributed on a Hotelling line of unit length that describes product differentiation between PS and PPS, on which PS is located at 0 whereas PPS is located at 1. Underline that $\alpha = 1$ when piracy does not apply (benchmark case).

The software market is divided into the low market and the high market. We consider that these two sub-markets are interconnected so that the PS monopolist is not able to discriminate among users according to their type and location. Our justification is that it is quite difficult - at least costly - for the PS firm to identify unethical users from the whole population of users. Note that we allow a sub-market to have another size than the other one. This completes Shy and Thisse (1999) who restrict to equal sizes for sub-markets.

2.3 Market shares and utility functions

Each user derives gross utility r ($r > 0$) from adopting any software solution. She also gets a network externality benefit of joining the network of the adopters whose software products are compatible with hers. We assume this network externality to be linear and given by θN_j , where θ is the network parameter ($\theta > 0$) and N_j is the mass of users who adopt the software product j to which compatibility extends. Remind that PS and PPS are *de facto* compatible since PPS copies derive from PS. In contrast, PS (thus PPS) is not compatible with OSS unless the PS firm allows PS to be compatible with OSS. Define $n_{PS} = n_{PS}^e + n_{PS}^u \equiv D_{PS}$ the share of the users who legally adopt PS, where n_{PS}^e (resp. n_{PS}^u) is the share of the ethical users (resp. unethical users) who adopt PS. Similarly, define n_{OSS} the share of the users of adopt OSS and

n_{PPS} the share of the users of adopt PPS. From our specification of the whole population of software users, the share of the users who adopt a software product which is compatible with PS is given by $N_{PS} = n_{PS} + n_{PPS}$ if the PS firm does not allow software compatibility towards OSS. In this latter situation, the share of the users who adopt software which is compatible with OSS is $N_{OSS} = n_{OSS}$. If PS and OSS products turn to be compatible, assuming that both low market and high market are fully-served, we have $N_{PS} = N_{OSS} = 1$.

The net utility of a software user of type e located at $x \in [0, 1]$ on the high market is therefore given by

$$U_x \equiv \begin{cases} r - x + \theta N_{PS} - p & \text{if adopts PS} \\ r - (1 - x) + \theta N_{OSS} & \text{if adopts OSS} \end{cases} \quad (1)$$

Similarly, the net utility of a software user of type u indexed by $y \in [0, 1]$ on the low market is given by

$$U_y \equiv \begin{cases} r - y + \theta N_{PS} - p & \text{if adopts PS} \\ r - (1 - y) + \theta N_{PS} - s & \text{if adopts PPS} \end{cases} \quad (2)$$

Note that we normalize transport cost to 1. Also, we make the following assumption.

Assumption 1 *The gross utility r and network parameter θ are bounded so that $r > 1$ and $\theta \in]0, 1[$.*

Assumption 1 allows the low market and the high market to be fully-served and shared in both benchmark and piracy cases.

The timing of the model is as follows.

- At stage 1, the PS firm establishes its software compatibility strategy, that is, decides to extend PS compatibility to OSS or not (*compatibility strategy*)
- At stage 2, the PS firm selects its protection level s towards PPS (*protection strategy*)
- At stage 3, the PS firm chooses its price p for PS (*price strategy*). Each user adopts one of the three software products that are available on the market and the PS firm makes profit π .

The model is solved by backward induction. We assume that users have rational expectations

about the way they put value on their adoption decision, by correctly forecasting the market shares of each software provider at equilibrium. We start by considering the benchmark case in which piracy does not apply and we identify the strategies of the PS firm and its ensuing profit in this framework (section 3). We next introduce piracy and we present the results of the model in the piracy case (section 4). Section 5 develops a comparison of the results of the benchmark and piracy cases and discusses welfare implications. Section 6 concludes.

3 Competition without piracy

We first analyze competition between the PS firm and the OSS community when piracy does not apply. This provides a benchmark for further results' study. We derive the equilibrium outcomes under both cases of incompatibility and compatibility between PS and OSS. Note that the PS firm does not set any protection towards PPS copies here because the share of unethical users is 0. Substituting $s = 0$ and $\alpha = 1$ into (1) and (2), we define a two-stage game in which the PS firm firstly decides on its compatibility strategy towards OSS and secondly sets the price of PS.

3.1 Price subgame

We first focus on the price strategy of the PS firm, considering its compatibility decision to be given. We successively consider the case in which PS and OSS are incompatible solutions (3.1.1) and that in which PS and OS are compatible ones (3.1.2).

3.1.1 Incompatibility between PS and OSS

When the two software solutions are incompatible, software network sizes are given by $N_{PS,B}^{INC} = n_{PS,B}^{INC}$ and $N_{OSS,B}^{INC} = n_{OSS,B}^{INC}$, with $N_{PS,B}^{INC} + N_{OSS,B}^{INC} = 1$ under assumption 1. Let $\tilde{x}_B^{INC}$ be the location of the ethical user who is indifferent between PS and OSS in the benchmark case. Formally $\tilde{x}_B^{INC}$ must solve $-x + \theta x - p_B^{INC} = -(1-x) + \theta(1-x)$, which after some simplifications, yields

$$\tilde{x}_B^{INC}(p_B^{INC}) = \frac{1}{2} \left(\frac{1 - \theta - p_B^{INC}}{1 - \theta} \right)$$

The market share of the PS firm is thus $n_{PS,B}^{INC} = \tilde{x}_B^{INC}$ whereas that of the OSS community is $n_{OS,B}^{INC} = 1 - \tilde{x}_B^{INC}$. Consequently, the demand for the PS solution is expressed by $N_{PS,B}^{INC} = \tilde{x}_B^{INC}$,

that is,

$$N_{PS,B}^{INC}(p_B^{INC}) = \frac{1}{2} \left(\frac{1 - \theta - p_B^{INC}}{1 - \theta} \right)$$

Given that the market is covered, the demand for OSS is $N_{OSS,B}^{INC} = 1 - N_{PS,B}^{INC}$. The profit of the PS firm is $\pi_B^{INC} = N_{PS,B}^{INC}(p_B^{INC}) p_B^{INC}$. From the first order condition, and solving out for p_B^{INC} , we obtain²

$$p_B^{INC*} = \frac{(1 - \theta)}{2} \quad (3)$$

and optimal market share and profit are simply

$$n_{PS,B}^{INC*} = \frac{1}{4}, \quad n_{OSS,B}^{INC*} = \frac{3}{4}, \quad \pi_B^{INC*} = \frac{(1 - \theta)}{8} \quad (4)$$

3.1.2 Compatibility between PS and OSS

When the two software solutions are compatible, software network sizes are given by $N_{PS,B} = N_{OSS,B} = 1$. Let $\tilde{x}_B^C$ be the location of the ethical user who is indifferent between PS and OSS in the benchmark case. Formally $\tilde{x}_B^C$ must solve $-x + \theta - p_B^C = -(1 - x) + \theta$, which yields

$$\tilde{x}_B^C(p_B^C) = \frac{(1 - p_B^C)}{2}$$

The market share of the PS firm is $n_{PS,B}^C = \tilde{x}_B^C$ whereas the market share of the PS is $n_{OSS,B}^C = 1 - \tilde{x}_B^C$. The network size of the PS firm and the OSS community is equal to $N_{PS,B}^{INC} = N_{OSS,B}^{INC} = 1$. The profit of the PS firm is defined by $\pi_B^C = n_{PS,B}^C p_B^C$. From the first order condition, we obtain³

$$p_B^{C*} = \frac{1}{2} \quad (5)$$

and optimal market share and profit are

$$n_{PS,B}^{C*} = \frac{1}{4}, \quad n_{OSS,B}^{C*} = \frac{3}{4}, \quad \pi_B^{C*} = \frac{1}{8} \quad (6)$$

²The second order condition is satisfied, i.e. $\frac{\partial^2 \pi_B^{INC}}{\partial p_B^{INC2}} = -\frac{1}{1-\theta} < 0$

³The second order condition is satisfied, i.e. $\frac{\partial^2 \pi_B^C}{\partial p_B^{C2}} = -\frac{1}{1} < 0$

3.2 Compatibility choice

We now solve the first-stage game in which the PS firm defines its compatibility strategy. The following proposition characterizes the optimal compatibility decision of the PS firm.

Proposition 1 *When piracy does not apply, the PS firm always provides compatibility towards the OSS solution, i.e. $\pi_B^{C*} > \pi_B^{INC*}$.*

Proof. $\Delta\pi_B^* = \pi_B^{C*} - \pi_B^{INC*} = \frac{1}{8}\theta > 0$ ■

In the benchmark case, the PS firm is always better off providing compatibility for its solution towards the OSS one. Moreover, the higher the value for θ , the more it benefits from compatibility between PS and OSS products. The intuition of proposition 1 is that providing software compatibility provides higher valuation for software users. The willingness-to-pay for PS product increases and the PS firm is thus able to charge higher level of price without losing market shares over the OSS community. Its profit consequently increases compared to the situation in which both software suppliers provide incompatible software solutions.

4 Competition with piracy

In the benchmark model, we have conducted analysis in the absence of piracy. We extend our benchmark model to understand how the introduction of piracy impacts on the pricing and compatibility decision of the PS firm, as well as on the protection strategy of the PS firm towards PPS. Note that two markets are now considered, that is, a high market in which ethical users keep on trading-off between PS and OSS products and a high market in which unethical users decide to adopt between PS and PPS solutions. We derive the equilibrium outcomes under the two compatibility regimes between PS and OSS solutions.

4.1 Price subgame

4.1.1 Incompatibility

Define $\tilde{x}^{INC}$ (resp. $\tilde{y}^{INC}$) the location of the ethical user (resp. unethical user) who is indifferent between PS and OSS in the high market (resp. PS and PSS in the low market) when PS and OSS solutions are not compatible. In the case in which both markets are fully-covered and both

PS firm and OSS community provide incompatibles standards, network sizes N_{PS}^{INC} and N_{OSS}^{INC} are defined as follows:

$$N_{PS}^{INC} = n_{PS}^{INC} + n_{PPS}^{INC} = \alpha \tilde{x}^{INC} + (1 - \alpha) \quad ; \quad N_{OSS} = n_{OSS}^{INC} = \alpha (1 - \tilde{x}^{INC})$$

The location $\tilde{x}^{INC} \in (0, 1)$ of the marginal ethical user who is indifferent about adopting either PS or OSS must solve $r - x + \theta(\alpha x + (1 - \alpha)) - p^{INC} = r - (1 - x) + \theta(\alpha(1 - x))$. In a similar fashion, the location $\tilde{y}^{INC} \in (0, 1)$ of the marginal ethical user who is indifferent about adopting either PS or PPS must solve $-y + \theta(\alpha x + (1 - \alpha)) - p^{INC} = -(1 - y) + \theta(\alpha x + (1 - \alpha)) - s$. We eventually obtain

$$\tilde{x}^{INC} = \frac{(1 - p - 2\theta\alpha + \theta)}{2(1 - \theta\alpha)}, \quad \tilde{y}^{INC} = \frac{(1 - p + s)}{2}$$

The demand for PS solution is given by the sum of the market share of the PS firm that comes from ethical users and the market share of the PS firm that comes from unethical users, that is, $n_{PS}^{INC} = \alpha \tilde{x}^{INC} + (1 - \alpha) \tilde{y}^{INC}$. Some calculations yield

$$n_{PS}^{INC} = \frac{-p(1 - \theta\alpha(1 - \alpha)) + s(1 - \alpha)(1 - \theta\alpha) + (1 - \theta\alpha^2)}{2(1 - \theta\alpha)}$$

The profit function of the PS firm being given by $\pi_B^{INC} = n_{PS}^{INC} p^{INC} - \frac{1}{2} s^2$, we get from the first order condition of the latter objective function the following optimal price function⁴:

$$p^{INC}(s) = \frac{s(1 - \alpha)(1 - \theta\alpha) + (1 - \theta\alpha^2)}{2(1 - \theta\alpha(1 - \alpha))} \quad (7)$$

The market share of the PS firm on both markets is defined by

$$n_{PS}^{INC}(s) = \frac{s(1 - \alpha)(1 - \theta\alpha) + (1 - \theta\alpha^2)}{4(1 - \theta\alpha)} \quad (8)$$

and its profit function is now expressed as

$$\pi^{INC}(s) = \frac{(s - s\alpha - \theta\alpha^2 - s\theta\alpha + s\theta\alpha^2 + 1)^2}{2(4 - 4\theta\alpha)(\theta\alpha^2 - \theta\alpha + 1)} - \frac{1}{2} s^2 \quad (9)$$

⁴The second order condition requires $\frac{\partial^2 \pi_{PS}^{INC}(p^{INC}, s)}{\partial p^{INC2}} = -\frac{1 - \theta\alpha(1 - \alpha)}{1 - \theta\alpha} < 0$, which is verified.

4.1.2 Compatibility

In the case in which both markets are fully-covered and both PS and OS producers provide compatible standards, network sizes N_{PS}^C and N_{OSS}^C are defined as follows:

$$N_{PS}^C = N_{OSS}^C = 1$$

The location $\tilde{x}^C \in (0, 1)$ of the marginal ethical user who is indifferent about adopting either PS or OSS must solve $-x + \theta - p^C = -(1 - x) + \theta$. In a similar fashion, the location $\tilde{y}^C \in (0, 1)$ of the marginal ethical user who is indifferent about adopting either PS or PPS must solve $-y + \theta - p = -(1 - y) + \theta - s$. After some simple calculations, we obtain

$$\tilde{x}^C = \frac{1 - p}{2}, \tilde{y}^C = \frac{1 - p + s}{2}$$

From above, the demand for PS solution is once again given by the sum of the market share of the PS firm that comes from ethical users and the market share of the PS firm that comes from unethical users, that is, $n_{PS}^C = \alpha \tilde{x}^C + (1 - \alpha) \tilde{y}^C$. We eventually get

$$n_{PS}^C = \frac{s - p - s\alpha + 1}{2}$$

The profit function of the PS firm being given by $\pi^C = n_{PS}^C p^C - \frac{1}{2} s^2$, we get from the first order condition of the latter objective function the following optimal price function⁵

$$p^C(s) = \frac{s - s\alpha + 1}{2} \tag{10}$$

The market share of the PS firm on both markets is defined by

$$n_{PS}^C(s) = \frac{1}{4}(s + 1 - s\alpha) \tag{11}$$

and its profit function is now expressed as

$$\pi^C(s) = \left(\frac{s + 1 - s\alpha}{4} \right) \left(\frac{s - s\alpha + 1}{2} \right) - \frac{1}{2} s^2 \tag{12}$$

⁵The second order condition requires $\frac{\partial^2 \pi_{PS}^C(p^C, s)}{\partial p^{C2}} = -1 < 0$, which is verified.

4.2 Protection subgame

Given the price strategy of the PS firm, we turn to the second stage of the game in the case of both compatibility regimes.

Under the incompatibility regime, the PS firm defines its protection level s^{INC*} which maximizes its profit function $\pi^{INC}(s)$ given by (9). Optimal protection level is given so that the first order condition $\frac{\partial \pi^{INC}(s)}{\partial s} = 0$ is fulfilled and can eventually be derived to be

$$s^{INC*} = \frac{(1 - \alpha)(1 - \theta\alpha^2)}{\alpha(2 - \alpha) + 3(1 - \theta\alpha) + \theta\alpha^2(\alpha + 2)} \quad (13)$$

Similarly, under the compatibility regime, the PS firm defines its protection level s^{C*} which maximizes its profit function $\pi^C(s)$ given by (12). Optimal protection level is given so that the first order condition $\frac{\partial \pi^C(s)}{\partial s} = 0$ is fulfilled and is

$$s^{C*} = \frac{1 - \alpha}{2\alpha(1 - \alpha) + 3} \quad (14)$$

Under assumption 1, we verify that s^{INC*} and s^{C*} are unique optimal solutions since second order conditions are met, i.e., $\frac{\partial^2 \pi^{INC}(s)}{\partial s^2} < 0$ and $\frac{\partial^2 \pi^C(s)}{\partial s^2} < 0$. Note that the level of protection set by the PS firm under the compatibility regime does not depend on the strength of the network effect θ whereas that under the incompatibility regime does. This is because the mutualization of the software network does not provide any competitive advantage to one of the provider over the other and does not lead the PS firm to integrate it into its optimal protection strategy. The optimal protection levels which are set by the PS firm however depend on the size of both high and low markets (i.e., α).

Lemma 1 *When piracy applies, there exists a level $\tilde{\alpha} \in]0, 1[$ ($\tilde{\alpha} \simeq 0.4827$) under which the level of protection which is set by the firm is higher under the incompatibility regime than under the compatibility one (i.e., $s^{INC*} > s^{C*}$), whatever the value for θ is. Otherwise, the PS firm sets higher level of software protection under the compatibility regime than under the incompatibility one (i.e., $s^{C*} > s^{INC*}$), regardless of θ .*

Proof. See Appendix. ■

The size of both low and high markets is important for this result since the protection levels under compatibility and incompatibility regimes depend on it. When α is low (i.e., $0 < \alpha < \tilde{\alpha}$),

the competition for unethical users is more intense because the PS firm is more sensitive to the threat of the piracy channel. As a result, the PS firm defines stronger protection strategy under the incompatibility regime because it allows it to attract more unethical users, thus making them purchasing PS rather than adopting PPS. Indeed, the PS firm engages to fiercer competition towards the piracy channel, thus all the more increasing its protection level than the willingness-to-pay of the ethical users for PS decreases, resulting from incompatibility between PS and OSS solutions. In contrast, when the size of the high market tends to increase (i.e., values of α tend to be higher), the PS firm turns to better consider competition with the OSS community than the piracy channel. It consequently decreases its protection level vis-a-vis the diffusion of the PPS alternative as much as the willingness-to-pay of the ethical users for PS increases. Dealing with a double-threat from both the OS community and the piracy channel, we find that the PS firm all the more accomodates the piracy channel that it selects software incompatibility towards the OSS solution.

4.3 Compatibility choice

We now solve the first stage game in which the PS firm establishes its compatibility decision.

Substituting (13) into (7) and (8) yields

$$p^{INC*} = \frac{2(1 - \theta\alpha^2)}{\alpha(2 - \alpha) + 3(1 - \theta\alpha) + \theta\alpha^2(\alpha + 2)} \quad (15)$$

and

$$n_{PS}^{INC*} = \frac{(1 - \theta\alpha) + \theta^2\alpha^3(1 + \alpha)}{(1 - \theta\alpha)(\alpha(2 - \alpha) + 3(1 - \theta\alpha) + \theta\alpha^2(\alpha + 2))} \quad (16)$$

whereas substituting (14) into (10) and (11) yields

$$p^{C*} = \frac{2}{\alpha(2 - \alpha) + 3} \quad (17)$$

and

$$n_{PS}^{C*} = \frac{1}{\alpha(2 - \alpha) + 3} \quad (18)$$

Optimal profits under incompatibility and compatibility regimes are obtained respectively

by replacing (13) into (9) and (14) into (12).

$$\pi^{INC*} = \frac{(1 - \theta\alpha^2)^2}{2(1 - \theta\alpha)(\alpha(2 - \alpha) + 3(1 - \theta\alpha) + \theta\alpha^2(\alpha + 2))} \quad (19)$$

and

$$\pi^{C*} = \frac{1}{2} \frac{1}{\alpha(2 - \alpha) + 3} \quad (20)$$

Proposition 2 characterizes the optimal compatibility decision of the PS firm when piracy is introduced.

Proposition 2 *When piracy applies, thresholds $0 < \underline{\alpha} < \bar{\alpha} < 1$ and $\bar{\theta} \in]0, 1[$ exist such that:*

- (i) *if $0 \leq \alpha \leq \underline{\alpha}$, the PS firm provides incompatibility towards the OSS solution;*
- (ii) *if $\underline{\alpha} < \alpha \leq \bar{\alpha}$, the PS firm provides compatibility towards the OSS solution when $\theta \in]0, \bar{\theta}[$, incompatibility otherwise;*
- (iii) *if $\bar{\alpha} < \alpha < 1$, the firm provides compatibility towards the OSS solution.*

Proof. See Appendix. ■

When the size of the high market is low, the PS firm has no interest in mutualizing its software network with that of the OSS community, since it gives it a strategic advantage over the diffusion of the OSS product. As α increases, the PS firm revises its strategy by accomodating the OSS community through compatibility provision so as to less suffer from the threat of the OSS community. In the specific case in which $\underline{\alpha} < \alpha \leq \bar{\alpha}$ and $\theta \in]\bar{\theta}, 1[$, the PS firm defines an incompatibility strategy to increase the overall demand for the PS solution in both markets, thus reaching out higher-leveled profit than that which would result from software compatibility between PS and OSS solutions. However note here that such an increase of the overall demand for PS leads to the decrease of the price of the PS solution, which impact is nevertheless downplayed from a profit point of view. In all the other cases, we find that the PS firm defines the compatibility strategy which allows it to better relax overall price competition on both markets, yet losing market share over its competitors. As long as α increases, the willingness-to-pay of the users for the PS solution decreases and the PS firm defines the compatibility strategy which allows it to charge the highest level possible for p^* , for a given value of α .

5 Comparative analysis

In this section we compare the equilibrium results of the model across the two compatibility regimes without and with piracy. Doing so, we provide some tentative policy guidelines with respect to fighting against piracy and allowing software compatibility between PS and OSS solutions. We first hold a comparative analysis to investigate to what extent the PS firm may increase its profit when designing its compatibility strategy towards OSS while piracy does/does not apply (5.1). We then turn to a welfare analysis to identify potential conflicts of interest between the objective of the PS firm and that of the social planner (5.2).

5.1 Profits

Proposition 3 highlights the impact of both compatibility regime and piracy on the level of profit of the PS firm.

Proposition 3 *The PS firm generates higher profit when piracy applies.*

Proof. *See Appendix.* ■

Whether through compatibility or incompatibility, we find that the introduction of the piracy channel allows the firm to increase its profit. The mechanisms at stake are as follows. When piracy applies, selecting between software compatibility and incompatibility enables the PS firm to simultaneously level overall price competition down and gain overall market shares over both OSS and PPS providers. Moreover, we can show that the PS firm benefits from both software incompatibility and piracy when (i) the size of the high market is low whatever the value of network parameter θ (i.e., $0 < \alpha \leq 0.6601$ and $0 < \theta < 1$) and (ii) the size of the high market is intermediate while the value of network parameter θ is high (i.e., $0.6601 < \alpha \leq 0.7281$ and $\underline{\theta}_1 < \theta < 1$, with $\underline{\theta}_1 \in]0, 1[$). In all the other cases, the PS firm is better off providing software compatibility, still benefiting from piracy. Note that decreasing price and increasing market share requires the PS firm to set somehow high level for protection, which yet allows it to reach out higher level for profit. Also note that selecting software incompatibility when piracy does not apply always leads to the lowest outcome for the PS firm.

5.2 Welfare

We examine the incentives of the social planner to prevent piracy. To this purpose, we define welfare as the sum of the users' surplus and the PS firm's profit, that is, $W = \pi + US$. When the benchmark case, the users' surplus is given by $US_B = \int_{x=0}^{\tilde{x}} r - x + \theta N_{PS} - p \, dx + \int_{\tilde{x}}^1 r - (1 - x) + \theta N_{OSS} \, dx$. When piracy applies, it extends to $US = \alpha \int_{x=0}^{\tilde{x}} r - x + \theta N_{PS} - p \, dx + \alpha \int_{\tilde{x}}^1 r - (1 - x) + \theta N_{OSS} \, dx + (1 - \alpha) \int_{y=0}^{\tilde{y}} r - y + \theta N_{PS} - p \, dy + (1 - \alpha) \int_{\tilde{y}}^1 r - (1 - y) + \theta N_{PS} - s \, dy$. Proposition 4 shows to what extent both compatibility regime and piracy affect the profit of the PS firm.

Proposition 4 *Welfare is always higher when the PS firm provides compatibility towards the OSS solution and piracy is prevented, whatever values for α and θ are, i.e., $W_B^{C*} > \{W^{C*}, W^{INC*}, W_B^{INC*}\}$.*

Proof. See Appendix. ■

The result pointed out in proposition 4 directly derives from proposition 3. Indeed, relaxing overall price competition allows the PS firm to extract higher-level consumers' surplus, which is obviously detrimental of the adopters of the PS product. The willingness-to-pay of the users of PS all the more increases than the PS firm provides software compatibility towards the OSS solution, all other things being equal. Besides, the demand for the OSS solution decreases, which negatively impacts on the surplus of the users who adopt the OSS solution. On the PS users' side, the effect of software compatibility towards OSS is neutral whereas it is positive for the PS firm which benefits from an increase in its product's price. Finally, so as to keep on charging a high level for price, the PS firm sets a high level of protection that is harmful the surplus of the adopter of the PPS copy. Taken all the effects at stake together, we find that the ability of the PS firm to select its protection level is detrimental to all the software users, although the monopolist can extract a higher level of profit. Put it differently, the gain the PS firm generates from piracy and/or software incompatibility on one side is lower than the loss software users have to support on the other side.

We therefore identify an original result which is rarely pointed out in the literature dealing with digital piracy. Whereas a large number of contributions agree on the fact that piracy is likely to generate positive effects which may be beneficial to both ethical and unethical users, we find that public actions should be led to prevent the diffusion of PPS copies. It has nevertheless to be pointed out that such public actions should be designed so that the PS monopolist has also

incentives to provide software compatibility towards the OSS solution. As this is not always the case (e.g., for 'low' values for α , see proposition 3), a conflict of interest may arise. This conflict is here atypical since it refers to a situation in which the piracy benefits to the monopolist whereas it is detrimental at the same time at the overall social level. Our explanation is that allowing the PS firm to set its compatibility/incompatibility strategy gives it better weapons to extract higher volume of users' surplus, then reaching out higher profits although protection costs - yet weaker - have to be supported.

6 Conclusion

In this paper we have analyzed the impact on a PS monopolist's firm and on social welfare of the activities of an open source community and a piracy channel. The analysis has been conducted developing a model in which the PS firm deals with the threat of both OSS solutions and software copies. In our framework software network externalities allow the PS firm to design suitable strategies to better fight against the diffusion of the OSS product and/or that of the software copy. In addition to traditional pricing concerns, we have also here focused on two strategies the PS firm may employ, namely compatibility/incompatibility strategy towards OSS solutions and anti-copy protection towards software copies. Our goal has been to identify the PS firm's 'best enemy' in settings in which piracy applies or not, then intending to describe competition in a software industry where users are somehow aware of the existence of an OSS (resp. illegal) alternative to the PS solution, namely ethical users (resp. unethical users), which are displayed on a high market (resp. low market). Key issues have notably been to understand the way competition between the PS firm and the OS community is affected by the existence of the illegal version of PS when software network externalities are at stake, as well as the mechanisms at work when the PS firm can set its price, compatibility and protection strategies. Our intuition has been that the PS monopolist can manipulate network externalities so as to design appropriate price and protection strategies, depending on the size of the ethical users over the unethical ones.

Dealing with the motives of the PS firm to develop its compatibility strategy, we first show that providing software compatibility towards the OSS solution is always the monopolist's best choice when piracy does not apply. This result nevertheless does not hold when the piracy

channel is introduced. When piracy applies, compatibility (resp. incompatibility) is the firm's best strategy only when the size of the low market is very low (resp. very high), whatever the value of the network parameter is. Otherwise, the likelihood of the firm to provide compatibility towards OSS depends on the value of the network parameter. A second - and main - result is that the diffusion of software copies enables the PS firm to reach out higher profit than when piracy is prevented. We indeed find that allowing the PS firm to calibrate its compatibility strategy leads it to level overall price competition down while gaining overall market share over its competitors. Although the PS firm may have to support higher costs in order to provide higher-leveled software protection towards the diffusion of software copies, the increase in its revenues overcomes that in such extra costs. From a regulatory point of view, the results of our model stress that welfare is always higher when the PS firm provides compatibility towards OSS while piracy is prevented. We therefore identify a potential original conflict of interest between the PS firm and the social planner which has not been likely to be highlighted in the literature. Indeed, although the software compatibility regime provides higher-leveled outcomes from both private and social sides, our results suggest that welfare decreases when piracy applies whereas profit increases in such a framework. Our results go in line in a large body of the literature dealing with digital piracy inasmuch as we once again identify another case of 'profitable' piracy. However, letting the monopolist defining its compatibility and protection strategies in addition to its price one is found to be detrimental in terms of welfare. This obviously once again raises complex concerns about policies and social efficiency in the case of digital piracy.

The study we have carried out has nevertheless some limitations. First of all, we have considered the size of the high market (i.e., α) to be given. Introducing endogeneity for α would allow us to better understand the impact of the penetration of piracy on the ability of the PS firm to benefit from it when the social planner defines optimal anti-piracy enforcement policies. We could secondly add a R&D-setting so as to downplay the symmetry in the intrinsic qualities of the three types of software in the model we have developed. One may finally find appropriate to turn to an oligopolistic framework in which several PS producers compete to verify if our results hold. This leaves room for further research to better understand the impact of digital piracy in the software industry, in which traditional software firms have now to deal with atypical sources of innovation.

7 Appendix

Proof of lemma 1. Maximizing the profit function of the firm with respect to its protection level against piracy under the incompatibility regime, we get the following first order condition:

$$\frac{\partial \pi^{INC}}{\partial s} = 0 \Leftrightarrow -\frac{(3s + \alpha + 2s\alpha - s\alpha^2 + \theta\alpha^2 - \theta\alpha^3 - 3s\theta\alpha + 2s\theta\alpha^2 + s\theta\alpha^3 - 1)}{4(\theta\alpha^2 - \theta\alpha + 1)} = 0 \quad (21a)$$

The second order condition implies that

$$\frac{\partial^2 \pi^{INC}}{\partial s^2} < 0 \Leftrightarrow -\frac{(2\alpha - \alpha^2 - 3\theta\alpha + 2\theta\alpha^2 + \theta\alpha^3 + 3)}{4(\theta\alpha^2 - \theta\alpha + 1)} < 0 \quad (21b)$$

, which can be shown to be met inasmuch as $\theta\alpha^2 - \theta\alpha + 1 = \theta\alpha^2 + (1 - \theta\alpha) > 0$ and $2\alpha - \alpha^2 - 3\theta\alpha + 2\theta\alpha^2 + \theta\alpha^3 + 3 = \alpha((2 - \alpha) + 2\theta\alpha + \theta\alpha^2) + 3(1 - \theta\alpha) > 0$. Solving for s in (21a) yields

$$s^{INC*} = \frac{(1 - \alpha)(1 - \theta\alpha^2)}{2\alpha - \alpha^2 - 3\theta\alpha + 2\theta\alpha^2 + \theta\alpha^3 + 3} \quad (21c)$$

, which obviously defines a positive level from above. Substituting this expression into (9) provides the expression of the firm's profit under the incompatibility regime in (19). Similarly, maximizing the profit function of the firm with respect to its protection level against piracy under the compatibility regime, we get the following first order condition

$$\frac{\partial \pi^C}{\partial s} = 0 \Leftrightarrow \frac{1}{4}s\alpha^2 - \frac{1}{4}\alpha - \frac{1}{2}s\alpha - \frac{3}{4}s + \frac{1}{4} = 0 \quad (22a)$$

The second order condition implies that

$$\frac{\partial^2 \pi^C}{\partial s^2} < 0 \Leftrightarrow -\frac{2\alpha - \alpha^2 + 3}{4} < 0 \quad (22b)$$

, which is once again shown to be fulfilled because $2\alpha - \alpha^2 + 3 = -(\alpha - 3)(\alpha + 1) > 0$ for any value of $\alpha \in [0, 1]$. Solving for s in (22a) yields

$$s^C = \frac{1 - \alpha}{2\alpha - \alpha^2 + 3} \quad (22c)$$

, which obviously defines a positive level from above. Substituting this expression into (12) provides the expression of the firm's profit under the compatibility regime in (20). The differential

between the protection equilibria under the two regimes is therefore computed as

$$\Delta s = s^{INC} - s^C = \frac{\alpha(1-\alpha)}{2\alpha^2 - 2\alpha - 3} \frac{\alpha - 3\theta + 5\theta\alpha + 3\theta\alpha^2 - 2\theta\alpha^3}{2\alpha - \alpha^2 - 3\theta\alpha + 2\theta\alpha^2 + \theta\alpha^3 + 3}$$

Given that $2\alpha - \alpha^2 - 3\theta\alpha + 2\theta\alpha^2 + \theta\alpha^3 + 3 > 0$, $\alpha(1-\alpha) > 0$ and $2\alpha^2 - 2\alpha - 3 = 2(\alpha + \frac{1}{2})(\alpha - \frac{3}{2}) < 0$ for any value of $\alpha \in [0, 1]$, the sign of Δs is given by that of $\alpha - 3\theta + 5\theta\alpha + 3\theta\alpha^2 - 2\theta\alpha^3$. Since we have assumed that $\alpha \in]0, 1[$, it can be proved that the sign of this expression is always positive for any value of α defined so that $\alpha < \tilde{\alpha} \simeq 0.4827$, negative otherwise. ■

Proof of proposition 2. Comparing between (19) and (20) we get the differential between the profit equilibria under the two regimes:

$$\begin{aligned} \Delta\pi &= \pi^{INC} - \pi^C \\ &= \frac{(1 - \theta\alpha^2)^2}{2(1 - \theta\alpha)(\alpha(2 - \alpha) + 3(1 - \theta\alpha) + \theta\alpha^2(\alpha + 2))} - \frac{1}{2} \frac{1}{\alpha(2 - \alpha) + 3} \\ &= \frac{\theta\alpha}{2(-\alpha^2 + 2\alpha + 3)} \frac{(2\alpha^3 - 6\alpha^2 - 6\alpha - 3\theta\alpha + 2\theta\alpha^2 + 4\theta\alpha^3 + 2\theta\alpha^4 - \theta\alpha^5 + 6)}{(-\theta^2\alpha^4 - 2\theta^2\alpha^3 + 3\theta^2\alpha^2 + 2\theta\alpha^3 - 6\theta\alpha - \alpha^2 + 2\alpha + 3)} \end{aligned}$$

The sign of this expression is given by that of $\frac{(2\alpha^3 - 6\alpha^2 - 6\alpha - 3\theta\alpha + 2\theta\alpha^2 + 4\theta\alpha^3 + 2\theta\alpha^4 - \theta\alpha^5 + 6)}{(-\theta^2\alpha^4 - 2\theta^2\alpha^3 + 3\theta^2\alpha^2 + 2\theta\alpha^3 - 6\theta\alpha - \alpha^2 + 2\alpha + 3)}$, whose analysis is somewhat involved. Complex calculations - for which additional material is available upon request - would show that thresholds $\underline{\alpha}$, $\bar{\alpha}$ and $\bar{\theta}$ exist such that four cases are identified, namely case 1, case 2, case 3 and case 4. In case 1, when $0 \leq \alpha \leq \underline{\alpha}$, whatever the value of $\theta \in]0, 1[$ is, $\Delta\pi > 0$. In case 2, when $\bar{\alpha} < \alpha < 1$, whatever the value of $\theta \in]0, 1[$ is, $\Delta\pi < 0$. In case 3, when $\underline{\alpha} < \alpha < \bar{\alpha}$ and $0 < \theta < \bar{\theta}$, $\Delta\pi < 0$. In case 4, when $\underline{\alpha} < \alpha < \bar{\alpha}$ and $\bar{\theta} < \theta < 1$, $\Delta\pi > 0$. These four latter cases define the likelihood of the firm to provide compatibility towards the OSS solution or not to. ■

Proof of proposition 3. To demonstrate proposition 3, we start by showing that the level of profit the firm obtains when it provides incompatibility towards the OSS solution while piracy does not apply is always lower than at least one of the other possible outcomes. To do so, let denote $\Delta\pi_1 = \pi_B^{INC*} - \pi_B^{C*}$, $\Delta\pi_2 = \pi_B^{INC*} - \pi^{INC*}$ and $\Delta\pi_3 = \pi_B^{INC*} - \pi^{C*}$. From proposition 1 and its proof, we already know that $\Delta\pi_1 < 0$. The likelihood of the firm to benefit from

piracy when it provides incompatibility towards the OSS product is given by the sign of $\Delta\pi_2$.

$$\begin{aligned}\Delta\pi_2 &= \frac{(1-\theta)}{8} - \frac{(1-\theta\alpha^2)^2}{2(1-\theta\alpha)(\alpha(2-\alpha)+3(1-\theta\alpha)+\theta\alpha^2(\alpha+2))} \\ &= -\frac{(1-\alpha)(\theta^3\alpha^3+3\theta^3\alpha^2-5\theta^2\alpha^3-9\theta^2\alpha^2-6\theta^2\alpha+2\theta\alpha^2+11\theta\alpha+3\theta-\alpha+1)}{8(-\theta^2\alpha^4-2\theta^2\alpha^3+3\theta^2\alpha^2+2\theta\alpha^3-6\theta\alpha-\alpha^2+2\alpha+3)}\end{aligned}$$

The sign of $\Delta\pi_2$ is given by that of $-\frac{(\theta^3\alpha^3+3\theta^3\alpha^2-5\theta^2\alpha^3-9\theta^2\alpha^2-6\theta^2\alpha+2\theta\alpha^2+11\theta\alpha+3\theta-\alpha+1)}{(-\theta^2\alpha^4-2\theta^2\alpha^3+3\theta^2\alpha^2+2\theta\alpha^3-6\theta\alpha-\alpha^2+2\alpha+3)}$. After some rearrangements, we find that $\theta^3\alpha^3+3\theta^3\alpha^2-5\theta^2\alpha^3-9\theta^2\alpha^2-6\theta^2\alpha+2\theta\alpha^2+11\theta\alpha+3\theta-\alpha+1$ can be reexpressed by $(1-\alpha)+(\theta\alpha)(\theta\alpha-1)(\theta\alpha-8)+(2\theta\alpha^2)(1-\theta\alpha)+(3\theta)\left((\theta-1)^2+\alpha(1-\theta\alpha^2)\right) > 0$ and that $-\theta^2\alpha^4-2\theta^2\alpha^3+3\theta^2\alpha^2+2\theta\alpha^3-6\theta\alpha-\alpha^2+2\alpha+3$ can be rewritten as $(\theta\alpha)^2(1-\alpha^2)+(2\theta\alpha^3)(1-\alpha)+\alpha(1-\theta)+(1-\theta\alpha)(3-2\theta\alpha) > 0$. It results that $\Delta\pi_2 < 0$. The computation of $\Delta\pi_3$ leads to

$$\Delta\pi_3 = \frac{(1-\theta)}{8} - \frac{1}{2\alpha(2-\alpha)+3} = \frac{(3\theta-2\alpha+\alpha^2+2\theta\alpha-\theta\alpha^2+1)}{8(\alpha+1)(\alpha-3)}$$

We deduce from this that the sign of $\Delta\pi_3$ is given by that of $-(3\theta-2\alpha+\alpha^2+2\theta\alpha-\theta\alpha^2+1)$, which can also be expressed by $-(3\theta+\theta\alpha(2-\alpha)+(1-\alpha)^2) < 0$. Thus, $\Delta\pi_3 < 0$. Consequently, we find that providing incompatibility towards the OSS product when piracy does not apply never allows the firm to generate the highest profit level amongst the four outcomes. We next have to compare π_B^{C*} , π^{C*} and π^{INC*} to eventually demonstrate proposition 3. To do so, let us also show that $\pi^{C*} > \pi_B^{C*}$ whatever the values of $\alpha \in [0, 1]$ and $\theta \in]0, 1[$ are. The differential between these two profit levels can be expressed by $\Delta\pi_4 = \pi_B^{C*} - \pi^{C*}$, with

$$\Delta\pi_4 = \frac{1}{2\alpha(2-\alpha)+3} - \frac{1}{8} = \frac{1}{8} \frac{(1-\alpha)^2}{2\alpha-\alpha^2+3}$$

From the proof of lemma 1, it is easy to see that $\Delta\pi_4 < 0$. Consequently, we find that there are three ways of ranking π_B^{C*} , π^{C*} and π^{INC*} , namely (i) $\pi^{C*} > \pi^{INC*} > \pi_B^{C*}$, (ii) $\pi^{C*} > \pi_B^{C*} > \pi^{INC*}$ and (iii) $\pi^{INC*} > \pi^{C*} > \pi_B^{C*}$. From this, we find that the firm generates the highest profit level when piracy applies, provided that it adopts a suitable compatibility strategy to do so. Indeed, complex calculations - for which additional material is available upon request - would show that the firm benefits from both software incompatibility and piracy when the size of the high market is low whatever the value of network parameter θ (i.e., $0 < \alpha \leq 0.6601$

and $0 < \theta < 1$) or when the size of the high market is intermediate while the value of network parameter θ is high (i.e., $0.6601 < \alpha \leq 0.7281$ and $\underline{\theta}_1 < \theta < 1$, with $\underline{\theta}_1 \in]0, 1[$). Otherwise, the firm is found to be better off providing software compatibility, once again benefiting from piracy. ■

Proof of proposition 4. The surplus enjoyed by software users when the firm provides incompatibility (resp. compatibility) towards the OSS product when piracy does not apply is given by US_B^{INC*} (resp. US_B^{C*}), given as follows:

$$\begin{aligned} US_B^{INC*} &= \int_0^{\frac{1}{4}} \left(r - x + \frac{\theta}{4} - \left(\frac{1-\theta}{2} \right) \right) dx + \int_{\frac{1}{4}}^1 \left(r - (1-x) + \frac{\theta}{4} \right) dx \\ &= r + \frac{3}{8}\theta - \frac{7}{16} \end{aligned}$$

$$\begin{aligned} US_B^{C*} &= \int_0^{\frac{1}{4}} \left(r - x + \theta - \frac{1}{2} \right) dx + \int_{\frac{1}{4}}^1 (r - (1-x) + \theta) dx \\ &= r + \theta - \frac{7}{16} \end{aligned}$$

From the latter expressions, we derive the welfare levels which are generated when the firm provides incompatibility (resp. compatibility) towards the OSS product when piracy does not apply:

$$\begin{aligned} W_B^{INC*} &= \left(r + \frac{3}{8}\theta - \frac{7}{16} \right) + \frac{(1-\theta)}{8} = r + \frac{1}{4}\theta - \frac{5}{16} \\ W_B^{C*} &= \left(r + \theta - \frac{7}{16} \right) + \frac{1}{8} = r + \theta - \frac{5}{16} \end{aligned}$$

Defining $\Delta W_1 = W_B^{C*} - W_B^{INC*}$, we can easily show that $W_B^{C*} > W_B^{INC*}$ whatever the value of $\theta \in]0, 1[$ is:

$$\Delta W_1 = W_B^{C*} - W_B^{INC*} = \left(r + \theta - \frac{5}{16} \right) - \left(r + \frac{1}{4}\theta - \frac{5}{16} \right) = \frac{3}{4}\theta > 0$$

Turning to the cases in which piracy is introduced, we can define the surplus levels that are reached out by users, depending on the compatibility strategy of the firm. When the firm

provides incompatibility towards the OSS product, users' surplus writes as

$$\begin{aligned}
US^{INC*} = & \alpha \int_0^{\tilde{x}^{INC*}} (r - x + \theta (\alpha \tilde{x}^{INC*} + (1 - \alpha)) - p^{INC*}) dx \\
& + (1 - \alpha) \int_0^{\tilde{y}^{INC*}} (r - y + \theta (\alpha \tilde{x}^{INC*} + (1 - \alpha)) - p^{INC*}) dy \\
& + \alpha \int_{\tilde{x}^{INC*}}^1 (r - (1 - x) + \theta (\alpha (1 - \tilde{x}^{INC*}))) dx \\
& + (1 - \alpha) \int_{\tilde{y}^{INC*}}^1 (r - (1 - y) + \theta (\alpha \tilde{x}^{INC*} + (1 - \alpha)) - s^{INC*}) dy
\end{aligned}$$

, with

$$\tilde{x}^{INC*} = \frac{(2\theta^2\alpha^4 + 3\theta^2\alpha^3 - 8\theta^2\alpha^2 + 3\theta^2\alpha - 3\theta\alpha^3 + \theta\alpha^2 + 7\theta\alpha - 3\theta + \alpha^2 - 2\alpha - 1)}{2(\theta\alpha - 1)(\alpha(2 - \alpha) + 3(1 - \theta\alpha) + \theta\alpha^2(\alpha + 2))}$$

and

$$\tilde{y}^{INC*} = \frac{(\alpha - \alpha^2 - 3\theta\alpha + 3\theta\alpha^2 + 2\theta\alpha^3 + 2)}{2(\alpha(2 - \alpha) + 3(1 - \theta\alpha) + \theta\alpha^2(\alpha + 2))}$$

When the firm provides compatibility towards the OSS product, users' surplus is

$$\begin{aligned}
US^{C*} = & \alpha \int_0^{\tilde{x}^{C*}} (r - x + \theta - p^{C*}) dx + (1 - \alpha) \int_0^{\tilde{y}^{C*}} (r - y + \theta - p^{C*}) dy \\
& + \alpha \int_{\tilde{x}^{C*}}^1 (r - (1 - x) + \theta) dx + (1 - \alpha) \int_{\tilde{y}^{C*}}^1 (r - (1 - y) + \theta - s^{C*}) dy
\end{aligned}$$

, with

$$\tilde{x}^{C*} = \frac{1}{2} \frac{\alpha(2 - \alpha) + 1}{(\alpha(2 - \alpha) + 3)}$$

and

$$\tilde{y}^{C*} = \frac{1}{2} \frac{2 - \alpha}{3 - \alpha}$$

Ensuing welfare levels are consequently given by

$$\begin{aligned}
W^{INC*} &= US^{INC*} + \pi^{INC*} \\
&= \alpha \int_0^{\tilde{x}^{INC*}} (r - x + \theta (\alpha \tilde{x}^{INC*} + (1 - \alpha))) dx \\
&\quad + (1 - \alpha) \int_0^{\tilde{y}^{INC*}} (r - y + \theta (\alpha \tilde{x}^{INC*} + (1 - \alpha))) dy \\
&\quad + \alpha \int_{\tilde{x}^{INC*}}^1 (r - (1 - x) + \theta (\alpha (1 - \tilde{x}^{INC*}))) dx \\
&\quad + (1 - \alpha) \int_{\tilde{y}^{INC*}}^1 (r - (1 - y) + \theta (\alpha \tilde{x}^{INC*} + (1 - \alpha)) - s^{INC*}) dy
\end{aligned}$$

and

$$\begin{aligned}
W^{C*} &= US^{C*} + \pi^{INC*} \\
&= \alpha \int_0^{\tilde{x}^{C*}} (r - x + \theta) dx + (1 - \alpha) \int_0^{\tilde{y}^{C*}} (r - y + \theta) dy \\
&\quad + \alpha \int_{\tilde{x}^{C*}}^1 (r - (1 - x) + \theta) dx + (1 - \alpha) \int_{\tilde{y}^{C*}}^1 (r - (1 - y) + \theta - s^{C*}) dy
\end{aligned}$$

Further manipulations and one-by-one comparative statics lead to severe complications. Nevertheless, denoting $\Delta W_2 = W_B^{C*} - W^{INC*}$ and $\Delta W_3 = W_B^{C*} - W^{C*}$, we can eventually find that $\Delta W_2 > 0$ and $\Delta W_3 > 0$, which completes the proof of proposition 4 (further demonstration material is available upon request). ■

8 References

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