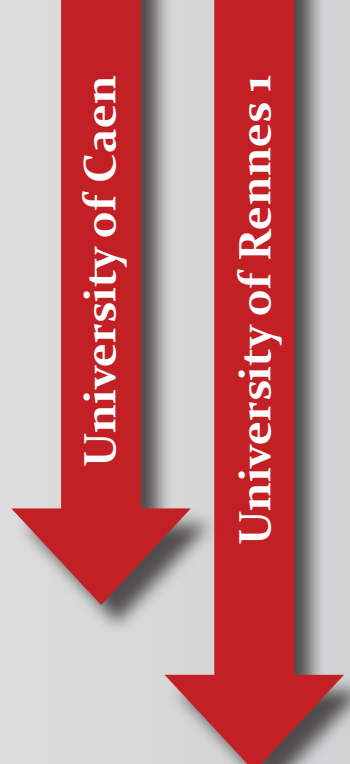




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# Sequential screening with privately known characteristics of cost distribution

**Daniel Danau**

*University of Caen Basse-Normandie, CREM CNRS UMR 6211, France*

**Annalisa Vinella**

*Università degli Studi di Bari «Aldo Moro», Italy*

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# Sequential screening with privately known characteristics of cost distribution

Daniel Danau\*

Annalisa Vinella<sup>†</sup>

## Abstract

We consider a sequential screening problem where, in the contracting stage, the agent has private information on *both* the expected value and the spread of the unit cost of production. As the principal's marginal surplus function becomes less concave / more convex in consumption units, information rents and quantity distortions in the optimal contract reflect progressively stronger concerns with the agent being eager to misrepresent the spread rather than the expected value. As long as marginal surplus is not very convex, relevant incentives to lie on each of the two information dimensions taken separately go in the same direction as in sequential screening problems where only the expected value, or only the spread, is privately known. Otherwise, unusual incentives come to matter. None of the contractual solutions, which are found for different principal's preferences, reduces to familiar sequential screening mechanisms (Riordan and Sappington, 1987; Courty and Li, 2000). The solution is reminiscent of a multidimensional screening mechanism (Armstrong and Rochet, 1999) only if marginal surplus is sufficiently convex.

*Keywords:* Sequential screening; multidimensional screening; expected cost; spread; marginal surplus function

*J.E.L. Classification Numbers:* D82

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\*Université de Caen Basse-Normandie, Centre de Recherche en Economie et Management, 19 rue Claude Bloch, 14000 Caen. E-mail: daniel.danau@unicaen.fr

<sup>†</sup>University of Bari, Department of Economics and Quantitative methods, Via C. Rosalba 53, 70124 Bari (Italy). E-mail: a.vinella@dse.uniba.it

# 1 Introduction

## MOTIVATION AND AIM

In the literature on agency problems, there is now a number of studies on contractual design in situations where the agent observes privately a signal about the distribution of some variable which matters in the relationship with the principal. That signal is taken to be either the expected value of the variable (as in Riordan and Sappington [34], Courty and Li [8], Eso and Szentes [13], Kraemer and Strausz [23]) or, alternatively, the spread of the variable around the mean (as in Courty and Li [8], Dai *et al.* [9], Hoffmann and Inderst [19]).<sup>1</sup> However, it is natural that, if an agent can form more accurate forecasts than the principal, then he will enjoy an information advantage on *both* the characteristics of the distribution of the relevant variable, namely the expected value and the spread, rather than only one of them.<sup>2</sup>

In the delegation of public works or services, including transportation, energy, water and information technology, there are many concerns about errors made in the forecast of future demand and costs, and it is generally argued that errors widely mirror incentive problems (see Flyvbjerg [14]). Looking at transportation policy and planning, Flyvbjerg [14] and Flyvbjerg *et al.* [15] - [16] find that estimations are largely inaccurate and argue that an important part of inaccuracy is explained by strategic misrepresentation. To understand why and how delegated firms might want to manipulate forecasts, it should be considered that they present feasibility analyses to delegating authorities. Such analyses contain an estimate of the entire distribution of the unknown variable, including both the expected value and the spread, on the basis of which contractual terms are then determined. Having these situations in mind, in this study we attempt to understand the incentives to camouflage forecasts that an agent might have *vis-à-vis* the principal when he observes privately both the mean and the spread of the distribution of the unknown variable, which we take to be the cost of production. We further aim at assessing the implications that this informational advantage has on contractual design.

In environments where the agent observes a signal about the distribution of the relevant variable before the contract is signed, it is very likely that he will also learn the realized value in a later stage. A sequential learning process takes place, involving that it is optimal for the principal to screen the agent in subsequent stages.<sup>3</sup> Studying sequential screening in monopoly franchising, Riordan and Sappington [34] show that an information rent is to be conceded to the franchisee-firm for not exaggerating the expected cost in the contracting stage and for not pretending to operate at a high cost in a later stage, if a low cost is realized instead. To contain that rent, production is optimally decreased below its efficient level for any cost realization, provided that the expected cost is high.<sup>4</sup> Referring to other contexts, Courty and Li [8] and a

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<sup>1</sup>Specifically, in the former bunch of papers, distributions are ranked in the sense of first-order stochastic dominance; in the latter, they are ranked in the sense of mean-preserving spread.

<sup>2</sup>In line with this is the observation made by Hoffmann and Inderst [19] in the conclusion to their study.

<sup>3</sup>As Myerson [33] shows, the principal should require the agent to provide a report every time he acquires some new piece of information during the execution of the contract.

<sup>4</sup>In the literature on delegation of activities developed thereafter, the concern with how to screen a delegated firm that acquires new information in subsequent stages is little present. That literature evolves in two main

few articles thereafter find that, when the agent is privately informed about the spread of the variable, he has incentives to understate it. Consequently, the principal gives up an information rent when the values of the unknown variable are little spread around the mean and, to contain that rent, she sets the associated volumes of trade such that the gap between them is decreased below its efficient level. It remains unclear which incentives an agent has to (mis)represent the expected value and the spread of the unknown variable in situations where he observes the two of them privately, and how contractual design is then affected. We contribute to the literature by exploring these issues.

Our analytical framework is as follows. The principal offers a contract to an agent for the provision of some good or service and, as standard in sequential screening models, the parties fully commit to the contract so that individual rationality can be required from an *ex-ante* perspective only. Four cost distributions are known to be possible; in each distribution the cost can take two values which are symmetric around the mean and can be realized with equal probabilities. The agent observes the distribution privately before contracting with the principal. Thus, the principal handles four possible types when the contract is drawn up, leading to eight possible cost values thereafter.<sup>5</sup> By considering this information structure, we bridge the literature on sequential screening problems with that on multidimensional information issues where, in the contracting stage, the agent holds more than one piece of information concerning either one activity (Armstrong [1], Asker and Cantillon [3]) or two activities (Dana [10], Armstrong and Rochet [2]). In so doing, we bring novel elements to either domain of literature. On the one hand, unlike in studies on sequential screening, distributions can be ranked neither in the sense of first-order stochastic dominance nor in the sense of mean-preserving spread.<sup>6</sup> On the other hand, unlike in multidimensional screening models, private knowledge is about the characteristics of some distribution. The circumstance that the two pieces of information pertain to one distribution causes analytical complications and introduces specificities in contractual design as compared to static multidimensional settings.

## MAIN RESULTS

**Information rents and production distortions** In standard screening problems with one dimension of private information *à la* Baron and Myerson [5], higher-order types have stronger incentives to report adjacent than non-adjacent lower-order types. This is because the

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strands. The first strand includes models where the firm decides whether to take some action before learning the true cost, or the decision is delayed until after the cost is realized (Kjerstad and Vagstad [21], Board [6], Mougeot and Naegelen [32]). The second strand includes models on cost overruns, where the firm is privately informed on the expected cost in the delegation stage, whereas the true cost is observed publicly in a subsequent stage (Spulber [37], Chen and Smith [7]). The issue of cost overruns is also argued to be critical in recent studies on public-private partnerships. However, in the latter, the expected cost is commonly known, whereas its realization is observed privately by the firm (Laffont [25], Guasch *et al.* [17] - [18], Iossa and Martimort [20], Danau and Vinella [11]).

<sup>5</sup>The literature typically assumes that the values of the distribution of the unknown variable are drawn from a continuous range rather than a discrete set. In our study, restricting attention to discrete values is functional to determining the whole bulk of solutions to the screening problem and to developing a comparative analysis.

<sup>6</sup>The fact that having no complete ordering of distributions complicates screening problems is fairly known in the literature. See Rochet and Stole [35] for a comment on this point.

information rents they would receive for not mimicking non-adjacent types are included in those they receive for not mimicking adjacent types, hence they anyway appropriate those benefits. The same occurs in screening problems with multiple but uncorrelated information dimensions, as illustrated in Armstrong and Rochet [2]. In fact, a screening problem with this information structure is simply twice a replica of a unidimensional screening problem. In our framework, although mean and spread are uncorrelated in each cost distribution, this kind of result cannot arise. There are various possible combinations of binding incentive constraints, not necessarily corresponding to adjacent types; accordingly, there are various possible contractual solutions to the principal's screening problem. Which exact solution arises depends on the shape of the marginal surplus function. This follows from the circumstance that the two pieces of information blended together in each type pertain to the same cost distribution.

By looking at the whole bulk of possible solutions to the screening problem, we deduce that, as the marginal surplus function becomes less concave / more convex, the principal is increasingly more concerned with eliminating incentives to mimic adjacent rather than non-adjacent types. Consequently, the less concave / more convex is marginal surplus, the more rents the principal must concede to elicit information on the spread rather than on the expected value of the cost. In turn, production distortions induced to contain rents conceded to prevent misrepresentation of the spread become increasingly more important than those induced to contain rents conceded to prevent misrepresentation of the expected cost. The screening problem reduces to a standard multidimensional screening problem, as in Armstrong and Rochet [2], if and only if the marginal surplus function is sufficiently convex. On the other hand, as we clarify below, there is no solution coinciding with the contractual allocation that is optimal in a sequential screening problem with unidimensional information.

**(Un)usual incentives to misrepresent information** The information issues the principal faces in our setting are similar, in general, to those detected in sequential screening models where private information concerns either the expected value or the spread of the unknown variable, not both. Broadly speaking, unless incentives to truth-tell are provided, the agent is tempted to exaggerate the expected cost *vis-à-vis* the principal; on the other hand, he is tempted to understate the spread of the cost. However, in our framework, "unusual" incentives appear when the marginal surplus function is sufficiently convex.

First, when dealing with a low-expected-cost but high-spread agent, the principal is more concerned with preventing understatement of the spread rather than exaggeration of the expected cost. This result suggests that a principal whose marginal surplus is sufficiently convex does not need to be particularly concerned with the incentives to overstate the expected cost of an agent displaying a low expected cost, as is usually the case in the literature, provided that the agent also observes a high spread. A more important issue comes from the temptation that an agent may have to understate the spread when this is high and the expected cost is low instead. The intuition is as follows. When marginal surplus is sufficiently convex, the principal prefers to associate well differentiated production levels to the two possible cost realizations

of each distribution. A high gap between production levels in a given cost distribution comes along with a low expected total cost. This motivates the agent to pretend that he forecasts a high total cost by understating the spread, rather than overstating the expected unit cost.

Second, provided the principal prefers a low-expected-cost low-spread agent to a high-expected-cost high-spread agent, the former type has incentives to exaggerate not only the expected cost but also the spread. This finding is in contrast with other sequential screening problems, where types are ordered in the sense of mean-preserving spread. It is rooted in the circumstance that, in addition to the preference on the production schedule previously described, the principal has also a second preference, *i.e.*, associate more dispersed production levels with more dispersed cost values within a given distribution. Designing a production schedule with this characteristic makes it more difficult to elicit information from a low-expected-cost low-spread agent, involving that a rent must be conceded to prevent exaggeration of the spread on top of the expected cost.

**No contractual solution collapses onto a sequential screening mechanism with unidimensional information** When presenting the first result, we emphasized that, in situations where the marginal surplus function is sufficiently convex, the optimal mechanism is similar to the contractual solution to a multidimensional screening problem. On the other hand, there is no such similarity with sequential screening mechanisms. Despite that "unusual" incentives to camouflage information are found to arise only when marginal surplus is sufficiently convex, as we illustrated, distortions are related to the possibility of cheating on the two information pieces at once in all solutions the screening problem can take. Indeed, when distorting expected output away from the efficient level for some type of agent to contain the rent given up to prevent lies on the expected cost, the principal can adjust the production levels associated with the two possible cost values to a different extent. The exact balance of the two adjustments cannot be chosen without also considering incentives to misrepresent the spread. In other words, a fundamental link is detected between the efficiency/rent-extraction trade-off that the principal faces due to the information problem on the expected cost and the efficiency/rent-extraction trade-off that she faces due to the information problem on the spread. Actually, this is the root of the plurality of solutions that the problem of the principal has, depending upon her preferences for the good.

## 1.1 Outline

The remainder of the article is organized as follows. In section 2, we describe the model and formalize the principal's problem. In section 3, we identify the two reduced problems on which we can rely to determine the solution to the general problem. The solution is characterized in section 4 for different shapes of the marginal surplus function. After presenting general results in section 5, we suggest how to make practical use of them in section 6. Section 7 offers a comparison of results with the mainly related literature. Section 8 concludes. Mathematical details are relegated to an appendix.

## 2 The model

Consider a principal (P) who delegates an activity to an agent for the provision of some good or service. Production of  $y$  units of the good is compensated with a payment of  $t$ . Consumption of  $y$  units yields a gross surplus of  $S(y)$ , where  $S(0) = 0$  and  $S(\cdot)$  is twice continuously differentiable and such that  $S'(\cdot) > 0$  and  $S''(\cdot) < 0$ . The relationship between P and the agent takes place in two stages. In the *first* stage, the contract is signed and parties fully commit to it. The agent has private information on the exogenous parameters of the distribution of the unit production cost, say due to the expertise previously acquired by running similar activities. The parameters of the distribution are the expected value  $\theta$  and the spread  $\sigma$ ; they are independent of each other. However, it is commonly known that  $\theta$  takes values in the set  $\{\theta_L, \theta_H\}$ , where  $\theta_H > \theta_L > 0$ , with probabilities  $\nu$  and  $1 - \nu$ ;  $\sigma$  takes values in the set  $\{\sigma_L, \sigma_H\}$ , where  $\sigma_H > \sigma_L > 0$ , with probabilities  $\mu$  and  $1 - \mu$ . In the *second* stage, the unit cost is realized. It is either  $\theta - \sigma$  or  $\theta + \sigma$  with equal probabilities. With this simple specification, the true cost is entirely determined by its mean value plus an independent noise.<sup>7</sup> The agent observes privately whether the true value of the unit cost diverges from the mean value in a good sense (" $-$ ") or in a bad sense (" $+$ "). Then, he produces the good and is compensated by P according to the contract.

### 2.1 The principal's problem

The objective of P is to attain the largest expected surplus net of the payment to the agent. To that end, P offers to the agent an incentive contract which includes, for each initial type  $ij \in \Upsilon \equiv \{LL, LH, HL, HH\}$ , the pair of allocations  $\left(\{y_{ij}, t_{ij}\}, \{\bar{y}_{ij}, \bar{t}_{ij}\}\right)$ , depending on whether the second-stage shock is good or bad (hence, respectively, the final cost is  $\theta_i - \sigma_j$  or  $\theta_i + \sigma_j$ ). Accordingly, the type- $ij$  profit is either  $\underline{\pi}_{ij} = t_{ij} - (\theta_i - \sigma_j)y_{ij}$  or  $\bar{\pi}_{ij} = \bar{t}_{ij} - (\theta_i + \sigma_j)\bar{y}_{ij}$ . To induce the agent to report the observed shock correctly in the second stage, conditional on the deliver of a truthful report  $ij$  in the first stage, P must design profits in compliance with the following *second-stage incentive constraints*:

$$\begin{aligned} \underline{ic}_{ij} &: \underline{\pi}_{ij} \geq \bar{\pi}_{ij} + 2\sigma_j\bar{y}_{ij}, \quad \forall ij \in \Upsilon \\ \bar{ic}_{ij} &: \bar{\pi}_{ij} \geq \underline{\pi}_{ij} - 2\sigma_j y_{ij}, \quad \forall ij \in \Upsilon. \end{aligned}$$

Meeting these constraints requires satisfying the monotonicity condition  $y_{ij} \geq \bar{y}_{ij}$  for all  $ij$ . Assuming no discounting, in expectation the agent obtains  $\Pi_{ij} = \frac{1}{2}(\underline{\pi}_{ij} + \bar{\pi}_{ij})$ .<sup>8</sup> To also induce the agent to report his type correctly in the first stage, P must design expected profits in

<sup>7</sup>The assumption that the stochastic variable is additive in mean and spread and that the latter are independent of each other is also made by Courty and Li [8], Eso and Szentes [13] (Example 1) and Hoffmann and Inderst [19]. Kraahmer and Strausz [23] model the cost as a sum of a signal privately known in the first stage and a random shock realized in the second stage, mainly focusing on the case where the distribution of the shock is independent of the signal.

<sup>8</sup>The assumption of no discounting is without loss of generality as payoffs depend on the second-stage production levels and transfers.

compliance with the following *first-stage incentive constraints*:<sup>9</sup>

$$\begin{aligned}
IC_{ij}^{i'j'} : \quad & \Pi_{ij} \geq \frac{1}{2} \max \left\{ \bar{\pi}_{i'j'} + (\theta_{i'} - \theta_i + \sigma_{j'} - \sigma_j) \bar{y}_{i'j'}; \underline{\pi}_{i'j'} + (\theta_{i'} - \theta_i - \sigma_{j'} - \sigma_j) \underline{y}_{i'j'} \right\} \\
& + \frac{1}{2} \max \left\{ \underline{\pi}_{i'j'} + (\theta_{i'} - \theta_i + \sigma_j - \sigma_{j'}) \underline{y}_{i'j'}; \bar{\pi}_{i'j'} + (\theta_{i'} - \theta_i + \sigma_{j'} + \sigma_j) \bar{y}_{i'j'} \right\}, \\
& \forall i, j, \forall i', j' \in \Upsilon.
\end{aligned}$$

This formulation takes into account that, when selecting the report to be made in the first stage, the agent anticipates that it will either truth-tell or lie in the second stage. Conditional on the first-stage type being  $ij$ , any second-stage cost value other than  $\theta_i - \sigma_j$  and  $\theta_i + \sigma_j$  has zero probability. This has two implications. First, in the second stage, P does not need to require the agent to report the cost realization but only the shock, namely, "−" or "+". The number of incentive constraints would otherwise be unnecessarily large. Second, the agent has the possibility of coordinating lies between first and second stage. This issue is common in sequential screening problems where the support of final values is type-dependent, involving that the number of first-stage incentive constraints is likely to be richer. To circumvent this difficulty, the literature systematically assumes that the support of final values is non-shifting across distributions.<sup>10</sup> In our setting, it is not possible to have a non-shifting support because expected value and spread are independent of each other, and, at the same time, the realized cost cannot be allowed to take negative values. It can nonetheless be shown that inter-temporally coordinated lies are unattractive for the agent when the degree of uncertainty about the expected cost is sufficiently small, *i.e.*  $\theta_H - \theta_L \leq 2\sigma_L$  (see appendix A). As we are interested to understand how the optimal contract in our framework diverges from usual sequential screening mechanisms, we focus on situations where this condition does hold so as to avoid concerns associated with off-the-equilibrium path lying.<sup>11</sup>

Once quantities are set to satisfy monotonicity conditions, transfers  $\underline{t}_{ij}$  and  $\bar{t}_{ij}$ , hence profits  $\underline{\pi}_{ij}$  and  $\bar{\pi}_{ij}$ , can be found such that, if the resulting expected profit  $\Pi_{ij}$  satisfies  $IC_{ij}^{i'j'}$ , then  $\underline{ic}_{ij}$  and  $\bar{ic}_{ij}$  are met as well. Let  $q_{ij} \equiv \frac{1}{2}(\underline{y}_{ij} + \bar{y}_{ij})$  be the expected output and  $r_{ij} \equiv \frac{1}{2}(\underline{y}_{ij} - \bar{y}_{ij})$  the expected output difference for type  $ij$ . The problem of P, denoted  $\Gamma$ , is formulated as follows:

$$\begin{aligned}
& \underset{\{q_{ij}, r_{ij}, \Pi_{ij}\}}{\text{Max}} \sum_{ij \in \Upsilon} \mathbb{E}_{ij} \left[ \left( \frac{1}{2} (S(q_{ij} + r_{ij}) + S(q_{ij} - r_{ij})) - (\theta_i q_{ij} - \sigma_j r_{ij})) \right) - \Pi_{ij} \right] \\
& \text{subject to} \\
& \begin{cases} r_{ij} \geq 0, \forall ij \\ IC_{ij}^{i'j'} : \Pi_{ij} \geq \Pi_{i'j'} + (\theta_{i'} - \theta_i) q_{ij} + (\sigma_j - \sigma_{j'}) r_{ij}, \forall ij, \forall i', j' \\ PC_{ij} : \Pi_{ij} \geq 0, \forall ij \end{cases}
\end{aligned}$$

<sup>9</sup>In our notation,  $ij \neq i'j'$  indicates that either  $i \neq i'$  or  $j \neq j'$  or both.

<sup>10</sup>An exception is the study of Khramer and Strausz [22].

<sup>11</sup>As remarked by Courty and Li [8] (footnote 7), in a setting with a continuum of values the assumption of non-shifting support could be relaxed to only require that supports of different distributions overlap sufficiently. The assumption we make is the counterpart for that requirement in a model with discrete types.



The payoff of P includes the expected surplus obtained, for all possible types, from consumption of  $q_{ij} + r_{ij}$  units of the good, if the shock is found to be good, and of  $q_{ij} - r_{ij}$  units of the good, if the shock is found to be bad. This surplus is diminished by the expected cost of production  $\theta_i q_{ij} - \sigma_j r_{ij}$  and the expected rent  $\Pi_{ij}$  to be conceded for information release. The incentive constraint  $IC_{ij}^{i'j'}$  ensures that an agent of type  $ij$  is not tempted to mimic some other type  $i'j'$  in the first stage, whether he is going to truthtell or lie in the second stage, as previously explained. The participation constraint  $PC_{ij}$  further ensures that an agent of type  $ij$  is willing to enter the contract with P, being allowed to break even when he is informed of his type but not yet informed of the final cost. We omit the incentive constraints whereby an agent of type  $ij$  has no incentives to misrepresent information in the second stage following a first-stage lie as, under the assumption previously made, they are implied by  $IC_{ij}^{i'j'}$ ,  $\underline{ic}_{ij}$ , and  $\overline{ic}_{ij}$ . This stems from the Revelation Principle for multistage games, according to which one can restrict attention to reports that are conditional on truthtelling in the previous period for information release to be induced in the subsequent period (Myerson [33]).

If participation constraints could be saturated for all types, then P would be able to effect the first-best allocation (henceforth, FB). Appending stars to denote FB values, this is defined by the following set of conditions:  $\Pi_{ij}^* = 0$ ,  $S'(q_{ij}^* + r_{ij}^*) = \theta_i - \sigma_j$ ,  $S'(q_{ij}^* - r_{ij}^*) = \theta_i + \sigma_j$ ,  $\forall ij$ . However, inspection of incentive constraints evidences that this allocation cannot be implemented. As usual in screening problems, quantity distortions will appear at optimum, trading off efficiency losses against rent extraction. What is peculiar to our framework is that this trade-off depends critically on the curvature of the marginal surplus function. Intuitively, if the expected quantity  $q_{ij}$  is distorted downwards at the aim of containing the rents designed to avoid cheating on  $\theta$ , then the expected surplus is decreased by an amount of  $\frac{1}{2} [S'(q_{ij} + r_{ij}) + S'(q_{ij} - r_{ij})]$ . This is not the only effect though. In addition, a second effect is caused on the expected difference between the surplus created when the shock is good, namely  $S(q_{ij} + r_{ij})$ , and the surplus created when the shock is bad, namely  $S(q_{ij} - r_{ij})$ . This effect is specifically measured by a marginal change of  $\frac{1}{2} [S'(q_{ij} + r_{ij}) - S'(q_{ij} - r_{ij})]$ . To see why this is important, observe that, in fact, this amount measures the marginal loss in expected surplus that follows when  $r_{ij}$  is decreased at the aim of containing the rents conceded to prevent lies on  $\sigma$ . Because of this, the trade-off between efficiency loss and extraction of the rents designed to induce truthtelling on  $\theta$  indirectly affects the trade-off between efficiency loss and extraction of the rents designed to induce truthtelling on  $\sigma$ , and *vice versa*. While output levels  $q_{ij} + r_{ij}$  and  $q_{ij} - r_{ij}$  are distributed symmetrically around their mean value of  $q_{ij}$ , surplus values associated with those levels are not, in general, and the resulting asymmetry depends on the curvature of  $S'(\cdot)$ .

Understanding how the shape of  $S'(\cdot)$  affects the contractual solution is a core issue in our study. This issue is found neither in sequential screening problems, where private information is unidimensional, nor in multidimensional screening problems, where the different pieces of information do not refer to the distribution of one same variable. To make progress with the analysis, we first investigate which incentive constraints are potentially binding in  $\Gamma$ .<sup>12</sup>

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<sup>12</sup>In consumption theory, the behaviour of the marginal utility function with respect to consumption units

### 3 Reduced problems

For any given type  $ij$ , the expected total cost, namely  $\theta_i q_{ij} - \sigma_j r_{ij}$ , increases with the expected unit cost  $\theta_i$  and decreases with the cost variability  $\sigma_j$ , provided that  $r_{ij} > 0$ . Thus, P is better off when she faces an agent with lower expected unit cost and/or higher spread. In particular, P prefers a high spread because production levels in each cost distribution must satisfy the second-stage monotonicity condition. This involves that, in each distribution, total costs, unlike unit costs, are asymmetric around the mean value, and that the divergence from the latter is more pronounced when the shock is good rather than bad.<sup>13</sup> It follows that types  $LH$  and  $HL$  are, respectively, the best and the worst type for P. It remains to clarify the ordering of types  $LL$  and  $HH$ . We make the reasonable assumption that the degree of uncertainty about the expected unit cost is more important than that about the spread. Formally, this is tantamount to requiring that  $\Delta\theta > \Delta\sigma$ , where  $\Delta\theta = \theta_H - \theta_L$  and  $\Delta\sigma = \sigma_H - \sigma_L$ . Consequently, P prefers a low-expected-cost and low-spread agent to a high-expected-cost and high-spread agent.<sup>14</sup> The overall type ordering is as follows:

$$LH \succ LL \succ HH \succ HL. \quad (1)$$

Remarkably, unlike in the literature on sequential screening, distributions  $ij$  are ordered neither in the sense of first-order stochastic dominance nor in the sense of mean preserving spread. The mean value of the unit cost is lower for types  $Lj$  than for types  $Hj$ ; it is equal for types  $iH$  and types  $iL$ . However, there is no stochastic dominance condition according to which the four distributions can be ordered altogether. Our stylized representation with discrete types enables us to overcome this limit with the identification of the type ordering in (1). Henceforth, for convenience, we refer to (1) as to an "efficiency" ordering; accordingly,  $LH$  is the most efficient type,  $HL$  the least efficient type, and so on.

Resting on (1), we are able to pin down the downward incentive constraints whereby more efficient types are not tempted to mimic less efficient types. Specifically, type  $LH$  might want to claim  $LL$ ,  $HH$ , or  $HL$ ; type  $LL$  might want to claim either  $HL$  or  $HH$ ; type  $HH$  might want to claim  $HL$ . By restricting attention to these incentives, we focus on a reduced problem, to be denoted  $\Gamma'$ , in which the participation constraint of the least efficient type is binding and all incentive constraints but the downward ones are assumed to be slack. Moreover, to keep the problem interesting, we take the monotonicity condition  $r_{ij} \geq 0$  to be strictly satisfied.

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(the counterpart of which is  $S'(\cdot)$  in our model) affects the trade-off between current consumption and savings for future consumption that an individual faces when maximizing her expected utility under uncertainty about future income (see Leland [27], Rothschild and Stiglitz [36], Drèze and Modigliani [12], Menegatti [31]). The results of that literature have no bite in our framework as the principal does not face a trade-off between first-stage and second-stage quantities, given that production (hence, consumption) takes place only in the second stage.

<sup>13</sup>The fact that P prefers a type with a higher spread around the mean is common in the literature.

<sup>14</sup>In a setting with a continuum of cost values and symmetric distributions (such as normal distributions), this assumption would involve that P prefers one distribution to another when cost values in the first distribution are all smaller than cost values in the second distribution.

The conditions under which this is the case will be identified at a later stage. Once we find the solution to  $\Gamma'$ , we will need to check that it solves  $\Gamma$  as well. Looking at a reduced problem at the aim of identifying the solution to the general problem is a similar approach to that usually followed in multidimensional screening problems (see Armstrong and Rochet [2], for instance). However, the circumstance that screening concerns two characteristics of one distribution in our model introduced additional complications in the analysis. This will become apparent in the sequel of the study. For the time being, we characterize the solution to  $\Gamma'$ .

**Lemma 1** *At the solution to  $\Gamma'$ , information rents are such that:*

$$\Pi_{HL} = 0 \tag{2a}$$

$$\Pi_{HH} = \Delta\sigma r_{HL} \tag{2b}$$

$$\Pi_{LL} = \beta\Pi_{LL,1} + (1 - \beta)\Pi_{LL,2} \tag{2c}$$

$$\Pi_{LH} = \gamma_1\Pi_{LH,1} + \gamma_2[\beta\Pi_{LH,2} + (1 - \beta)\Pi_{LH,3}] + \gamma_3\Pi_{LH,4}, \tag{2d}$$

where

$$\Pi_{LL,1} = \Delta\theta q_{HL} \quad \text{and} \quad \Pi_{LL,2} = \Delta\theta q_{HH} - \Delta\sigma(r_{HH} - r_{HL}) \tag{3}$$

together with

$$\Pi_{LH,1} = \Delta\theta q_{HH} + \Delta\sigma r_{HL}, \quad \Pi_{LH,2} = \Delta\theta q_{HL} + \Delta\sigma r_{LL}, \tag{4}$$

$$\Pi_{LH,3} = \Delta\theta q_{HH} + \Delta\sigma r_{HL} - \Delta\sigma(r_{HH} - r_{LL}) \quad \text{and} \quad \Pi_{LH,4} = \Delta\theta q_{HL} + \Delta\sigma r_{HL},$$

and  $\beta \in [0, 1]$ ,  $\gamma_z \in [0, 1]$ ,  $\forall z \in \{1, 2, 3\}$ ,  $\sum_{z \in \{1, 2, 3\}} \gamma_z = 1$ .

Identifying the downward incentive constraints that are relevant at the solution to  $\Gamma'$ , hence the exact information rents to be conceded, is tantamount to identifying the values of the parameters  $\beta$  and  $\gamma_z$ ,  $\forall z \in \{1, 2, 3\}$ . If the problem we study were a standard multidimensional screening problem with uncorrelated pieces of private information, then relevant incentive constraints would be those whereby more efficient types are not tempted to claim adjacent types.<sup>15</sup> Formally, the relevant incentive constraints would be  $IC_{LL}^{HH}$  and  $IC_{LH}^{LL}$ , meaning that we would have  $\beta = 0$  and  $\gamma_2 = 1$ . However, this does not need to be the case in our framework since identification of the type ordering does not straightforwardly involve identification of the binding incentive constraints. This will become apparent in the characterization of the solution below. Resting on Lemma 1, the quantity solution to  $\Gamma'$  can be determined as a function of  $\beta$  and  $\gamma_z$ .

**Lemma 2** *At the solution to  $\Gamma'$ , FB production levels are assigned to type LH. Production*

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<sup>15</sup>Analogous conclusion is reached in unidimensional screening problems with more than two types. Under some standard conditions, for high-ranked types facing more than one cheating possibility the strongest temptation is to mimic adjacent types, meaning that global incentive constraints are implied by local incentive constraints (see section 3.1 in Laffont and Martimort [26], for instance).

levels assigned to types  $LL$ ,  $HH$  and  $HL$ , respectively, satisfy:

$$\begin{aligned} S'(q_{LL} + r_{LL}) &= \theta_L - \sigma_L + \gamma_2 \frac{1-\mu}{\mu} \Delta\sigma \\ S'(q_{LL} - r_{LL}) &= \theta_L + \sigma_L - \gamma_2 \frac{1-\mu}{\mu} \Delta\sigma, \end{aligned}$$

$$\begin{aligned} S'(q_{HH} + r_{HH}) &= \theta_H - \sigma_H \\ &+ \frac{\nu}{1-\nu} \left\{ \left[ \gamma_1 + (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \right] \Delta\theta - (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \Delta\sigma \right\} \\ S'(q_{HH} - r_{HH}) &= \theta_H + \sigma_H \\ &+ \frac{\nu}{1-\nu} \left\{ \left[ \gamma_1 + (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \right] \Delta\theta + (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \Delta\sigma \right\} \end{aligned}$$

and

$$\begin{aligned} S'(q_{HL} + r_{HL}) &= \theta_H - \sigma_L \\ &+ \frac{\nu}{1-\nu} \left\{ \left[ \beta + (\gamma_2\beta + \gamma_3) \frac{1-\mu}{\mu} \right] \Delta\theta + \left[ 1 - \beta + (1 - \nu\gamma_2\beta) \frac{1-\mu}{\nu\mu} \right] \Delta\sigma \right\} \\ S'(q_{HL} - r_{HL}) &= \theta_H + \sigma_L \\ &+ \frac{\nu}{1-\nu} \left\{ \left[ \beta + (\gamma_2\beta + \gamma_3) \frac{1-\mu}{\mu} \right] \Delta\theta - \left[ 1 - \beta + (1 - \nu\gamma_2\beta) \frac{1-\mu}{\nu\mu} \right] \Delta\sigma \right\}. \end{aligned}$$

As expected, quantities are distorted away from the FB levels for all but the most efficient type in both states of nature. The next step is to check whether there are circumstances under which production levels as characterized in Lemma 2 are inconsistent with Lemma 1. It turns out that this is case when the degree of uncertainty about  $\theta$  is big relative to that about  $\sigma$ .

**Lemma 3** *At the solution to  $\Gamma$ , types  $HH$  and  $HL$  are assigned production levels characterized in Lemma 2 only if:*

$$\frac{\Delta\theta}{\Delta\sigma} < \frac{\frac{1}{\nu} + (\mu - \beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right)}{\left| 1 - \gamma_1 - (1 - \beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \right|}. \quad (5)$$

If (5) is satisfied, then those production levels are such that  $q_{HH} + r_{HH} > q_{HL} + r_{HL}$  and  $q_{HH} - r_{HH} < q_{HL} - r_{HL}$ .

FB efficiency requires that production levels assigned to type  $HH$  be more spread around the expected value of  $q_{HH}^*$  than are spread those assigned to type  $HL$  around the expected value of  $q_{HL}^*$ . That is,  $q_{HH}^* + r_{HH}^* > q_{HL}^* + r_{HL}^*$  and  $q_{HH}^* - r_{HH}^* < q_{HL}^* - r_{HL}^*$ . This is because type  $HH$  faces a higher spread of the cost around the mean value of  $\theta_H$ . Lemma 3 states that, when uncertainty about the mean is substantially more important than uncertainty about the spread (*i.e.*,  $\Delta\theta/\Delta\sigma$  is large), output levels no longer reflect these conditions. In that situation, to contain the rents designed for type  $LL$  not to exaggerate  $\theta$ , P would be more eager to distort the expected production values of types  $HH$  and  $HL$  rather than their expected

production differences. Specifically, she would like to decrease  $q_{HH}$  until the point where  $q_{HH} + r_{HH} < q_{HL} + r_{HL}$ , if the relevant temptation of type  $LL$  is to claim  $HH$ , and  $q_{HL}$  until the point where  $q_{HH} - r_{HH} > q_{HL} - r_{HL}$ , if its relevant temptation is to claim  $HL$ . In either case, such a pronounced distortion is not worth in  $\Gamma$  so that the solution to  $\Gamma'$  does not solve  $\Gamma$  as well. Take the relevant lie to be  $HH$ , for instance. Then, as  $q_{HH}$  is decreased,  $\Pi_{LL,2}$  may fall below  $\Pi_{LL,1}$ . However, at that point, any further reduction in  $q_{HH}$  is useless: type  $LL$  will anyway pocket the bigger rent. Similarly, when the relevant lie is  $HL$ , P cannot extract more rent by decreasing  $q_{HL}$  down to the point where  $\Pi_{LL,1}$  becomes smaller than  $\Pi_{LL,2}$ . Therefore, when  $\Delta\theta/\Delta\sigma$  is sufficiently large, the best for P is not to force distortions in  $q_{HH}$  and  $q_{HL}$  beyond the point where  $\Pi_{LL,1}$  and  $\Pi_{LL,2}$  are equal, hence  $IC_{LL}^{HL}$  and  $IC_{LL}^{HH}$  are both saturated. This leads us to consider the next reduced problem, denoted  $\Gamma''$ , which is tantamount to  $\Gamma'$  up to the two additional constraints  $q_{HL} = q_{HH}$  and  $r_{HL} = r_{HH}$ . Remarkably, bunching types  $HH$  and  $HL$  is a way for the principal to avoid inducing more important distortions than are useful to save on information rents. This result is unusual in the literature, where bunching of types typically arises when efficiency considerations conflict with monotonicity conditions imposed by the incentive constraints of the concerned types, as is the case in agency problems with countervailing incentives (see Lewis and Sappington [29] and Maggi and Rodriguez-Clare [30]).<sup>16</sup>

**Lemma 4** *At the solution to  $\Gamma''$ , production levels assigned to types  $LH$  and  $LL$  are characterized as in Lemma 2. Those assigned to type  $HH$  are characterized as follows:*

$$\begin{aligned} S'(q_{HH} + r_{HH}) &= \theta_H - \sigma_H + \frac{\nu}{(1-\nu)(1-\mu)}\Delta\theta + \frac{1-\gamma_2\nu}{1-\nu}\Delta\sigma \\ S'(q_{HH} - r_{HH}) &= \theta_H + \sigma_H + \frac{\nu}{(1-\nu)(1-\mu)}\Delta\theta - \frac{1-\gamma_2\nu}{1-\nu}\Delta\sigma. \end{aligned}$$

For type  $HL$ , they are such that  $q_{HL} = q_{HH}$  and  $r_{HL} = r_{HH}$ . Information rents are determined as in Lemma 1.

## 4 Characterization of the solution

The next step of analysis is to understand, in each of the two reduced problems, which incentive constraints are relevant for types  $LL$  and  $LH$ . To that end, as previously mentioned, we need to take the preferences of P into account. To keep the analysis tractable and make progress with it, we assume that the sign of  $S'''(\cdot)$  is invariant for all consumption levels for which marginal surplus is strictly positive.<sup>17</sup> The next lemma completes the list of properties

<sup>16</sup>The monotonicity condition imposed by  $IC_{HH}^{HL}$  and  $IC_{HL}^{HH}$  is  $r_{HH} \geq r_{HL}$ . As we show in appendix B.3, if (5) is violated, then the solution to  $\Gamma'$  is such that either  $q_{HH} + r_{HH} > q_{HL} + r_{HL}$  together with  $q_{HH} - r_{HH} > q_{HL} - r_{HL}$ , or  $q_{HH} + r_{HH} < q_{HL} + r_{HL}$  together with  $q_{HH} - r_{HH} < q_{HL} - r_{HL}$ . Neither of these pairs of conditions implies that the monotonicity condition is violated.

<sup>17</sup>This is a usual assumption in theoretical models on consumption decisions under uncertainty, where the shape of the marginal utility function is core to the trade-off between current and future consumption.

of the surplus function.

**Lemma 5**  $S'''(\cdot) < 0$  only if (i)  $\exists Y$  such that  $0 < Y < \infty$  and  $S'(y) = 0, \forall y \in [Y, \infty)$ , and (ii)  $\lim_{y \rightarrow 0} S''(y) > -\infty$ .

From Menegatti [31] we know that, when  $Y \rightarrow \infty$ ,  $S'(\cdot)$  is convex. In line with that result, for  $S'(\cdot)$  to be also possibly concave, it must be the case that  $Y$  is finite. The lemma states that, in addition, it is also necessary that  $\lim_{y \rightarrow 0} S''(y) > -\infty$ . Henceforth, we take the two conditions in Lemma 5 to be satisfied. This is without loss of generality in that it simply enables us to admit any shape of  $S'(\cdot)$  in the analysis. If any such condition were not met, then, out of the solutions presented below, we would only be left with those arising for  $S'(\cdot)$  convex. There are two further points to make when the conditions in Lemma 5 hold. First, the surplus function does not satisfy the Inada conditions, involving that an interior solution may not exist. However, as investigation of corner solutions would not add much insight to our study, we neglect them and take all solution quantities to belong to the interval  $(0, Y)$ . Second, if  $S'''(\hat{y})$  increases/decreases for some given  $\hat{y} \in (0, Y)$ , then so does  $S'''(y)$  for all  $y \in (0, Y)$ . Henceforth, for sake of simplicity, we use the notation  $S'''(\hat{y}) \equiv \zeta$  as a measure of the degree of concavity/convexity of  $S'(\cdot)$ .

**Lemma 6** Define  $S'(y) = a$ , and  $f(a) = y$  the inverse function of  $S'(y)$ . Suppose that  $a_1 > a_2$ ,  $a_3 > a_4$ ,  $a_2 > a_4$ . Then, the difference  $[f(a_4) - f(a_3)] - [f(a_2) - f(a_1)]$  increases with  $\zeta$ . Moreover:

- 1) if  $a_3 - a_4 < a_1 - a_2$ , then there exists at most one value  $\varepsilon_1 > 0$  such that  $f(a_4) - f(a_3) = f(a_2) - f(a_1)$  for  $\zeta = \varepsilon_1$ ;
- 2) if  $a_3 - a_4 \geq a_1 - a_2$ , then there exists at most one value  $\varepsilon_2 < 0$  such that  $f(a_4) - f(a_3) = f(a_2) - f(a_1)$  for  $\zeta = \varepsilon_2$ .

For brevity we use the notation  $S'(q_{ij} + r_{ij}) \equiv a_{ij}^+$  and  $S'(q_{ij} - r_{ij}) \equiv a_{ij}^-$ ,  $\forall ij \in \Upsilon$ , to refer to the quantity solution in Lemma 2 or Lemma 4, depending on whether problem  $\Gamma'$  or  $\Gamma''$  is concerned. The values  $f(a_{ij}^+)$  and  $f(a_{ij}^-)$  respectively represent the production levels  $q_{ij} + r_{ij}$  and  $q_{ij} - r_{ij}$  at the solution to  $\Gamma'$  or  $\Gamma''$ . Hence, for any two types  $ij$  and  $i'j'$ , one can refer to the wedges  $f(a_{ij}^+) - f(a_{i'j'}^+)$  and  $f(a_{ij}^-) - f(a_{i'j'}^-)$  to compare  $r_{ij}$  with  $r_{i'j'}$ , and to the wedges  $f(a_{ij}^+) - f(a_{i'j'}^+)$  and  $f(a_{i'j'}^-) - f(a_{ij}^-)$  to compare  $q_{ij}$  with  $q_{i'j'}$ . Accordingly, Lemma 6 identifies the exact link between the shape of the marginal surplus function and the information rents which are contained by distorting production away from efficient levels at the solution to each of the two reduced problems. Resting on Lemma 6, we can thus determine the relevant incentive constraints in the reduced problems  $\Gamma'$  and  $\Gamma''$ , as presented in Lemma 7 - 9 below.

**Lemma 7** Suppose that (5) holds. At the solution to  $\Gamma'$ ,  $\beta > 0$  if and only if  $\zeta \leq \zeta_1$ , where  $\zeta_1 > 0$  is the highest value of  $\zeta$  such that:

$$r_{HH} - r_{HL} = \frac{\Delta\theta}{\Delta\sigma} (q_{HH} - q_{HL}). \quad (6)$$

Provided  $\zeta \leq \zeta_1$  is equivalent to  $\Pi_{LL,1} \geq \Pi_{LL,2}$ , the lemma establishes which of the two possible rents accrues to type  $LL$  at the solution to  $\Gamma'$ , depending on the preferences of  $P$ . As marginal surplus becomes more convex,  $P$  is more and more concerned with the temptation of type  $LL$  to exaggerate not only the expected value (thus claiming  $HL$ ) but also the spread of the unit cost (thus claiming  $HH$ , instead). This reflects the increasingly stronger interest of  $P$  to recommend more output, in expected terms, from type  $HH$  than from type  $HL$ . (6) determines the value of  $\beta$  as a function of  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$ , which are yet to be identified; the exact value of  $\zeta_1$  depends on all these parameters. However, regardless of the particular values of the parameters, Lemma 7 suggests that there exists a unique value of  $\zeta$  that separates the range of values where  $\Pi_{LL,1} \geq \Pi_{LL,2}$  from that where this inequality is violated.

**Lemma 8** *Suppose that (5) holds. At the solution to  $\Gamma'$ ,  $\exists \varepsilon_H \in (\varepsilon_{LH}, 0)$  such that  $q_{HL} = q_{HH}$  if and only if  $\zeta = \varepsilon_H$ . There exists at most one value  $\varepsilon_L > 0$  such that  $r_{LL} = r_{HH}$ .*

- (i) *If  $\varepsilon_{LH} \leq \zeta \leq \varepsilon_H$ , then  $\gamma_3 = 1$ ,  $\gamma_1 = 0$  and  $\gamma_2 = 0$ .*
- (ii) *If  $\zeta \in (\varepsilon_H, \varepsilon_L)$ , then  $\gamma_3 = 0$ ,  $\gamma_1 > 0$  and  $\gamma_2 > 0$ .*
- (iii) *If  $\zeta \geq \varepsilon_L$ , then  $\gamma_3 = 0$ ,  $\gamma_1 = 0$  and  $\gamma_2 = 1$ .*

As compared to Lemma 7, the cases included in Lemma 8 reflect the circumstance that the most efficient type has more cheating possibilities than type  $LL$ . These cases are identified by considering that, according to Lemma 6, the bigger is  $\zeta$ , the higher the differences  $r_{LL} - r_{HL}$  and  $q_{HH} - q_{HL}$  are set. As long as marginal surplus is sufficiently concave (case (i)), these differences are both negative and  $IC_{LH}^{HL}$  is binding. By contrast, when  $S'(\cdot)$  is linear or nearly so (case (ii)), the differences  $r_{LL} - r_{HL}$  and  $q_{HH} - q_{HL}$  are positive but small and  $IC_{LH}^{HL}$  becomes slack, whereas  $IC_{LH}^{LL}$  and  $IC_{LH}^{HH}$  are both binding at the solution to  $\Gamma'$ . As pointed out with regards to Lemma 7, the exact values of  $\gamma_1$  and  $\gamma_2$  are not essential to understand the finding in (ii); it is yet worth mentioning that they are obtained from the condition  $\gamma_1 + \gamma_2 = 1$ , together with  $\Pi_{LH,1} = \Pi_{LH,2}$  if  $\varepsilon_L \leq \zeta_1$ , and with  $\Pi_{LH,1} = \Pi_{LH,3}$  if  $\varepsilon_L > \zeta_1$  and  $\zeta \in (\varepsilon_L, \zeta_1)$ . Lastly, when  $S'(\cdot)$  is sufficiently convex (case (iii)), the difference  $r_{LL} - r_{HL}$  is big relative to  $q_{HH} - q_{HL}$  and  $IC_{LH}^{HH}$  becomes slack in turn. Hence, in this case, the only concern of  $P$  with a  $LH$ -agent is that he might want to understate the spread.

We are now left with characterizing the solution to  $\Gamma''$  in situations where the values of  $\beta$  and  $\gamma_z$ ,  $\forall z$ , that are determined at the solutions in Lemma 7 and 8, are such that (5) is violated. Apart from forcing types  $HL$  and  $HH$  to produce the same output in order to ensure that type  $LL$  is given up the smallest affordable rent for not mimicking either such type, the relationship between the shape of the marginal surplus function and the incentives of type  $LH$  that matter for  $P$  follows a similar pattern to that described in previous lemmas. Since  $q_{HH} = r_{HH}$  and  $q_{HL} = r_{HL}$  in  $\Gamma''$ ,  $\gamma_3$  and  $\beta$  become irrelevant and they are conveniently set to zero. Accordingly, the following lemma is stated.

**Lemma 9** *Suppose that (5) is violated.  $\exists \widehat{\varepsilon}_H < 0$ ,  $\widehat{\varepsilon}_L > 0$  such that  $r_{LL} < r_{HH}$  if  $\zeta < \widehat{\varepsilon}_H$ ,  $r_{LL} = r_{HH}$  if  $\zeta \in [\widehat{\varepsilon}_H, \widehat{\varepsilon}_L]$ , and  $r_{LL} > r_{HH}$  if  $\zeta > \widehat{\varepsilon}_L$ .*

- (i) If  $\zeta < \widehat{\varepsilon}_H$ , then  $\gamma_3 = 0$ ,  $\gamma_1 = 1$  and  $\gamma_2 = 0$ .
- (ii) If  $\zeta \in [\widehat{\varepsilon}_H, 0]$ , then  $\gamma_3 = 0$ ,  $\gamma_1 > 0$  and  $\gamma_2 > 0$ .
- (iii) If  $\zeta > 0$ , then  $\gamma_3 = 0$ ,  $\gamma_1 = 0$  and  $\gamma_2 = 1$ .

As above, in case (ii), the exact values of  $\gamma_1$  and  $\gamma_2$  are found by setting  $\gamma_1 + \gamma_2 = 1$  and imposing  $r_{LL} = r_{HH}$  to the quantities pinned down in Lemma 4.

The next two lemmas complete the characterization of the contract.

**Lemma 10** *When (5) is satisfied, there exists at most one value  $\varepsilon_{LH} \in (-\infty, \varepsilon_H)$  such that if  $\zeta < \varepsilon_{LH}$  then, at the solution to  $\Gamma'$ , either  $r_{HL} > r_{LH}$  or  $q_{HL} > q_{LH}$ . When (5) is violated, there exists at most one value  $\widehat{\varepsilon}_{LH} \in (-\infty, \widehat{\varepsilon}_H)$  such that if  $\zeta < \widehat{\varepsilon}_{LH}$  then, at the solution to  $\Gamma''$ , either  $r_{HL} > r_{LH}$  or  $q_{HL} > q_{LH}$ .*

Resting on Lemma 10, one cannot exclude the possibility of output being set such that either  $r_{HL} - r_{LH} > 0$  or  $q_{HL} - q_{LH} > 0$  when marginal surplus is very concave, although the difference that is positive, out of those two, would be very small in that case.<sup>18</sup> With an output profile satisfying these conditions,  $IC_{HL}^{LH}$  would be violated in  $\Gamma$ . This brings an unnecessary complication, if it is considered that  $IC_{HL}^{LH}$  would be "almost satisfied" with either  $r_{HL} - r_{LH}$  or  $q_{HL} - q_{LH}$  very small, and that it is difficult to identify surplus functions with this kind of property. Thus, in the exposition of results below, we restrict attention to cases where  $\zeta \geq \max\{\varepsilon_{LH}, \widehat{\varepsilon}_{LH}\}$  so that:

$$r_{LH} \geq r_{HL} \quad \text{and} \quad q_{LH} \geq q_{HL}. \quad (7)$$

This restriction on  $\zeta$  does not affect the findings presented in Lemma 7 - 9 since  $\varepsilon_{LH} \in (-\infty, \varepsilon_H)$  and  $\widehat{\varepsilon}_{LH} \in (-\infty, \widehat{\varepsilon}_H)$ .

Lastly, recall that the second-period monotonicity condition was assumed to be strictly satisfied for all types. As a last step, we provide a sufficient condition under which this is actually the case in either reduced problem.

**Lemma 11** *If*

$$\frac{\sigma_H}{\sigma_L} < \frac{1}{1 - \mu(1 - \nu)}, \quad (8)$$

*then, at solution to both  $\Gamma'$  and  $\Gamma''$ ,  $r_{ij} > 0 \forall ij \in \Upsilon$ .*

When uncertainty about the spread is relatively large in a proportional sense so that (8) is violated, the contractual solution loses both the multidimensional and the sequential nature. Inducing information release on  $\sigma_j$  is so costly that P renounces to screen on it. Only private information on the expected unit cost is considered in contractual design. The principal's problem degenerates onto a static agency problem with two possible initial types, namely  $\theta_L$  and  $\theta_H$ , each being assigned a single production level, regardless of whether the true cost

<sup>18</sup>In the limit case where  $\zeta \rightarrow -\infty$ ,  $S'(\cdot)$  has a reversed L-shaped form with  $S'(y) \rightarrow S'(0), \forall y \in (0, Y)$ , involving that  $r_{HL} - r_{LH} \rightarrow 0$  and  $q_{HL} - q_{LH} \rightarrow 0$ .



realization is bigger or smaller than initially forecast. In light of this, all results to be drawn below refer to situations where uncertainty about the spread is relatively small in a proportional sense so that (8) is satisfied.

## 5 Results

To begin with, we check that the solution to each of the reduced problems previously presented solves the general problem as well.

**Proposition 1** *Assume that (8) is satisfied and that  $\zeta \geq \max\{\varepsilon_{LH}, \widehat{\varepsilon}_{LH}\}$ . For any given value of  $\zeta$  and the associated solution identified in Lemma 7 and 8, there exist values of  $\nu, \mu \in (0, 1)$  such that (5) is satisfied, and values of  $\nu, \mu \in (0, 1)$  such that (5) is violated.*

*If (5) is satisfied, then the solution to  $\Gamma'$  solves  $\Gamma$  as well.*

*If (5) is violated, then the solution to  $\Gamma''$  solves  $\Gamma$  as well.*

The proposition confirms that the general problem can actually be replaced by an appropriate relaxed problem. This evidences that the methodology adopted to solve multidimensional screening problems also applies to sequential screening problems with a discrete number of types. This is particularly useful in our setting where working with discrete types facilitates the task because, unlike in other sequential screening models, there is no complete ordering of distributions.<sup>19</sup> The following corollary pins down the agent's incentives to misrepresent information which are relevant for P, as can be deduced from the solutions in Lemma 7 - 9. With a general view of the relevant incentives, we will then be able to enucleate similarities and differences with other sequential screening problems.

**Corollary 1** *In the optimal contract, information rents and production levels reflect:*

(a) *incentives to overstate  $\theta$  : of type LL,  $\forall \Delta\theta/\Delta\sigma$ ; of type LH, if and only if  $\zeta \leq \varepsilon_{LL}$  when (5) holds, and if and only if  $\zeta \leq \widehat{\varepsilon}_{LL}$  when (5) is violated;*

(b) *incentives to understate  $\sigma$  : of types HH and LH,  $\forall \Delta\theta/\Delta\sigma, \forall \zeta$ ;*

(c) *incentives to overstate  $\sigma$  : of type LL, if and only if  $\zeta \geq \zeta_1$  when (5) holds, and  $\forall \zeta$  when (5) is violated.*

There are two main concerns of P in contractual design, namely the agent's temptation to overstate the expected unit cost and/or that to understate the spread (part (a) and (b) in the corollary). These preoccupations are similar to those usually identified in sequential screening models where the agent holds private information on either the expected value or the spread of the unknown variable. However, it is important to notice that, for sufficiently convex marginal surplus, exaggeration of the expected cost by a low-expected-cost high-spread agent is no longer an issue for P (point (a), cases where  $\zeta > \varepsilon_{LL}$  and  $\zeta > \widehat{\varepsilon}_{LL}$ ). In that situation,

<sup>19</sup>See also Rochet and Stole [35], who consider an example of sequential screening problem without complete ordering of distributions. In our model, unlike in their example, global incentive constraints are not necessarily implied by local incentive constraints.

indeed, the principal prefers to set more dispersed production levels for each cost distribution. With this output profile, understatement of  $\sigma$  becomes more worrisome than overstatement of  $\theta$ . Moreover, when marginal surplus is sufficiently convex, P is concerned with the possibility of the spread being overstated rather than understated. This preoccupation arises with regards to an agent facing not only a low spread but also a low expected cost (part (c)), a joint lie on the two information dimensions representing the strongest temptation for this type (part (a) and (c) altogether).

Corollary 1 emphasizes that different shapes of the marginal surplus function mirror, in fact, different preferences of P as to how the two production levels should be set for each cost distribution. The extent to which P is ready to distort production levels to contain agency costs, depending upon her preferences for the good, is in turn suggestive of how important private information on one dimension is for P relative to the other. This result is core to our investigation.

**Corollary 2** *For all values of  $\Delta\theta/\Delta\sigma$ , as  $\zeta$  increases, information rents in the optimal contract, and accordingly distortions induced in production levels to contain those rents, reflect progressively stronger concerns with the agent misrepresenting  $\sigma$  rather than  $\theta$ .*

On the one hand, the shape of  $S'(\cdot)$  dictates how dispersed production levels should be on efficiency grounds. Increasingly more dispersion is desirable as  $S'(\cdot)$  becomes less concave / more convex. On the other hand, for rent-extraction purposes, P prefers to choose production levels relatively close both across and within cost distributions because this helps her contain agency costs. Therefore, proceeding from concavity to convexity, the trade-off between efficiency, as expressed in expected terms over the two second-stage productions, and rent-extraction is initially loose and then progressively exacerbated. This reflects an increasing concern with the possibility of  $\sigma$  being misrepresented rather than  $\theta$  as marginal surplus becomes less concave / more convex. The next two corollaries complete the presentation of results.

**Corollary 3** *The optimal contract collapses onto a multidimensional (static) screening mechanism if and only if  $S'(\cdot)$  is sufficiently convex:*

- a) *in the sense of part (iii) in Lemma 8, if (5) is satisfied for  $\beta = 0$  and  $\gamma_2 = 1$ ;*
- b) *in the sense of part (iii) in Lemma 9, if (5) is violated for  $\beta = 0$  and  $\gamma_2 = 1$ .*

When marginal surplus is sufficiently convex, the contractual solution reduces to a multidimensional screening mechanism with uncorrelated types. This is because in that case  $\beta = 0$  and  $\gamma_2 = 1$ , involving that adjacent types are the only attractive lies. Results are more complex here since all possible degrees of concavity and convexity of the marginal surplus function are to be considered. In a multidimensional but static framework, production levels would be determined in the first stage, one for each type. Hence, there would be no need to evaluate efficiency losses associated with quantity distortions in expected terms. Once again, the shape of  $S'(\cdot)$  would be irrelevant in the identification of the rents to be optimally assigned to the various types, and all cases/solutions to  $\Gamma$  we identified for different curvatures of  $S'(\cdot)$  would

collapse onto one single case/solution. By this we do not mean that in multidimensional but static screening problems the principal's preferences never affect contractual features. Sometimes this does occur, yet in different ways. In environments where the agent executes two distinct activities for the principal and has private knowledge of the cost of each activity, it is the preference symmetry across activities (rather than the shape of the marginal surplus function) that matters in the optimal contractual choice (Armstrong and Rochet [2]).

**Corollary 4** *When  $\Delta\sigma \neq 0$  the optimal incentive contract does not reduce to a sequential (unidimensional) screening mechanism.*

The fact that the agent knows privately two pieces of information related to one distribution involves that the features of the optimal contract are more nuanced as compared to sequential screening mechanisms usually considered in the literature. One could detect similarities when the index  $\Delta\theta/\Delta\sigma$  is so large that (5) is violated, because bunching is then induced with respect to the spread. Another partial similarity lies in the no-distortion-at-the-top result. For instance, in Riordan and Sappington [34] and Courty and Li [8], the most desirable first-stage type (that would correspond to  $\theta_L$  in our model, if private information were only about  $\theta$ ) is assigned the FB trade volume, regardless of the second-stage state of nature. To some extent, this is found in our analysis as well in that the low-expected-cost agent is actually required to produce FB output when he faces a high spread. However, when the low-expected-cost agent faces a low spread instead, this is no longer the optimal policy in situations where marginal surplus is sufficiently convex. Overall, we cannot identify any solutions, among those presented in Lemma 7 - 9, which reduce to usual sequential screening mechanisms.

## 6 Discussion

A fundamental prediction of our analysis is that, in agency relationships with the information structure here represented, the characteristics of the optimal contract are related finely to the principal's preferences. We now discuss a possible application, having in mind the public utilities that inspired our investigation.

P can be regarded as the regulator of a firm providing some good or service in a monopolistic market. Her goal is to maximize expected consumer surplus net of the compensation owed to the firm. In this context, the marginal surplus function is the inverse demand function, expressing the willingness to pay for any given consumption level. A shift from concavity to convexity corresponds to a shift from less to more price-elastic demand. As an illustration, take  $S(y) = ky - y^{e+1}/(e+1)$ , where  $e, k > 0$ . Then,  $S'(y) \equiv p(y) = k - y^e$  is strictly concave in  $y$  if  $e > 1$ , linear if  $e = 1$ , strictly convex if  $e < 1$ . All else equal, the direct demand  $y(p) = (k - p)^{1/e}$  is less price-elastic the larger  $e$ , for any given  $p$ . When  $e$  is large, hence demand is little elastic, a price raise triggers a limited reduction in demanded units of the good. This means that the regulator can afford more important quantity distortions, which facilitates rent-extraction from the regulated firm. As  $e$  decreases, hence demand becomes more

elastic, a price raise triggers increasingly bigger reductions in demanded units. This means that the regulator can afford progressively smaller quantity distortions, which weakens her ability to extract rents from the regulated firm. This is in line with Corollary 2, according to which, as  $S'(\cdot)$  shifts from concave to convex,  $P$  becomes less prone to distort quantities away from FB levels and tolerates more important agency costs, thus conceding rents to eliminate incentives to mimic adjacent rather than nonadjacent types.

The relationship between the curvature of the marginal surplus function and the price-elasticity of market demand suggests a way to make functional use of the insights of our study along the current regulatory practice. Actually, in markets where the demand function is only vaguely known to regulators, the latter typically refer to elasticity estimates, which can be formed with more reasonable accuracy when little information is available on demand conditions. In regulator/firm hierarchies where the information structure is as represented in our model, the regulator could use elasticity estimates to identify the relevant information rents and set output accordingly. An illustration is provided here below.

**Example** Table 1 summarizes numerical results that obtain when  $S(y) = ky - y^{e+1}/(e+1)$  and  $k = 15$ ,  $\nu = \mu = 0.3$ ,  $\theta_L = 4$ ,  $\theta_H = 5$ ,  $\sigma_L = 3$ ,  $\sigma_H = 3.3$ ; further details on the development are relegated to appendix D. Condition (5) holds for the two values of  $e$  considered. Instead, condition (29) holds for  $e = 1$ , namely when  $S'(\cdot)$  is linear, whereas it is violated for  $e = 0.5$ , namely when  $S'(\cdot)$  is sufficiently convex in the sense of Lemma 7. Hence,  $\Pi_{LL,2} > \Pi_{LL,1}$  in the former case ( $\beta = 1$ );  $\Pi_{LL,2} < \Pi_{LL,1}$  in the latter ( $\beta = 0$ ). That is, as long as demand is little elastic,  $IC_{LL}^{HL}$  is binding and  $IC_{LL}^{HH}$  is slack; when demand is sufficiently elastic,  $IC_{LL}^{HH}$  is binding. In either scenario,  $\Pi_{LH,1} = \Pi_{LH,2}$  ( $\gamma_1 > 0$  and  $\gamma_2 > 0$ ).

$e$	$\gamma_1$	$\gamma_2$	Condition (5)	$\Delta\sigma(r_{HH} - r_{HL})$	$\Delta\theta(q_{HH} - q_{HL})$
1	0.826	0.174	$3.33 < 16.73$	0.75	0.5
0.5	0.948	0.052	$3.33 < 57.63$	15	16.5

Table 1: Numerical results

## 7 Relation to the literature

### Sequential screening

In a sequential screening problem where distributions are ordered in the sense of first-order stochastic dominance, Courty and Li [8] show that, under standard assumptions, distortions are reminiscent of those arising in static screening problems *à la* Baron and Myerson [5], with the caveat that types represent distributions rather than real values. That is, no distortion is induced for the most desirable type, regardless of the second-stage realization of the unknown variable; increasingly greater distortions are induced for progressively less desirable types. This result is in line with that previously obtained by Riordan and Sappington [34] in a context of

monopoly franchising. Courty and Li [8] further assess that, when distributions are ordered in the sense of mean-preserving spread, the allocation is fully efficient for the biggest-spread type, whereas distortions are induced for lower-spread types. Our findings highlight that in situations where types are drawn from four distributions, sharing the same mean and the same spread two by two, not only are distortions greater for types with greater expected unit cost and smaller spread; they are also related to the curvature of the marginal surplus function, in the way identified in Corollary 2.

### **Multidimensional screening**

Armstrong and Rochet [2] offer a user guide to solve multidimensional screening problems where the number of types is discrete, pointing out that it is very difficult to obtain closed-form solutions in models with a continuum of types. The strategy is to rely on a reduced problem where only downward incentive constraints (namely, incentive constraints whereby more desirable types are unwilling to mimic less desirable types) are potentially binding, and to check that the solution to the reduced problem solves the general problem as well. Our analysis unveils that, when the problem is both multidimensional and sequential, the way in which incentives to misrepresent the two first-stage information dimensions interact, induces an unusual dependency of contractual features on principal's preferences, which makes the identification of the solution pattern more complex.

### **Bunching of types**

Both in multidimensional and in sequential screening problems bunching of types is sometimes identified. In a multidimensional screening model with discrete types, Armstrong [1] shows that pooling may arise if the worst type in the principal's ordering is tempted to mimic its adjacent better-ranked type. Bunching these types affords weaker distortions to the principal when satisfying their incentive constraints. In a sequential screening model with discrete types, Krahmer and Strausz [22] assess that bunching is likely to appear as one moves away from the usual assumption that the support of final values is independent of first-stage private information. To contain the rent conceded to prevent off-the-equilibrium path lying, the principal would like to induce distortions for the types representing attractive reports, which are yet too pronounced to satisfy the monotonicity condition imposed by the incentive constraints of those types. Bunching them is thus necessary in the same fashion as in agency problems with countervailing incentives (such as Lewis and Sappington [29] and Maggi and Rodriguez-Clare [30]). While in Armstrong [1] the monotonicity condition imposed by incentive compatibility prevents the principal from limiting distortions, in Krahmer and Strausz [22] and in the studies on countervailing incentives it prevents the principal from raising distortions aimed to contain information rents. Interestingly, in our screening problem, which is both multidimensional and sequential, the result that high-spread types may be optimally bunched is more in line with findings of multidimensional screening models. Indeed, as in Armstrong [1], bunching types

is a way for the principal to limit distortions that would not grant further savings on agency costs. This unveils a reason for bunching in a sequential screening problem other than the possibility of the agent lying off the equilibrium path, which our assumptions ruled out. Unlike in Armstrong [1], however, bunching in our framework does not reflect the mutual temptation of adjacent types to mimic each other, which hardens a monotonicity condition warranting incentive compatibility of the solution. Instead, it mirrors the circumstance that, for some type, both adjacent and non-adjacent lower-ranked types represent attractive reports, involving that, for that type, more incentive constraints are relevant at once.

## 8 Conclusion

Considering a sequential screening problem with multidimensional information in the first stage, we showed that the identification of the optimal solution is less immediate than in typical multidimensional screening problems. Complications arise because the two first-stage pieces of information are tied together in the distribution of the unknown variable. To characterize contractual features in this framework, it was fundamental to detect and understand their dependency on the principal's preferences.

We assessed that there is no contractual solution collapsing onto the solution to a sequential screening problem where only one characteristic of the distribution is privately known. On the other hand, the contractual solution displays similarities with a multidimensional screening mechanism when the marginal surplus function is sufficiently convex. In regulated markets where demand is elastic to price, this case would be most relevant for regulatory bodies. Consumption theory suggests that this is actually what one should expect most often in practice. It is noteworthy that this is also the case where private information on the spread is most important. Our analysis predicts that the primary concern of a principal with sufficiently convex marginal surplus should be to contain information rents conceded to prevent cheating on the spread of the cost, rather than on its expected value, and that distortions in production should mirror this concern.

We focused on a simple setting where only four distributions are possible, each including only two values symmetrically distributed around the mean. On the one hand, this makes our analysis widely applicable. In particular, each distribution has the characteristics of a two-periods random walk. On the other hand, the essential content of our results would not be different if cost values were distributed asymmetrically around the mean. The pattern of contractual solutions associated with different curvatures of the marginal surplus function would not be dissimilar from the one we identified.

Our investigation was inspired to situations where public activities are delegated to firms which might have incentives to manipulate forecasts of initially unknown variables *vis-à-vis* appointing and/or regulatory authorities. In practice, such activities are now typically awarded to firms by means of tendering procedures. To account for this, we could consider an auction mechanism rather than looking at an incentive contract. Our choice was not reductive though.

The insights of our work would not change in that environment since in our model, as in Riordan and Sappington [34], there would be separability between number of bidders and contractual allocation.<sup>20</sup> Moreover, while we focused on a full-commitment framework, delegation of public activities sometimes occurs in limited-commitment environments where firms might camouflage forecasts in the contracting stage in view of a later renegotiation. To eliminate the perspective of contractual renegotiation, hence incentives to strategic misrepresentation related to that, one can think of the principal as being able to credibly engage in future enforcement in the presence of solid institutions, and of the agent as being motivated to comply with the contract in the presence of cancellation fees or *ex-post* participation constraints.<sup>21</sup> Contract design under *ex-post* participation constraints is analyzed by Spulber [37] and Chen and Smith [7] in a setting where the firm has private information on the cost distribution in the contracting stage but the cost realization is publicly observed in a later stage. Because our aim was to explore the impact of multidimensional private information in a sequential screening problem, embodying *ex-post* participation constraints would have been beyond the scope of the present study.<sup>22</sup> This is on our research agenda.

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<sup>20</sup>The proof is available upon request.

<sup>21</sup>For a discussion on commitment in continuing relationships see Baron and Besanko [4].

<sup>22</sup>Focusing on the canonical sequential screening model, Krahmer and Strausz [24] show that the principal no longer benefits from eliciting private information sequentially if the agent has an *ex-post* outside option.

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## A Proof of condition $\Delta\theta \leq 2\sigma_L$

In this proof, we denote  $IC_{ij}^{i'j'(-,+)}$ ,  $IC_{ij}^{i'j'(-)}$  and  $IC_{ij}^{i'j' (+)}$  the first-stage incentive constraint whereby type  $ij$  is unwilling to lie in the first stage, anticipating that in the second stage he will lie, respectively, whatever the shock, only when the shock is "−", and only when the shock is "+." We further define for this proof  $F \equiv \Pi_{i'j'} + (\theta_{i'} - \theta_i) q_{i'j'} + (\sigma_j - \sigma_{j'}) r_{i'j'}$ . Accordingly  $IC_{ij}^{i'j'}$  (as expressed in  $\Gamma$ ) is rewritten as  $\Pi_{ij} \geq F$ .  $IC_{ij}^{i'j'(-,+)}$ ,  $IC_{ij}^{i'j'(-)}$  and  $IC_{ij}^{i'j' (+)}$  are developed as follows:

$$\begin{aligned} IC_{ij}^{i'j'(-,+)} &: \Pi_{ij} \geq F - 2\sigma_j r_{i'j'}, \\ IC_{ij}^{i'j'(-)} &: \Pi_{ij} \geq F - \frac{1}{2} \left( \underline{\pi}_{i'j'} - \bar{\pi}_{i'j'} - 2\sigma_{j'} \underline{y}_{i'j'} \right) + (\theta_i - \theta_{i'} - \sigma_j - \sigma_{j'}) r_{i'j'} \\ IC_{ij}^{i'j'(+)} &: \Pi_{ij} \geq F + \frac{1}{2} \left( \underline{\pi}_{i'j'} - \bar{\pi}_{i'j'} - 2\sigma_{j'} \bar{y}_{i'j'} \right) + (\theta_{i'} - \theta_i - \sigma_{j'} - \sigma_j) r_{i'j'}. \end{aligned}$$

We see that  $IC_{ij}^{i'j'(-,+)}$  is implied by  $IC_{ij}^{i'j'}$ ; moreover,  $IC_{ij}^{i'j'(-)}$  and  $IC_{ij}^{i'j' (+)}$  are both implied by  $IC_{ij}^{i'j'}$  if and only if:

$$2\sigma_{j'} \underline{y}_{i'j'} + 2(\theta_i - \theta_{i'} - \sigma_j - \sigma_{j'}) r_{i'j'} \leq \underline{\pi}_{i'j'} - \bar{\pi}_{i'j'} \leq 2\sigma_{j'} \bar{y}_{i'j'} - 2(\theta_{i'} - \theta_i - \sigma_{j'} - \sigma_j) r_{i'j'}$$

It is immediate to check that the range of values  $\left[ 2\sigma_{j'} \underline{y}_{i'j'} + 2(\theta_i - \theta_{i'} - \sigma_j - \sigma_{j'}) r_{i'j'}, 2\sigma_{j'} \bar{y}_{i'j'} - 2(\theta_{i'} - \theta_i - \sigma_{j'} - \sigma_j) r_{i'j'} \right]$  does exist and that values of  $\underline{\pi}_{i'j'} - \bar{\pi}_{i'j'}$  satisfying  $\underline{iC}_{i'j'}$  and  $\bar{iC}_{i'j'}$  can be drawn from that range if and only if  $\theta_i - \theta_{i'} - \sigma_j - \sigma_{j'} \leq 0$ . This condition is satisfied  $\forall ij$  and  $i'j'$  if  $\Delta\theta \leq 2\sigma_L$ .

## B Proofs of lemmas

### B.1 Proof of Lemma 1

Incentive constraints in  $\Gamma$  are written as follows:

$$\Pi_{LL} \geq \Pi_{HL} + \Delta\theta_{qHL} \quad (\text{IC1})$$

$$\Pi_{LL} \geq \Pi_{LH} - \Delta\sigma r_{LH} \quad (\text{IC2})$$

$$\Pi_{LL} \geq \Pi_{HH} + \Delta\theta_{qHH} - \Delta\sigma r_{HH} \quad (\text{IC3})$$

$$\Pi_{HL} \geq \Pi_{LL} - \Delta\theta_{qLL} \quad (\text{IC4})$$

$$\Pi_{HL} \geq \Pi_{HH} - \Delta\sigma r_{HH} \quad (\text{IC5})$$

$$\Pi_{HL} \geq \Pi_{LH} - \Delta\theta_{qLH} - \Delta\sigma r_{LH} \quad (\text{IC6})$$

$$\Pi_{LH} \geq \Pi_{HH} + \Delta\theta_{qHH} \quad (\text{IC7})$$

$$\Pi_{LH} \geq \Pi_{LL} + \Delta\sigma r_{LL} \quad (\text{IC8})$$

$$\Pi_{LH} \geq \Pi_{HL} + \Delta\theta_{qHL} + \Delta\sigma r_{HL} \quad (\text{IC9})$$

$$\Pi_{HH} \geq \Pi_{LH} - \Delta\theta_{qLH} \quad (\text{IC10})$$

$$\Pi_{HH} \geq \Pi_{HL} + \Delta\sigma r_{HL} \quad (\text{IC11})$$

$$\Pi_{HH} \geq \Pi_{LL} - \Delta\theta_{qLL} + \Delta\sigma r_{LL}. \quad (\text{IC12})$$

From the definition of  $\Gamma'$ , any of the constraints (IC1), (IC3), (IC7), (IC8), (IC9) and (IC11) is potentially binding. Replacing  $\Pi_{HL} = 0$ , the rents in (2a) - (2d) are obtained.

### B.2 Proof of Lemma 2

Using (2a) - (2d) the objective function in  $\Gamma'$  is rewritten as:

$$\begin{aligned} & \sum_{ij \in \Upsilon} E_{ij} \left[ \frac{1}{2} \left( S(\underline{y}_{ij}) + S(\bar{y}_{ij}) \right) - (\theta_i q_{ij} - \sigma_j r_{ij}) \right] \\ & - \nu \mu [\beta \Delta\theta_{qHL} + (1 - \beta) (\Delta\sigma r_{HL} + \Delta\theta_{qHH} - \Delta\sigma r_{HH})] \\ & - (1 - \nu) (1 - \mu) \Delta\sigma r_{HL} \\ & - \nu (1 - \mu) \{ \gamma_1 (\Delta\sigma r_{HL} + \Delta\theta_{qHH}) \\ & + \gamma_2 [\beta \Delta\theta_{qHL} + (1 - \beta) (\Delta\sigma r_{HL} + \Delta\theta_{qHH} - \Delta\sigma r_{HH}) + \Delta\sigma r_{LL}] \\ & + \gamma_3 (\Delta\theta_{qHL} + \Delta\sigma r_{HL}) \}, \end{aligned} \quad (9)$$

from which the first-order conditions in Lemma 2 are obtained.

### B.3 Proof of Lemma 3

Denote:

$$\phi(\beta) \equiv \frac{\frac{1}{\nu} - (\beta - \mu) \left( \gamma_2 + \frac{\mu}{1 - \mu} \right)}{1 - (\gamma_1 + \gamma_2) + \gamma_2 \beta - \frac{(1 - \beta)\mu}{1 - \mu}}. \quad (10)$$

The numerator of  $\phi(\beta)$  is strictly positive so that  $\phi(\beta) \neq 0$ . The denominator is either positive or negative depending on the value of  $\beta$ . Suppose  $\beta$  is such that  $\phi(\beta) > 0$ . Then, from Lemma

2,  $\underline{y}_{HH} > \underline{y}_{HL}$  if and only if:

$$\frac{\Delta\theta}{\Delta\sigma} > -\phi(\beta), \quad (11)$$

which is true. Moreover,  $\bar{y}_{HL} > \bar{y}_{HH}$  if and only if  $\phi(\beta) > 1$  and

$$\frac{\Delta\theta}{\Delta\sigma} < \phi(\beta). \quad (12)$$

Suppose now that  $\phi(\beta) < 0$ . Then, from Lemma 2,  $\bar{y}_{HL} > \bar{y}_{HH}$  and so equivalently:

$$\frac{\Delta\theta}{\Delta\sigma} > \phi(\beta). \quad (13)$$

Moreover,  $\underline{y}_{HH} > \underline{y}_{HL}$  if and only if  $-\phi(\beta) > 1$  and

$$\frac{\Delta\theta}{\Delta\sigma} < -\phi(\beta). \quad (14)$$

Take  $\frac{\Delta\theta}{\Delta\sigma} \geq |\phi(\beta)|$ . First suppose that  $\phi(\beta) > 0$ . Then, (11) is satisfied whereas (12) is violated. Under Lemma 2,  $\underline{y}_{HH} > \underline{y}_{HL}$  and  $\bar{y}_{HL} \leq \bar{y}_{HH}$ . It follows that  $\Pi_{LL,1} < \Pi_{LL,2}$  and so  $\beta = 0$ . Since  $\phi(0) < 0$ , the hypothesis that  $\phi(\beta) > 0$  leads to a contradiction. Next suppose that  $\phi(\beta) < 0$ . Then, (13) holds whereas (14) does not. Under Lemma 2,  $\bar{y}_{HL} > \bar{y}_{HH}$  and  $\underline{y}_{HH} \leq \underline{y}_{HL}$ . It implies that  $\Pi_{LL,1} > \Pi_{LL,2}$  and so  $\beta = 1$ . However,  $\phi(1) > 0$ , which contradicts the hypothesis that  $\phi(\beta) < 0$ . Therefore, when  $\frac{\Delta\theta}{\Delta\sigma} \geq |\phi(\beta)|$  or, equivalently, (5) is violated, quantities pinned down in Lemma 2 do not solve  $\Gamma$ .

## B.4 Proof of Lemma 4

Using  $\underline{y}_{HH} = \underline{y}_{HL}$  and  $\bar{y}_{HL} = \bar{y}_{HH}$ , the rents in Lemma 1 are rewritten as:

$$\begin{aligned} \Pi_{HL} &= 0; \quad \Pi_{HH} = \Delta\sigma r_{HH}; \quad \Pi_{LL} = \Delta\theta q_{HH} \\ \Pi_{LH} &= \gamma_1 (\Delta\theta q_{HH} + \Delta\sigma r_{HH}) + \gamma_2 (\Delta\theta q_{HH} + \Delta\sigma r_{LL}). \end{aligned} \quad (15)$$

The agent's expected rent is equal to  $\nu\Delta\theta q_{HH} + (1-\mu) \{[1 - (1-\gamma_1)\nu] r_{HH} + \nu\gamma_2 r_{LL}\} \Delta\sigma$ . Replacing this expression, the objective function in  $\Gamma$  is rewritten as:

$$\begin{aligned} \sum_{ij \in \Upsilon} E_{ij} \left[ \frac{1}{2} \left( S(\underline{y}_{ij}) + S(\bar{y}_{ij}) \right) - (\theta_i q_{ij} - \sigma_j r_{ij}) \right] \\ - \nu\Delta\theta q_{HH} - (1-\mu) \{[1 - (1-\gamma_1)\nu] r_{HH} + \nu\gamma_2 r_{LL}\} \Delta\sigma, \end{aligned}$$

from which the first-order conditions in Lemma 4 are obtained.

## B.5 Proof of Lemma 5

Recall the assumption that  $S'''(y)$  has the same sign  $\forall y \in [0, Y]$ . Suppose that  $S'''(y) < 0$ ,  $\forall y \in [0, Y]$ . Then,  $S''(y) \leq S''(0)$  implying that  $S'(y) = S'(0) + \int_0^y S''(x) dx \leq S'(0) + yS''(0)$ . The assumption that  $S''(0)$  is finite and, at the same time,  $Y \rightarrow \infty$  contradicts the assumption that  $S'(y) > 0$  when  $y = Y$ . Thus, it is necessary that either  $y$  is finite (implicitly,  $Y$  is finite) or  $S''(0) = -\infty$  or both. Since  $S'(0)$  is finite, the hypothesis that  $S''(0) = -\infty$  contradicts

as well the assumption that  $S'(y) > 0$ . Hence, it is necessary that  $Y < \infty$  together with  $S''(0) > -\infty$ .

## B.6 Proof of Lemma 6

As  $f(a) = y$  is the inverse function of  $S'(\cdot)$ , we have:

$$S''(y) = \frac{1}{f'(a)}. \quad (16)$$

Using

$$f(a_2) - f(a_1) = - \int_{a_2}^{a_1} f'(z) dz, \quad \forall a_1, a_2,$$

we can write:

$$(f(a_4) - f(a_3)) - (f(a_2) - f(a_1)) = - \int_{a_4}^{a_3} f'(v) dv - \left( - \int_{a_2}^{a_1} f'(z) dz \right), \quad \forall a_1, a_2, a_3, a_4 \quad (17)$$

This is non-negative if and only if:

$$- \int_{a_4}^{a_3} f'(v) dv \geq - \int_{a_2}^{a_1} f'(z) dz \quad (18)$$

when  $a_4 < a_3 < a_2 < a_1$ , and if and only if:

$$- \int_{a_4}^{a_2} f'(v) dv \geq - \int_{a_3}^{a_1} f'(z) dz \quad (19)$$

when  $a_4 < a_2 < a_3 < a_1$ .

Take  $a_4 < a_3 < a_2 < a_1$  (the proof is analogous, *mutatis mutandis*, for  $a_4 < a_2 < a_3 < a_1$ ).

(I) If  $S'''(y) \geq 0$ ,  $\forall y$ , then  $S''(q_1) \geq S''(q_2)$ ,  $\forall y_1 \geq y_2$ . Using (16), it follows that  $f'(v) \leq f'(z) \Leftrightarrow (-f'(v)) \geq (-f'(z))$ ,  $\forall v, z$  such that  $v < z$ .

If  $a_3 - a_4 \geq a_1 - a_2$ , then (18) holds.

If  $a_3 - a_4 < a_1 - a_2$ , then (18) holds if and only if  $\exists \varepsilon > 0$  such that  $(-f'(v)) - (-f'(z)) > \varepsilon$  for some  $v \in [a_4, a_3]$  and  $z \in [a_2, a_1]$ . Equivalently,  $\exists \eta < 0$  such that  $f''(a) > \eta$  at least for some  $a \in [a_4, a_1]$ . Equivalently,  $S'''(y) > \lambda$  for some  $\lambda > 0$  and  $y \in [f(a_1), f(a_4)]$ . As  $S'(0)$  and  $S'(Y)$  are fixed, a raise (resp., a decrease) in  $S'''(q)$  for some  $y \in (0, Y)$  involves a raise (resp., a decrease) in  $S'''(y)$ ,  $\forall y \in (0, Y)$ , and in particular in  $S'''(\hat{y}) \equiv \zeta$ . Hence, the condition  $S'''(y) > \lambda$  can be rewritten as  $\zeta > \varepsilon_1$  for some  $\varepsilon_1 > 0$ .

(II) If  $S'''(\cdot) < 0$ ,  $\forall y$ , then  $-f'(v) < -f'(z)$ ,  $\forall v, z$  such that  $v < z$ .

If  $a_3 - a_4 < a_1 - a_2$ , then (18) is violated.

If  $a_3 - a_4 \geq a_1 - a_2$ , then (18) holds if and only if  $\exists \eta > 0$  such that  $f''(a) < \eta$  at least for some  $a \in [a_4, a_1]$ . Equivalently,  $S'''(y) > \lambda$  for some  $\lambda < 0$  and  $y \in [f(a_1), f(a_4)]$ . As in case (a), this is equivalent to  $\zeta > \varepsilon_2$  for some  $\varepsilon_2 < 0$ .

From (I) and (II) we see that, if  $a_3 - a_4 < a_1 - a_2$ , then a necessary and sufficient condition for (18) to hold is that  $\zeta > \varepsilon_1$  for some  $\varepsilon_1 > 0$ ; if  $a_3 - a_4 \geq a_1 - a_2$ , then a necessary and sufficient condition for (18) to hold is that  $\zeta > \varepsilon_2$  for some  $\varepsilon_2 < 0$ .

It remains to show that  $(f(a_4) - f(a_3)) - (f(a_2) - f(a_1))$  increases with  $\zeta$ . An increase in  $\zeta$  reflects a higher difference  $(-f'(v)) - (-f'(z))$ ,  $\forall v, z$  such that  $v < z$ . Then, as  $a_4 < a_3$ ,  $a_2 < a_1$  and  $a_3 < a_1$  by definition, for any given values of  $a_1, a_2, a_3, a_4$  satisfying these

conditions, the right-hand side of (17) increases with  $\zeta$ .

## B.7 Proof of Lemma 7

In this proof we denote  $\chi \equiv \beta\gamma_2 + \gamma_3 - \frac{(1-\beta)\mu}{1-\mu}$ . Using (30), together with  $\underline{y}_{HH} > \underline{y}_{HL}$  and  $\bar{y}_{HL} > \bar{y}_{HH}$  in Lemma 6 (as from the proof of Lemma 10), we see that if  $\chi > 0$ , then  $\exists \varepsilon_2 < 0$  such that  $q_{HH} < q_{HL}$  if and only if  $\zeta < \varepsilon_2$ ; if  $\chi < 0$ , then  $\exists \varepsilon_1 > 0$  such that  $q_{HH} < q_{HL}$  if and only if  $\zeta < \varepsilon_1$ .

Take  $\chi > 0$  and  $\zeta < \varepsilon_2$  or, alternatively,  $\chi < 0$  and  $\zeta < \varepsilon_1$  so that, by Lemma 6, we have  $q_{HH} < q_{HL}$ . Suppose that  $\beta < 1$ . Recall from Lemma 1 that  $\beta < 1$  if and only if:

$$\Delta\sigma(r_{HH} - r_{HL}) \leq \Delta\theta(q_{HH} - q_{HL}).$$

Further recall from the proof of Lemma 10 that  $r_{HH} > r_{HL}$ . Thus, the above inequality holds only if  $q_{HH} > q_{HL}$ , which contradicts the hypothesis that  $\beta < 1$ .

First check whether  $\chi > 0$  for  $\zeta < \varepsilon_2$  and  $\beta = 1$ . With  $\beta = 1$ ,  $\chi > 0$  if and only if  $\gamma_2 + \gamma_3 > 0$ . Hence,  $\exists \varepsilon_2 = \varepsilon_H < 0$  such that if  $\zeta < \varepsilon_H$ , then  $\beta = 1$  and  $q_{HH} < q_{HL}$ . Next check whether  $\chi < 0$  for  $\zeta < \varepsilon_1$  and  $\beta = 1$ . With  $\beta = 1$ ,  $\chi < 0$  if and only if  $\gamma_2 + \gamma_3 < 0$ , which is impossible. Hence,  $\nexists \varepsilon_1 > 0$  such that, if  $\zeta \in (\varepsilon_H, \varepsilon_1)$ , then  $q_{HH} < q_{HL}$ . Consequently,  $q_{HH} > q_{HL}$  for any  $\zeta > \varepsilon_H$ .

Take  $\zeta > \varepsilon_{HH}$  so that  $q_{HH} > q_{HL}$ . From Lemma 6 it follows that, as  $\zeta$  increases,  $q_{HH} - q_{HL}$  becomes progressively larger than  $r_{HH} - r_{HL}$ . Hence,  $\exists \zeta_1 > \varepsilon_H$  such that (29) is violated and so  $\beta = 0$ . Moreover, because  $q_{HH} > q_{HL}$ , it is  $\gamma_3 = 0$ . Replacing into  $\chi$ , we find that  $\chi < 0$ . It is impossible to have  $\chi < 0$  and  $q_{HH} > q_{HL}$  for  $\zeta < \varepsilon_1$  where  $\varepsilon_1 > 0$  (Lemma 6). Hence,  $\zeta_1 > 0$ .

## B.8 Proof of Lemma 8

### B.8.1 Proof of (i)

From the proof of Lemma 7,  $\zeta < \varepsilon_H \Rightarrow \beta = 1$  and  $\chi \equiv \beta\gamma_2 + \gamma_3 - \frac{(1-\beta)\mu}{1-\mu} = \gamma_2 + \gamma_3 > 0$ . From Lemma 1  $\gamma_3 > 0$  if and only if  $q_{HL} \geq q_{HH}$  and  $r_{LL} \geq r_{HL}$ . From the proof of Lemma 7  $\zeta < \varepsilon_{HH} \Rightarrow q_{HL} > q_{HH}$ . From Lemma 2  $S'(\bar{y}_{LH}) - S'(\bar{y}_{LL}) = \left(1 + \gamma_2 \frac{1-\mu}{\mu}\right) \Delta\sigma > 0$  so that  $\bar{y}_{LL} > \bar{y}_{LH}$ . If  $\bar{y}_{LH} \geq \bar{y}_{HL}$ , then  $\bar{y}_{LL} > \bar{y}_{HL}$ . The assumption that the solution to  $\Gamma'$  is the solution to  $\Gamma$  as well implies that  $\bar{y}_{LH} \geq \bar{y}_{HL}$  (see the proof of Lemma 10 below). Hence  $\bar{y}_{LL} > \bar{y}_{HL}$ . From Lemma 2, for  $\beta = 1$  we find:

$$S'(\underline{y}_{HL}) - S'(\underline{y}_{LL}) = \left\{1 + \frac{\nu}{1-\nu} \left[1 + (\gamma_2 + \gamma_3) \frac{1-\mu}{\mu}\right]\right\} \Delta\theta + \frac{(1-\gamma_2)(1-\mu)}{\mu(1-\nu)} \Delta\sigma,$$

from which  $\underline{y}_{LL} > \underline{y}_{HL}$ . Then

$$(S'(\underline{y}_{HL}) - S'(\underline{y}_{LL})) - (S'(\bar{y}_{HL}) - S'(\bar{y}_{LL})) = 2 \frac{(1-\gamma_2)(1-\mu)}{\mu(1-\nu)} \Delta\sigma. \quad (20)$$

As  $\gamma_2 \leq 1$ , the right-hand side of (20) is non-negative. Using this in Lemma 6, together with the inequalities  $\underline{y}_{LL} > \underline{y}_{HL}$  and  $\bar{y}_{LL} > \bar{y}_{HL}$ , we deduce that  $r_{HL} > r_{LL}$  if and only if  $\zeta < \varepsilon_2$  for some  $\varepsilon_2 = \varepsilon_{LL} \leq 0$ .

Overall, if  $\zeta < \min\{\varepsilon_{LL}, \varepsilon_H\}$  then both  $q_{HL} > q_{HH}$  and  $r_{HL} > r_{LL}$ , so that  $\gamma_3 = 1$ .

(I) Suppose  $\exists \zeta$  such that  $q_{HH} > q_{HL}$  and  $r_{LL} < r_{HL}$ , involving that  $\varepsilon_H < \varepsilon_{LL}$  and  $\zeta \in$

$(\varepsilon_H, \varepsilon_{LL})$ . Then:

$$\Delta\theta(q_{HH} - q_{HL}) > \Delta\sigma(r_{LL} - r_{HL}). \quad (21)$$

We supposed that  $\zeta < \varepsilon_{LL}$ , where  $\varepsilon_{LL} < \zeta_1$ , and we know that  $\beta > 0$  for  $\zeta < \zeta_1$ . From Lemma 1 and from (21) it follows that  $\gamma_1 = 1$  and so  $\gamma_2 = 0$  and  $\chi = -\frac{(1-\beta)\mu}{1-\mu} < 0$ . This is in contradiction with the condition  $q_{HH} > q_{HL}$ . Hence,  $\nexists \zeta$  such that  $q_{HH} > q_{HL}$  and  $r_{LL} < r_{HL}$ . We have  $\varepsilon_H \geq \varepsilon_{LL}$ .

(II) Suppose that  $\exists \zeta < \zeta_1$  such that  $q_{HH} < q_{HL}$  and  $r_{LL} > r_{HL}$ , involving that  $\varepsilon_H > \varepsilon_{LL}$  and  $\zeta \in (\varepsilon_{LL}, \varepsilon_H)$ . Then:

$$\Delta\sigma(r_{LL} - r_{HL}) > \Delta\theta(q_{HH} - q_{HL}). \quad (22)$$

Knowing that  $\beta = 1$  for  $\zeta < \varepsilon_H$ , from Lemma 1 and from (22) it follows that  $\gamma_2 = 1$ . However, using (20) in Lemma 6, we see that  $\gamma_2 = 1 \Rightarrow \varepsilon_{LL} = 0$ . Since  $\varepsilon_H < 0$ , the interval  $(\varepsilon_{LL}, \varepsilon_H)$  does not exist. Therefore,  $\varepsilon_H \leq \varepsilon_{LL}$ .

From (I) and (II) it follows that  $\varepsilon_{LL} = \varepsilon_H$ .

### B.8.2 Proof of (ii)

Take  $\zeta \in (\varepsilon_H, \zeta_1)$  so that  $\gamma_3 = 0$  and  $\beta > 0$ .

Suppose that  $\gamma_1 = 1$ . Then, as previously shown, we have  $\chi = -\frac{(1-\beta)\mu}{1-\mu} < 0$ , involving that  $q_{HH} < q_{HL}$ . This contradicts the result that  $q_{HH} > q_{HL}$  for  $\zeta \in (\varepsilon_H, \zeta_1)$ . Hence, we have  $\gamma_1 < 1$  and so  $\gamma_2 > 0$ .

Suppose that  $\gamma_2 = 1$ . Then,  $\chi = \beta - \frac{(1-\beta)\mu}{1-\mu}$ . We see that  $\chi \leq 1$ . Specifically,  $\chi = 1$  if and only if  $\beta = 1$ . Since for  $\zeta = \varepsilon_H$  we have  $\beta = 1$  and  $\gamma_3 = 1$  so that  $\chi = 1$ , when  $\gamma_2 = 1 \exists \beta > 0$  such that  $q_{HH} > q_{HL}$ . Moreover, for  $\gamma_2$  to be equal to 1, (22) must hold. Replacing  $\gamma_2 = 1$  into (20) and applying Lemma 6, we see that  $r_{LL} - r_{HL} > 0$  if and only if  $\zeta > 0$ . Hence, (22) does hold and so  $\gamma_2 = 1$  only if  $\zeta > 0$ . Therefore, there exists at most one value  $\varepsilon_L > 0$  such that, if  $\zeta \in (\varepsilon_H, \varepsilon_L)$ , then  $\gamma_1 > 0$  and  $\gamma_2 > 0$ ; if  $\zeta \in (\varepsilon_L, \zeta_1)$ , then  $\gamma_2 = 1$ . If  $\nexists \varepsilon_L$  in  $(\varepsilon_H, \zeta_1)$ , then  $\gamma_1 > 0$  and  $\gamma_2 > 0 \forall \zeta \in (\varepsilon_H, \zeta_1)$ .

Take now  $\zeta \geq \zeta_1$ . From Lemma 7 we know that  $\beta = 0$ , hence:

$$\Delta\theta(q_{HH} - q_{HL}) > \Delta\sigma(r_{HH} - r_{HL}).$$

Suppose that  $\gamma_1 = 1$ . As above, we have  $\chi < 0$ , which contradicts the result that  $q_{HH} > q_{HL} \forall \zeta > \varepsilon_H$ . Hence,  $\gamma_1 < 1$  and so  $\gamma_2 > 0$ .

Suppose that  $\gamma_2 = 1$ . From Lemma 1 this is the case if and only if  $r_{LL} > r_{HH}$ . From Lemma 2:

$$S'(\underline{y}_{HH}) - S'(\underline{y}_{LL}) = \Delta\theta - \Delta\sigma + \frac{\nu}{1-\nu} \left[ \frac{\Delta\theta}{1-\mu} - \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \Delta\sigma \right] - \gamma_2 \frac{1-\mu}{\mu} \Delta\sigma \quad (23)$$

$$S'(\bar{y}_{HH}) - S'(\bar{y}_{LL}) = \Delta\theta + \Delta\sigma + \frac{\nu}{1-\nu} \left[ \frac{\Delta\theta}{1-\mu} + \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \Delta\sigma \right] + \gamma_2 \frac{1-\mu}{\mu} \Delta\sigma. \quad (24)$$

Since  $S'(\bar{y}_{HH}) > S'(\bar{y}_{LL}) \forall \gamma_1, \gamma_2, \beta$  and since  $S''(\cdot) < 0$ , we have  $\bar{y}_{LL} > \bar{y}_{HH}$ . Then,  $r_{LL} > r_{HH}$  only if  $\underline{y}_{LL} > \underline{y}_{HH}$ . This is equivalent to:

$$\frac{\Delta\theta}{\Delta\sigma} > 1 + \frac{\gamma_2 \frac{1-\mu}{\mu} - \gamma_1 \frac{\nu}{1-\nu}}{1 + \frac{\nu}{1-\nu} \left[ \gamma_1 + \frac{\mu}{1-\mu} \left( 1 + \gamma_2 \frac{1-\mu}{\mu} \right) \right]} \quad (25)$$

For  $\gamma_2 = 1$  this is rewritten as:

$$\frac{\Delta\theta}{\Delta\sigma} > 1 + \frac{1}{\frac{\mu}{1-\mu} \left(1 + \frac{\nu}{(1-\nu)(1-\mu)}\right)} > 1, \quad (26)$$

which is not necessarily satisfied. If (7) is violated, then  $\underline{y}_{LL} \leq \underline{y}_{HH}$  and so  $\gamma_2 < 1 \forall \zeta$ . If (7) is satisfied, then Lemma 6 applies and  $\exists \varepsilon_L$  such that  $r_{LL} > r_{HH}$  if and only if  $\zeta > \varepsilon_L$ .

We are left with showing that  $\varepsilon_L > 0$ . For  $\gamma_2 = 1$  the right-hand side of (20) vanishes. Applying Lemma 6,  $r_{LL} > r_{HL}$  if and only if  $\zeta > 0$ . First take  $\zeta \in (\varepsilon_H, \zeta_1)$  so that  $\beta > 0$ . Then,  $\gamma_2 = 1$  if and only if (22) is satisfied. Since  $q_{HH} > q_{HL} \forall \zeta > \varepsilon_H$ , (22) can be satisfied only if  $\zeta > 0$ . Hence,  $\varepsilon_L > 0$ , provided that  $\varepsilon_L$  exists. Next take  $\zeta \geq \zeta_1$  and  $\varepsilon_L \in (\zeta_1, \infty)$ . Then  $\varepsilon_L > 0$ .

## B.9 Proof of Lemma 9

As  $\underline{y}_{HH} = \underline{y}_{HL}$  and  $\bar{y}_{HH} = \bar{y}_{HL}$ , the solution to  $\Gamma''$  is found with  $\gamma_3 = 0$  and  $\beta = 0$ . From (15) in the proof of Lemma 4,  $\gamma_1 > 0 \Leftrightarrow r_{HH} \geq r_{LL}$  and  $\gamma_1 = 1 \Leftrightarrow r_{HH} > r_{LL}$ .

Resting on Lemma 4, we can compute:

$$\begin{aligned} S'(\underline{y}_{HH}) - S'(\underline{y}_{LL}) &= \frac{1 - (1 - \nu)\mu}{(1 - \nu)(1 - \mu)} \Delta\theta + \frac{(1 - \gamma_2)\nu}{1 - \nu} \Delta\sigma \\ S'(\bar{y}_{HH}) - S'(\bar{y}_{LL}) &= \frac{1 - (1 - \nu)\mu}{(1 - \nu)(1 - \mu)} \Delta\theta - \frac{(1 - \gamma_2)\nu}{1 - \nu} \Delta\sigma. \end{aligned}$$

These are both strictly positive, hence  $\underline{y}_{LL} > \underline{y}_{HH}$  and  $\bar{y}_{LL} > \bar{y}_{HH}$ . Computing:

$$S'(\underline{y}_{HH}) - S'(\underline{y}_{LL}) - (S'(\bar{y}_{HH}) - S'(\bar{y}_{LL})) = 2 \frac{(1 - \gamma_2)\nu}{1 - \nu} \Delta\sigma$$

and applying Lemma 6,  $r_{HH} > r_{LL}$  if and only if  $\zeta \leq \varepsilon(\gamma_2)$  for some  $\varepsilon(\gamma_2)$  depending on  $\gamma_2$ . We see that  $\varepsilon(0) < 0 = \varepsilon(1)$ . Denoting  $\varepsilon(0) = \widehat{\varepsilon}_H$ , we conclude that  $\gamma_2 = 0$  if  $\zeta \leq \widehat{\varepsilon}_H$ ;  $0 < \gamma_2 < 1$  if  $\zeta \in (\widehat{\varepsilon}_H, 0)$ ;  $\gamma_2 = 1$  if  $\zeta \geq 0$ .

## B.10 Proof of Lemma 10

Suppose that (IC1) and (IC11) are both binding in  $\Gamma'$  and in  $\Gamma''$ . Then, (IC2) is rewritten as  $\Pi_{LH} - \Pi_{HL} \leq \Delta\sigma r_{LH} + \Delta\theta q_{HL}$ . It holds jointly with (IC9) only if  $r_{LH} \geq r_{HL}$ . (IC10) is rewritten as  $\Pi_{LH} - \Pi_{HL} \leq \Delta\sigma r_{HL} + \Delta\theta q_{LH}$ . It holds jointly with (IC9) only if  $q_{LH} \geq q_{HL}$ .

From Lemma 2:

$$\begin{aligned} S'(\underline{y}_{HL}) - S'(\underline{y}_{LH}) &= \Delta\theta + \Delta\sigma \\ &+ \frac{\nu}{1 - \nu} \left\{ \left[ \beta + (\beta\gamma_2 + \gamma_3) \frac{1 - \mu}{\mu} \right] \Delta\theta + \left[ 1 - \beta + (1 - \beta\gamma_2\nu) \frac{1 - \mu}{\nu\mu} \right] \Delta\sigma \right\}, \end{aligned} \quad (27)$$

$$\begin{aligned} S'(\bar{y}_{HL}) - S'(\bar{y}_{LH}) &= \Delta\theta - \Delta\sigma \\ &+ \frac{\nu}{1 - \nu} \left\{ \left[ \beta + (\beta\gamma_2 + \gamma_3) \frac{1 - \mu}{\mu} \right] \Delta\theta - \left[ 1 - \beta + (1 - \beta\gamma_2\nu) \frac{1 - \mu}{\nu\mu} \right] \Delta\sigma \right\}. \end{aligned} \quad (28)$$

From (27) and (28), we see that  $S'(y_{HL}) - S'(y_{LH}) > S'(\bar{y}_{HL}) - S'(\bar{y}_{LH})$ . From Lemma 2, we also have  $\underline{y}_{LH} > \underline{y}_{HL}$ . It follows from Lemma 6 that if  $\bar{y}_{LH} > \bar{y}_{HL}$ , then  $r_{LH} \geq r_{HL}$  if and only if  $\zeta \geq \varepsilon_2$  for some  $\varepsilon_2 = \varepsilon_{LH} < 0$ . Similarly, if  $\bar{y}_{LH} < \bar{y}_{HL}$ , then it is  $q_{LH} \geq q_{HL}$  if and only if  $\zeta \geq \varepsilon_{LH}$  for some  $\varepsilon_2 = \varepsilon_{LH} < 0$ .

It remains to show that (IC1) and (IC11) are both binding in  $\Gamma'$  and in  $\Gamma''$  when  $\zeta < \varepsilon_{LH}$  so that the two conditions  $r_{LH} \geq r_{HL}$  and  $q_{LH} \geq q_{HL}$  are both necessary in  $\Gamma'$  and in  $\Gamma''$ . (IC1) is binding in  $\Gamma'$  if and only if (IC3) is implied by (IC1). Equivalently:

$$\Delta\sigma(r_{HH} - r_{HL}) \geq \Delta\theta(q_{HH} - q_{HL}). \quad (29)$$

From Lemma 3, and provided that (5) holds, we have  $\underline{y}_{HH} > \underline{y}_{HL}$  together with  $\bar{y}_{HL} > \bar{y}_{HH}$  so that  $r_{HH} > r_{HL}$ . Being based on Lemma 2, we compute:

$$(S'(\underline{y}_{HL}) - S'(\underline{y}_{HH})) - (S'(\bar{y}_{HH}) - S'(\bar{y}_{HL})) = 2 \left( \beta\gamma_2 + \gamma_3 - \frac{(1-\beta)\mu}{1-\mu} \right) \frac{\nu}{(1-\nu)\mu} \Delta\theta. \quad (30)$$

As from the proof of Lemma 7 below, the right-hand side of (30) is strictly positive. Using this in Lemma 6, together with  $\underline{y}_{HH} > \underline{y}_{HL}$  and  $\bar{y}_{HL} > \bar{y}_{HH}$ , we see that  $q_{HH} > q_{HL}$  if and only if  $\zeta > \varepsilon_2$  for some  $\varepsilon_2 = \varepsilon_H < 0$ . Since  $q_{LH} > q_{HH}$ , from the definition of  $\varepsilon_{LH}$  and  $\varepsilon_H$  it follows that  $\varepsilon_H > \varepsilon_{LH}$ . Therefore,  $q_{HH} < q_{HL}$  for  $\zeta < \varepsilon_{LH}$ . Together with  $r_{HH} > r_{HL}$ , this involves that (29) is satisfied. Hence, (IC1) is binding in  $\Gamma'$ . Furthermore, (IC1) is binding in  $\Gamma''$  by definition of the programme. (IC11) is binding in both  $\Gamma'$  and  $\Gamma''$  by definition of the two programmes.

## B.11 Proof of Lemma 11

From Lemma 2, at the solution to  $\Gamma'$  :

$$\begin{aligned} S'(\bar{y}_{LL}) - S'(\underline{y}_{LL}) &= 2 \left( \sigma_L - \gamma_2 \frac{1-\mu}{\mu} \Delta\sigma \right) \\ S'(\bar{y}_{HH}) - S'(\underline{y}_{HH}) &= 2 \left[ \sigma_H + (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \frac{\nu}{1-\nu} \Delta\sigma \right] \\ S'(\bar{y}_{HL}) - S'(\underline{y}_{HL}) &= 2 \left[ \sigma_L - \frac{\nu}{1-\nu} \left[ 1-\beta + (1-\beta\gamma_2\nu) \frac{1-\mu}{\nu\mu} \right] \Delta\sigma \right] \end{aligned}$$

We see that  $S'(\bar{y}_{HH}) > S'(\underline{y}_{HH})$ . To have  $S'(\bar{y}_{LL}) > S'(\underline{y}_{LL}) \forall \gamma_2$ , it suffices that  $\sigma_L > \frac{1-\mu}{\mu} \Delta\sigma$ . To have  $S'(\bar{y}_{HL}) > S'(\underline{y}_{HL}) \forall \beta$  and  $\gamma_2$ , it suffices that  $\sigma_L > \frac{1}{1-\nu} \left( \nu + \frac{1-\mu}{\mu} \right) \Delta\sigma$ . Hence, the three conditions hold jointly if:

$$\sigma_L > \frac{1}{1-\nu} \left( \nu + \frac{1-\mu}{\mu} \right) \Delta\sigma \Leftrightarrow \frac{\sigma_H}{\sigma_L} < \frac{1}{1-\mu(1-\nu)}.$$

From Lemma 9, at the solution to  $\Gamma''$  :

$$S'(\bar{y}_{HH}) - S'(\underline{y}_{HH}) = 2 \left( \sigma_H - \frac{1-\gamma_2\nu}{1-\nu} \Delta\sigma \right),$$

so that  $S'(\bar{y}_{HH}) > S'(\underline{y}_{HH}) \Leftrightarrow \sigma_H > \frac{1-\gamma_2\nu}{1-\nu} \Delta\sigma$ . This holds  $\forall \gamma_2$  if  $\frac{\sigma_H}{\sigma_L} < \frac{1}{\nu}$ , which is implied by  $\frac{\sigma_H}{\sigma_L} < \frac{1}{1-\mu(1-\nu)}$ .



## C Proof of Proposition 1

### C.1 Ranges of values of the ratio $\frac{\Delta\theta}{\Delta\sigma}$

Recall from the proof of Lemma 3 that (5) is rewritten as  $\frac{\Delta\theta}{\Delta\sigma} < |\phi|$  where  $\phi$  is defined by (10).

As long as  $\zeta \leq \varepsilon_H$  so that  $\beta = 1$  and  $\gamma_3 = 1$  (proof of Lemma 8), we have:  $\phi = \frac{1}{\nu} - \mu > 1$ . When  $\zeta \in (\varepsilon_H, \min\{\varepsilon_L, \zeta_1\})$  so that  $\gamma_1 + \gamma_2 = 1$  and  $\beta > 0$  (proof of Lemma 8), we have:

$$\phi = \frac{\frac{1}{\nu} - (\beta - \mu) \left( \gamma_2 + \frac{\mu}{1-\mu} \right)}{\gamma_2\beta - \frac{(1-\beta)\mu}{1-\mu}}.$$

There are many combinations of values of  $\beta, \gamma_2, \nu, \mu$  such that  $\phi > 1$ . In particular, if  $\beta = 1$ , then  $\phi > 1$  if and only if  $\gamma_2 \leq \frac{1-\nu\mu}{\nu[1+(1-\mu)]}$ , the right-hand side of this inequality being positive. If  $\gamma_2 = 1$ , then  $\phi > 1$  if and only if  $\mu < \beta < \frac{1-(1-2\nu)\mu}{2\nu}$ , which is satisfied for various combinations of values of  $\nu$  and  $\mu$ .

When  $\zeta > \zeta_1$  so that  $\beta = 0$  (proof of Lemma 8), we further have:

$$|\phi| = \mu + \left( \gamma_2 + \frac{1}{\nu\mu} \right) (1 - \mu) > 1.$$

Overall,  $\forall \zeta, \exists \nu, \mu$  such that  $|\phi| > 1$ . Hence, it is possible to identify an interval  $(1, |\phi|)$  and an interval  $[|\phi|, \infty)$  within which the ratio  $\frac{\Delta\theta}{\Delta\sigma}$  takes values  $\forall \zeta$ .

### C.2 Incentive constraints in $\Gamma$

#### C.2.1 Incentive constraints omitted in $\Gamma'$

The incentive constraints included in  $\Gamma$  but omitted in  $\Gamma'$  are the upward constraints (IC2), (IC4), (IC5), (IC6), (IC10), (IC12).

Check (IC2). Take  $\zeta \leq \varepsilon_H$ . From Lemma 8,  $\zeta \leq \varepsilon_H \Rightarrow \gamma_3 = 1$ . From Lemma 1,  $\gamma_3 = 1$  implies that (IC9) is binding. (IC2) is rewritten as:

$$\Pi_{LL} \geq \Pi_{HL} + \Delta\theta q_{HL} - \Delta\sigma (r_{LH} - r_{HL}).$$

This is implied by (IC1) if  $r_{LH} \geq r_{HL}$ . From Lemma 10 this is the case if  $\zeta \geq \varepsilon_{LH}$ , which is true by assumption. Take  $\zeta > \varepsilon_H$ . From Lemma 8,  $\zeta > \varepsilon_H \Rightarrow \gamma_2 > 0$ . From Lemma 1,  $\gamma_2 > 0$  implies that (IC8) is binding. If  $\beta > 0$ , then (IC1) is binding as well so that (IC8) is rewritten as:

$$\Pi_{LH} = \Pi_{HL} + \Delta\theta q_{HL} + \Delta\sigma r_{LL},$$

and (IC2) rewritten as:

$$\Pi_{LL} \geq \Pi_{HL} + \Delta\theta q_{HL} - \Delta\sigma (r_{LH} - r_{LL}). \quad (31)$$

From Lemma 2:

$$S'(\bar{y}_{LH}) - S'(\bar{y}_{LL}) = S'(y_{LL}) - S'(y_{LH}) = \left( 1 + \gamma_2 \frac{1-\mu}{\mu} \right) \Delta\sigma \quad (32)$$

so that  $\bar{y}_{LL} > \bar{y}_{LH}$  and  $\underline{y}_{LH} > \underline{y}_{LL}$ . Hence  $r_{LH} > r_{LL}$ , involving that (IC1) implies (31).

Check (IC4). Take  $\zeta \leq \zeta_1$ . From Lemma 7,  $\zeta \leq \zeta_1 \Rightarrow \beta > 0$ . From Lemma 1,  $\beta > 0$  implies that (IC1) is binding. (IC4) is rewritten as  $q_{LL} \geq q_{HL}$ . From Lemma 2:

$$\begin{aligned} S'(\underline{y}_{HL}) - S'(\underline{y}_{LL}) &= \frac{\nu}{1-\nu} \left\{ \left[ \beta + \frac{1-\nu}{\nu} + \frac{(\beta\gamma_2 + \gamma_3)\mu}{1-\mu} \right] \Delta\theta \right. \\ &\quad \left. + \left[ 1 - \beta + [1 - \gamma_2(1 - (1-\beta)\nu)] \frac{1-\mu}{\nu\mu} \right] \Delta\sigma \right\} \\ S'(\bar{y}_{LH}) - S'(\bar{y}_{LL}) &= \left( 1 + \gamma_2 \frac{1-\mu}{\mu} \right) \Delta\sigma. \end{aligned}$$

Using  $S''(\cdot) < 0$  and  $S'(\underline{y}_{HL}) > S'(\underline{y}_{LL})$ , we deduce that  $\underline{y}_{LL} > \underline{y}_{HL}$ . Similarly,  $\bar{y}_{LL} > \bar{y}_{LH}$ . As  $\zeta \geq \varepsilon_{LH}$  (by assumption) and  $\zeta \geq \varepsilon_{LH} \Rightarrow \bar{y}_{LH} > \bar{y}_{HL}$  (Lemma 10), it follows that  $\bar{y}_{LL} > \bar{y}_{HL}$ . Therefore,  $q_{LL} > q_{HL}$ , involving that (IC4) is satisfied.

Check (IC5). Knowing that (IC11) is binding in  $\Gamma'$ , (IC5) is rewritten as  $r_{HH} \geq r_{HL}$ . Since  $\underline{y}_{HH} > \underline{y}_{HL}$  and  $\bar{y}_{HL} > \bar{y}_{HH}$  in  $\Gamma'$ , when the solution to  $\Gamma'$  solves  $\Gamma$  (Lemma 3),  $r_{HH} > r_{HL}$ . Hence, (IC5) is satisfied.

Check (IC6). If  $\zeta \leq \zeta_1 \Leftrightarrow \beta > 0$  (Lemma 3), then (IC1) is binding. (IC6) is rewritten as:

$$\Pi_{LL} \geq \Pi_{LH} - \Delta\sigma r_{LH} - \Delta\theta(q_{LH} - q_{HL}). \quad (33)$$

Since  $q_{LH} \geq q_{HL}$  when  $\zeta > \varepsilon_{LH}$  (Lemma 10) and since  $\zeta > \varepsilon_{LH}$  by assumption, (33) is implied by (31).

Check (IC10). Knowing that (IC11) is binding in  $\Gamma'$ , (IC10) is rewritten as:

$$\Pi_{HL} \geq \Pi_{LH} - \Delta\theta q_{LH} - \Delta\sigma r_{HL}. \quad (34)$$

First take  $\zeta \leq \varepsilon_H \Rightarrow \gamma_3 = 1$  (Lemma 8). It follows that (IC9) is binding (Lemma 1). (34) is rewritten as  $q_{LH} \geq q_{HL}$ . This is satisfied if  $\zeta \geq \varepsilon_{LH}$  (Lemma 10), which is true by assumption.

Next take  $\zeta > \varepsilon_H$ . As long as  $\beta < 1$ , (IC3) is binding (Lemma 1). (IC2) is rewritten as:

$$\Pi_{HH} \geq \Pi_{LH} - \Delta\theta q_{HH} - \Delta\sigma(r_{LH} - r_{HH}).$$

This implies (IC10) if and only if:

$$\Delta\theta(q_{LH} - q_{HH}) \geq \Delta\sigma(r_{LH} - r_{HH}). \quad (35)$$

From Lemma 2:

$$S'(\bar{y}_{HH}) - S'(\bar{y}_{LH}) = \Delta\theta + \frac{\nu}{1-\nu} \left\{ \left[ \gamma_1 + (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \right] \Delta\theta + (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \Delta\sigma \right\},$$

from which  $\bar{y}_{LH} > \bar{y}_{HH}$ . Resting on the definition of  $q_{ij}$  and  $r_{ij}$ , the following equivalence holds:

$$q_{ij} - q_{i'j'} \geq r_{ij} - r_{i'j'} \Leftrightarrow \bar{y}_{ij} > \bar{y}_{i'j'}, \quad \forall ij, i'j'. \quad (36)$$

Using (36) for types  $LH$  and  $HH$ , together with  $\Delta\theta > \Delta\sigma$  (by assumption), (35) is satisfied,

hence so is (IC10). Now take  $\beta = 1$  so that (IC1) is binding. (IC8) is rewritten as:

$$\Pi_{LH} \geq \Pi_{HL} + \Delta\sigma q_{HL} + \Delta\sigma r_{LL}.$$

This is binding because  $\gamma_2 > 0$  when  $\zeta > \varepsilon_H$  (Lemma 8). Thus, (IC10) becomes:

$$\Pi_{HH} \geq \Pi_{HL} + \Delta\theta (q_{HL} - q_{LH}) + \Delta\sigma r_{LL}.$$

(IC11) being binding, (IC10) is further rewritten as:

$$\Delta\theta (q_{LH} - q_{HL}) \geq \Delta\sigma (r_{LL} - r_{HL}). \quad (37)$$

If  $\gamma_2 < 1$ , then  $\Delta\theta (q_{HH} - q_{HL}) = \Delta\sigma (r_{LL} - r_{HL})$  (Lemma 1). From Lemma 2:

$$S'(\underline{y}_{HH}) - S'(\underline{y}_{LH}) = \Delta\theta + \frac{\nu}{1-\nu} \left\{ \left[ \gamma_1 + (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \right] \Delta\theta - (1-\beta) \left( \gamma_2 + \frac{\mu}{1-\mu} \right) \Delta\sigma \right\},$$

from which  $\underline{y}_{LH} > \underline{y}_{HH}$ . Together with  $\bar{y}_{LH} > \bar{y}_{HH}$  (as previously shown), it involves that  $q_{LH} > q_{HH}$ . Hence:

$$\Delta\theta (q_{LH} - q_{HL}) > \Delta\theta (q_{HH} - q_{HL}) = \Delta\sigma (r_{LL} - r_{HL}),$$

involving that (IC10) is satisfied. If  $\gamma_2 = 1$ , then (IC8) is binding. (IC10) is rewritten as:

$$\Pi_{HH} \geq \Pi_{LL} + \Delta\sigma r_{LL} - \Delta\theta q_{LH}.$$

This is implied by (IC12) if  $q_{LH} \geq q_{LL}$ . From (32) we have  $\bar{y}_{LL} > \bar{y}_{LH}$ ,  $\underline{y}_{LH} > \underline{y}_{LL}$  and  $(S'(\underline{y}_{LL}) - S'(\underline{y}_{LH})) - (S'(\bar{y}_{LH}) - S'(\bar{y}_{LL})) = 0$ . Applying Lemma 6,  $q_{LH} \geq q_{LL}$  if and only if  $\zeta \geq 0$ . From the proof of Lemma 8,  $\gamma_2 = 1$  only if  $\zeta > 0$ . Hence, (IC10) is necessarily satisfied for  $\gamma_2 = 1$ .

Check (IC12). As (IC11) is binding, (IC12) is rewritten as:

$$\Pi_{HL} \geq \Pi_{LL} - \Delta\theta q_{LL} - \Delta\sigma (r_{HL} - r_{LL}).$$

This is implied by (IC4) if  $r_{HL} \geq r_{LL}$ . By Lemma 8, this inequality holds when  $\zeta \leq \varepsilon_H$ . Hence, (IC12) is satisfied for  $\zeta \leq \varepsilon_H$ .

Taking now  $\zeta > \varepsilon_H$ , when  $\beta > 0$  (IC1) is binding (Lemma 1). (IC12) is rewritten as:

$$\Pi_{HH} \geq \Pi_{HL} + \Delta\theta (q_{HL} - q_{LL}) + \Delta\sigma r_{LL}.$$

As (IC11) is binding, this further becomes:

$$\Delta\theta (q_{LL} - q_{HL}) \geq \Delta\sigma (r_{LL} - r_{HL}). \quad (38)$$

Recall that  $\zeta > \varepsilon_H \Rightarrow \gamma_2 > 0$  and  $\gamma_3 = 0$  (Lemma 8). Further recall from the proof of Lemma 7 that  $\bar{y}_{LL} > \bar{y}_{HL}$ . Using (36) for types  $LL$  and  $HL$ , together with  $\Delta\theta > \Delta\sigma$ , we see that (38) is satisfied.

Next take  $\beta = 0$  so that (IC3) is binding. (IC12) is rewritten as:

$$\Delta\theta (q_{LL} - q_{HH}) \geq \Delta\sigma (r_{LL} - r_{HH}). \quad (39)$$

From the proof of Lemma 8,  $\bar{y}_{LL} > \bar{y}_{HH}$ . Using (36) for types  $LL$  and  $HL$ , together with  $\Delta\theta > \Delta\sigma$ , we see that (39) is satisfied.

### C.2.2 Incentive constraints omitted in $\Gamma''$

These constraints are the same as in  $\Gamma'$ .

Check (IC2). First take  $\zeta \leq \hat{\varepsilon}_H$  so that  $\gamma_1 = 1$  (Lemma 9). (IC7) is binding (Lemma 1). (IC2) is rewritten as:

$$\Pi_{LL} \geq \Pi_{HH} + \Delta\theta q_{HH} - \Delta\sigma r_{LH}. \quad (40)$$

Since  $r_{LH} \geq r_{HH}$  when  $\zeta > \hat{\varepsilon}_{LH}$  (Lemma 10) and since  $\zeta > \hat{\varepsilon}_{LH}$  by assumption, (40) is implied by (IC3). Next take  $\zeta > \hat{\varepsilon}_H$  so that  $\gamma_2 > 0$  (Lemma 9). (IC8) is binding (Lemma 1). (IC2) is rewritten as  $r_{LH} \geq r_{LL}$ . Since  $\bar{y}_{LL} > \bar{y}_{LH}$  and  $\underline{y}_{LH} > \underline{y}_{LL}$  at the solution to  $\Gamma'$  (as previously proved) and since in  $\Gamma''$  production levels of types  $LL$  and  $LH$  are determined by the same conditions as in  $\Gamma'$  (Lemma 9), it is  $r_{LH} > r_{LL}$ . Hence, (IC2) holds.

Check (IC4). (IC1) being binding in  $\Gamma''$ , (IC4) is rewritten as  $q_{LL} \geq q_{HL}$ . From the proof of Lemma 9,  $\underline{y}_{LL} > \underline{y}_{HH}$  and  $\bar{y}_{LL} > \bar{y}_{HH}$ . By definition of  $\Gamma''$ ,  $q_{HH} = q_{HL}$ . Hence,  $q_{LL} > q_{HH} = q_{HL}$  and so (IC4) is satisfied.

Check (IC5). As (IC11) is binding in  $\Gamma''$ , (IC5) is rewritten as  $r_{HH} \geq r_{HL}$ . By definition of  $\Gamma''$ ,  $r_{HH} = r_{HL}$  so that (IC5) is satisfied.

Check (IC6). As (IC1) is binding, (IC6) is rewritten as (33). Since  $q_{LH} \geq q_{HL}$  when  $\zeta > \hat{\varepsilon}_{LH}$  (Lemma 10) and since  $\zeta > \hat{\varepsilon}_{LH}$  by assumption, (33) is implied by (31).

Check (IC10). As (IC11) is binding in  $\Gamma''$ , (IC10) is rewritten as:

$$\Pi_{HL} \geq \Pi_{LH} - \Delta\theta q_{LH} - \Delta\sigma r_{HL}.$$

Take  $\zeta \leq \hat{\varepsilon}_H$  so that  $\gamma_3 = 1$  (Lemma 8). (IC9) is binding (Lemma 1). (34) is rewritten as  $q_{LH} \geq q_{HL}$ . This is satisfied if  $\zeta \geq \hat{\varepsilon}_{LH}$  (Lemma 10), which is true by assumption. Now take  $\zeta > \hat{\varepsilon}_H$ . (IC3) being binding at the solution to  $\Gamma''$ , (IC2) is rewritten as:

$$\Pi_{HH} \geq \Pi_{LH} - \Delta\theta q_{HH} - \Delta\sigma (r_{LH} - r_{HH}).$$

This implies (IC10) if and only if:

$$\Delta\theta (q_{LH} - q_{HH}) \geq \Delta\sigma (r_{LH} - r_{HH}). \quad (41)$$

From Lemma 9:

$$S'(\bar{y}_{HH}) - S'(\bar{y}_{LH}) = \left(1 + \frac{\nu}{(1-\nu)(1-\mu)}\right) \Delta\theta - \frac{1-\gamma_2\nu}{1-\nu} \Delta\sigma$$

This is non-negative if and only if:

$$\frac{\Delta\theta}{\Delta\sigma} \geq \frac{(1-\gamma_2\nu)(1-\mu)}{\nu + (1-\nu)(1-\mu)},$$

which is true as  $\frac{(1-\gamma_2\nu)(1-\mu)}{(1-\nu)(1-\mu)+\nu} < 1$  and  $\frac{\Delta\theta}{\Delta\sigma} > 1$ . Hence,  $\bar{y}_{LH} > \bar{y}_{HH}$ . Using this in (36),  $q_{LH} - q_{HH} > r_{LH} - r_{HH}$ . Thus, (41) is satisfied and so is (IC10) as well.

Check (IC12). (IC1) is binding, hence (IC12) is rewritten as:

$$\Pi_{HH} \geq \Pi_{HL} + \Delta\theta (q_{HL} - q_{LL}) + \Delta\sigma r_{LL}.$$

As also (IC11) is binding, this is further rewritten as:

$$\Delta\theta (q_{LL} - q_{HL}) \geq \Delta\sigma (r_{LL} - r_{HL}),$$

which is in turn equivalent to:

$$\Delta\theta (q_{LL} - q_{HH}) \geq \Delta\sigma (r_{LL} - r_{HH}).$$

Recall from the proof of Lemma 7 that  $\bar{y}_{LL} > \bar{y}_{HH}$ . Using this in (36),  $q_{LL} - q_{HH} > r_{LL} - r_{HH}$ . Together with  $\Delta\theta > \Delta\sigma$ , it involves that (IC12) is satisfied.

## D On the development of the example

The surplus function is  $S(q) = kq - \frac{q^{1+e}}{1+e}$ , hence  $S'(q) = k - q^e$  and quantities in Lemma 2 are characterized as follows:

$$\begin{aligned} \underline{y}_{LH} &= [a - (\theta_L - \sigma_H)]^{\frac{1}{e}} \\ \bar{y}_{LH} &= [a - (\theta_L + \sigma_H)]^{\frac{1}{e}} \\ \underline{y}_{LL} &= \left[ k - \left( \theta_L - \sigma_L + \gamma_2 \frac{1 - \mu}{\mu} \Delta\sigma \right) \right]^{\frac{1}{e}} \\ \bar{y}_{LL} &= \left[ k - \left( \theta_L + \sigma_L - \gamma_2 \frac{1 - \mu}{\mu} \Delta\sigma \right) \right]^{\frac{1}{e}} \\ \underline{y}_{HH} &= \left[ k - \left( \theta_H - \sigma_H + \gamma_1 \frac{\nu}{1 - \nu} \Delta\theta \right) \right]^{\frac{1}{e}} \\ \bar{y}_{HH} &= \left[ k - \left( \theta_H + \sigma_H + \gamma_1 \frac{\nu}{1 - \nu} \Delta\theta \right) \right]^{\frac{1}{e}} \\ \underline{y}_{HL} &= \left[ k - \left( \theta_H - \sigma_L + \nu \frac{\mu + \gamma_2 (1 - \mu)}{(1 - \nu) \mu} \Delta\theta + (1 - \mu) \frac{1 - (1 - \gamma_1) \nu}{(1 - \nu) \mu} \Delta\sigma \right) \right]^{\frac{1}{e}} \\ \bar{y}_{HL} &= \left[ k - \left( \theta_H + \sigma_L + \nu \frac{\mu + \gamma_2 (1 - \mu)}{(1 - \nu) \mu} \Delta\theta - (1 - \mu) \frac{1 - (1 - \gamma_1) \nu}{(1 - \nu) \mu} \Delta\sigma \right) \right]^{\frac{1}{e}}. \end{aligned}$$

In Table 2 we report the values that the quantities above take for  $e = 1$  and  $e = 0.5$ ,  $\gamma_1$  and  $\gamma_2$  being such that  $\Pi_{LH,1} = \Pi_{LH,2} \Leftrightarrow \Delta\theta(q_{HH} - q_{HL}) = \Delta\sigma(r_{LL} - r_{HL})$  and  $\gamma_1 + \gamma_2 = 1$ :

$e$	$\underline{y}_{HL}$	$\bar{y}_{HL}$	$\underline{y}_{HH}$	$\bar{y}_{HH}$	$\underline{y}_{LL}$	$\bar{y}_{LL}$	$\underline{y}_{LH}$	$\bar{y}_{LH}$
1	11.45	7.35	12.95	6.35	11.45	8.12	14.3	7.7
0.5	133.05	56.3	166.25	39.61	195	65	205	59.29

Table 2: Optimal quantities in the example