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Private or Public Law Enforcement? The Case of Digital Piracy Policies with Non-monitored Illegal Behaviors

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March 2014 - WP 2014-03

Working Paper

Private or Public Law Enforcement? The Case of Digital Piracy Policies with Non-monitored Illegal Behaviors*

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March 2014

Abstract

In the case of digital piracy should rights be publicly or privately enforced? The emergence of large-scale anti-piracy laws and the existence of non-monitored illegal channels raise important issues for the design of digital anti-piracy policies. In this paper, we study the impact of these two enforcement settings (public *vs.* private) in the presence of an illegal non-monitored outside option for users. Taking account of market outcomes, we show that in both cases, the optimal strategies of the legal seller and the monitoring authority leads to rejection of the outside option out while accommodating to the presence of illegal monitored channels. Compared to private enforcement, public enforcement generates higher monitoring levels and lower price levels. Public enforcement also generates greater (legal) welfare. However, we identify potential conflict of interests between the legal seller and the social planner when the efficiency of non monitored networks is low. We provide some insights into the role of supply side anti-piracy policies.

JEL Classification: D23, D78, K42

Keywords: copyright infringement, law enforcement, digital piracy, illegal file-sharing, illegal behavior deterrence

*This work has benefited from financial support from the M@RSOIN institute (ADAUPI project). We are grateful to participants in the 2013 European Policy for Intellectual Property Conference (Paris), 2013 Seminar on Digital piracy (Newark), 2013 Workshop on "Digital Piracy in Cultural Industries" (Rennes) and GREDEG 2014 seminar for their comments on earlier versions of this paper.

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1 Introduction

In its 2013 *Digital Music Report*, the IFPI (International Federation of the Phonographic Industry) estimates that "[in 2013] as many as a third of all internet users (32%) still regularly access unlicensed sites and underlines that "[their] markets remain rigged by illegal free music". For the industries concerned, such persistent online piracy trends are damaging the development of cultural industries and several legal initiatives have been implemented to try to tackle this issue. At the international level, several countries have agreed common standards to reinforce or better protect the rights of copyright holders (e.g., the 2011 *Anti-Counterfeiting Trade Agreement* - ACTA). While the ratification and thus implementation of the ACTA treaty is still ongoing, several countries have implemented their own anti-piracy measures. These fall into two categories - supply-side measures, which target the suppliers who allow illegal file-sharing, and demand-side measures which focus on individuals who share and consume illegal content. Examples of supply-side interventions are the Megaupload ban issued by the US Government in January 2012, and voluntary agreements between search engines and streaming websites on the one hand and copyright holders on the other hand not to rank websites or contents illegally displayed.

A specific demand-side intervention is the so-called "three-strike-law". It involves issuing a warning to an individual who infringes copyright, then suing it the infringement is repeated. The practical enforcement of this type of measures varies greatly among countries. New Zealand and France were among the first countries to implement such a measure and did so through a dedicated "independent administrative authority" - in France, *Hadopi or Haute Autorité pour la Diffusion des Oeuvres et la Protection des Droits sur Internet*, enacted by the French Government in 2008. A similar measure was more recently implemented in the US under the Copyright Alert System (CAS); it includes a larger number of warnings (6 instead of 3) and the penalty incurred is different (monetary fine in the case of France and potential restrictions on Internet connection in the US).¹ The main difference between the two systems is that the US system relies only on private players since it is implemented by a consortium of Internet Access Providers and copyright holders.² Other national initiatives involve the same distinction between private based (e.g. Ireland) and public-based enforcement mechanisms (e.g. South Korea, United Kingdom)

Academic debate on the effects of illegal downloading is very intense (see Waldfogel (2011) for a presentation of key issues). It revolves around both the theoretical mechanisms surrounding piracy (Belleflamme and Peitz (2010)) and how precisely these effects affect the industry from an empirical perspective (for instance, see Hammond (2014)). In

¹When enacted in 2008, the punishment under the Hadopi system was both a monetary fine and a shutdown of the infringers' Internet connection. No individual has been condemned to a connection shutdown and this punishment was abandoned in 2013. However, some infringers have been suited and given a monetary fine. Since the implementation of the CAS is more recent, to our knowledge, there are no official and exhaustive data on its implementation and effects.

²However, under the French system, copyright holders are not totally external to the policy enforcement mechanism since they have a strong influence on the list (number and names) of the titles monitored under supervision of the authority.

the theoretical literature, the design of anti-piracy policies has been well documented (see *e.g.* Banerjee et al. (2008), Choi et al. (2010) or Arai (2011)). However, less is known about the effects of various types of enforcement policies as implemented. Also, most studies oppose legal and illegal practices, the latter always being potentially monitored (although at some cost and to different degrees). A classical result in the literature on the economics of 'crime' is that monitoring some conducts affects the efficiency of legal conducts (Becker (1968)) and also provides incentives to generate new illegal behaviors to bypass existing regulations (Leung (1991)). The latter has been comprehensively documented in the empirical literature on digital piracy (Arnold et al. (2014)) but its impact on policy design and implementations have not been addressed directly. The present paper includes both these dimensions. First, we take full account of the opportunity for digital pirates to use an illegal non-monitored channel as well as both legal and illegal monitored channels to access digital goods. Second, we contrast two implementation settings (private *vs.* public enforcement) from which we derive the expected effect of the type of policy enforcement.

In this context, we show that the optimal strategies of the legal seller and the monitoring authority always lead to rejection of the illegal non-monitored channel, whatever the enforcement setting considered. However, this channel plays the role of a potential incumbent and affects the price that is charged by the legal seller. Since "supply side" anti-piracy policies often target illegal non-monitored channels by attempting to degrade their efficiency, we discuss the impact of this on price and monitoring strategies. The efficiency may move in different directions. On the one hand, supply-side policies might tend to lower it. On the other hand, Internet users continuously develop new illegal non-monitored channels as a reaction to existing regulations, moving their efficiency upwards. We find that increasing the efficiency of non-monitored channels always impacts negatively on price. However, the impact on monitoring intensity depends on the enforcement setting. In the public enforcement setting, an increase in the efficiency of the illegal non-monitored channels leads to higher monitoring intensity. In the private enforcement setting, monitoring intensity increases only if the illegal non-monitored channels are weakly efficient. Comparing the two settings, we find that the legal seller is not necessarily better off in the private enforcement setting compared to the public enforcement setting, because the monitoring authority exerts a higher control effort in the public enforcement setting. This conveys a positive externality on legal sellers when illegal non-monitored channels are weakly efficient. We also consider the effect of the two regimes on welfare, thus highlighting potential coordination issues when designing an enforcement setting.

Section 2 surveys the theoretical and empirical literature on digital piracy and anti-piracy enforcement. Section 3 presents the model. Section 4 presents the optimal strategies of both legal seller and monitoring authority, and their impacts on market outcomes and welfare. Section 5 discusses the results and concludes.

2 Related Literature

We draw on different topics in the literature. The first strand deals with theoretical models of digital piracy.³ These works try to identify the effects of digital piracy on users' behaviors and on the profits of copyright holders, with special emphasis on the conditions for piracy to be profitable for copyright holders. The standard framework is that of a monopolist selling a digital good that may exhibit network externalities but who at the same time is faced with some user communities that are able cheaply to reproduce the original good. The general finding in this literature is that introducing piracy often increases market segmentation. Through various types of externalities (direct or indirect sampling effects, club and network effects, provision of online or offline complementary items⁴), piracy can impact positively on the users' willingness to pay. In that respect, piracy may be profitable if there are substantial spillovers between the legal and illegal goods, depending on the artists' intrinsic characteristics⁵. However, in the long run, piracy may decrease the incentive to innovate and to supply variety (Piolatto and Schuett (2012)). This framework has been extended to account for competition (cf. Belleflamme and Picard (2007)) and for the intervention of commercial illegal players instead of user communities (Banerjee (2006), Martinez-Sanchez (2010)).

A second more general topic is related to economic analysis of illegal activities and public law enforcement to dissuade engagement in these activities. Since Becker (1968) influential article (Becker (1968)), many economists have become interested in analyzing in agents' incentives to engage in illegal activities, and in the type of sanctions that should be used to enforce public law efficiently (see Polinsky and Shavell (2000) for a survey of these issues). There is a body of work on economic analysis of traditional (maritime) piracy as illegal conduct (see *e.g.* Leeson (2010)). A relevant framework for our purposes is Guha (2012) who studies the effect of the monitoring intensity exerted by public authorities and the extent of piracy behaviors. Interestingly, they conclude to the existence of multiple equilibria in which less monitoring is sometimes more efficient. Although this setting is specific to maritime piracy, it provides several insights that can be translated to digital piracy. In particular, it shows that increasing anti-piracy monitoring may also depress the returns from the pirates' alternative occupation. In relation to digital piracy, these alternative occupations are of two types and correspond either to legal purchase of a digital good or to use of some non-monitored ways of acquiring this good. Our paper in some sense echoes the paper by Guha (2012) by introducing both possibilities.

Other studies mix these topics. In the context of commercial digital piracy, Banerjee (2003) examines government's role in restricting commercial piracy and shows that welfare maximization may or may not result in monitoring as the socially optimal outcome. However, the model targets the software industry where commercial piracy may be more widespread than in the case of cultural goods. In a more general setting, Banerjee (2011)

³See Belleflamme and Peitz (2010) for an excellent survey of digital piracy. See also Peitz and Waelbroeck (2006) for a survey of both theoretical and empirical aspects related to digital piracy.

⁴See *e.g.* Dewenter et al. (2012).

⁵See Bacache et al. (orth) for an empirical investigation.

shows that, in some cases, it may be better for the monopolist facing piracy to expand its output beyond the monopoly level rather than to invest in private detection mechanisms. Cremer and Pestieau (2009) also consider the effect of the degree of anti-piracy enforcement on welfare, and distinguishes three levels of enforcement. The highest level is that chosen by the private monopoly. The next level is the one chosen by a welfare-maximizing monopoly. The lowest level, which could be zero, is the monitoring level chosen by the monitoring authority when the good is sold and priced by a profit-maximizing monopoly.

This framework has been extended along several dimensions. A first extension concerns the interplay between private and public anti-piracy policies. In the context of digital experience goods, Banerjee et al. (2008) shows how public and private policies may interact in the case of software commercial piracy. There may be both public regulation (*e.g.*, fines imposed on illegal users when detected) and private regulation (*e.g.*, investment in anti-copying technology). The author shows that if monitoring is socially optimal, the anti-copying investment subgame perfect equilibrium does not guarantee that copying will be prevented. Conversely, if monitoring is not socially optimal, the anti-copying investment subgame perfect equilibrium may guarantee the prevention of copying. Choi et al. (2010) considers a similar issue in the context of competition, and shows that the optimal private protection level depends on the degree of substitution between private protection devices.

A second extension refers to the type of penalty incurred when copyright infringement is detected. Investigating the software market, Arai (2011) asks which court (*i.e.*, civil *vs.* penal) should deal with piracy and demonstrates that welfare is higher if fines are always enforced but is quite small. The last extension includes the possibility to target specific user categories when enforcing anti-piracy policies. Whenever possible, Harbaugh and Khemka (2010) shows that a targeted strategy may be more relevant for both legal sellers and for users than one that involved monitoring all categories of users.

Our paper adds to the theoretical literature on digital piracy.⁶ It differs from the existing literature in two ways. First, although many papers address the issue of digital piracy regulation, we did not find any work that considers the existence of an outside option to monitored piracy other than legal purchase, or no consumption. However, these alternative behaviors are likely to arise in the presence of a monitoring authority and motivate this assumption. Second, while many papers investigate the intensity and characteristics of digital piracy monitoring, few inquire about the impact of alternative enforcement settings. Our model attempts to fill these two gaps in the literature.

3 The Model

We introduce three types of agents in the model. Internet users that consume digital cultural goods (*e.g.*, music, movies). The monitoring authority in charge of monitoring

⁶Note that other papers try to assess the impacts of anti-piracy policy regulations using an experimental or empirical setting. See *e.g.* Adermon and Liang (2010), Arnold et al. (2014), Bhattacharjee et al. (2006), Danaher et al. (orth) or Maffioletti and Ramello (2004).

some illegal channels (*i.e.*, peer-to-peer protocol-based activities). The legal seller that provides the official and unique available legal version of the good.

3.1 Users

There is a continuum of N Internet users (further users) labeled i (with $i = 0, \dots, N$). Without loss of generality, let us suppose that $N = 1$ (hence, market shares are equal to the number of users). Users want to consume the digital content and derive from that a utility equal to v ($v > 0$). To do so, each user i selects one among three different distribution channels (further, channels).

The first potential strategy is to purchase the product legally (labeled B for *buying*). This strategy leads to the following utility level: $U_{B,i} = \alpha_B(v - p)$ with $i = 0, \dots, N$ where p ($p \geq 0$) denotes the price of the official digital good and α_B ($\alpha_B \in [0, 1]$) is the probability that users find the legal version *via* the legal channel.

The second possibility for the user to access digital content is to use a channel where activity is monitored. We denote this strategy by *monitored download* consumption (M). Hence, the use of the illegal channel is risky and leads to a fine f ($f > 0$) if detected. We suppose that users are risk-neutral and face a probability ϕ ($\phi \in [0, 1]$) of being detected using channel M. Further, we suppose that users have heterogeneous skills when downloading illegally. This is depicted by an individual expertise coefficient e_i that we assume to be uniformly distributed between 0 and N . For user i ($i = 1, \dots, N$), this translates into a probability e_i of downloading the searched good successfully when using the illegal non-monitored channel and of getting v . With the remaining probability $(1 - e_i)$, user i does not access the digital good she expects (*e.g.*, file is broken or damaged) and gets 0. Although a file that is downloaded from channel M may not meet the expectations of user i , it may be detected by the monitoring authority, incurring payment of a fine f ($f > 0$). We thus reach the following utility level when using the M channels⁷: $U_{M,i} = ve_i - \phi f$.

Finally, users may choose downloading or copying techniques to acquire digital files. These practices vary (*e.g.*, hand-to-hand copying using USB drives, private networks, newsgroups, illegal streaming websites, etc.). All these *alternative* practices (A) share the common feature of not currently being monitored by the monitoring authority. As for the illegal monitored channel, the utility user i derives from the use of channel A also depends on the users' individual expertise and is expressed by $U_{A,i} = \alpha_A(e_i)(v)$. Here, parameter α_A ($0 < \alpha_A < 1$) captures the relative efficiency of channel A.

Parameters α_k ($k = A, B, M$) all capture the availability of the digital goods which are searched for on the three different channels. Since piracy is a concern for commercial goods only, we consider that $\alpha_B = 1$, thus leaving aside other titles that are not currently commercially distributed, such as those distributed for free, or under alleged copyright terms or those whose copyright has expired. We then have $0 < \alpha_M < \alpha_A$, meaning that the illegal monitored channel displays greater efficiency (*e.g.*, in terms of track

⁷ $U_{M,i} = \alpha_M (((1 - \phi)e_i v + (1 - e_i)0) + \phi(e_i(v - f) + (1 - e_i)(-f))) = ve_i - \phi f$.

availability, download speed, and limitations) than the illegal non-monitored channel. From this viewpoint, we can distinguish these two channels qualitatively by assuming that $\alpha_A < \alpha_M = 1$. In order to focus only on relevant cases, we also assume $f > v > p$ for the punishment for potential infringers when caught, to be credible.

Using these assumptions and rearranging the terms, we have, for $i \in 0, 1$, with $0 < \alpha_A < 1$, $f > v > p \geq 0$:

$$\begin{aligned} U_{i,B} &= (v - p) \\ U_{i,M} &= ve_i - \phi f \\ U_{i,A} &= \alpha_A(e_i)(v) \end{aligned}$$

3.2 Legal seller and the monitoring authority

3.2.1 The legal seller and the monitoring authority

Several distribution strategies are used by content providers (*i.e.*, artists, resellers, platforms). Since here we are not interested in the internal organization of the music and/or movie industries, we only consider a single profit-motivated seller who distributes on the legal market at price p . Because production costs essentially are fixed costs, we normalize these costs to zero and profit simply is as pm_B (where m_B is the market share of the legal seller).

The monitoring authority is in charge of monitoring illegal activity⁸. It implements a monitoring strategy on the monitored illegal channel to track potential infringers. If caught, the infringer has a fine f ($f > 0$) imposed (assessed in monetary terms). Note that this fine may incorporate several types of sanctions such as Internet connection shutdown/restriction, legal penalties possibly including other administrative costs, and some psychological costs.

3.2.2 Monitoring activity in the Public *vs.* Private enforcement setting

Monitoring consists of sampling users on the monitored illegal channel with a probability ϕ . This measures the monitoring intensity enforced by the monitoring authority. Monitoring is costly and this cost depends on monitoring intensity. Let us denote by $\varsigma(\phi) = (a/2)\phi^2$ this monitoring cost (where a ($a > 0$) is a measure of the relative efficiency of the screening technology).

In the private enforcement setting, the legal seller is in charge of monitoring channel M. The monitoring intensity is deduced from a profit-maximizing reasoning where the benefits (revenues from the legal market) are balanced in relation to the monitoring costs. Thus, the objective of the legal seller is to maximize $\pi_{PR} = pm_B - \varsigma(\phi)$ with respect to ϕ , where π_{PR} is the profit of the legal seller in the private enforcement setting.

⁸Note that depending on the legal framework, this control authority may take diverse forms (authority, agency, etc.). For convenience, we use the term "monitoring authority" throughout this paper.

The public enforcement setting applies when the monitoring authority formulates its monitoring strategy independently. To do so, it considers the whole benefit from the legal market and maximizes the welfare generated by legal activities (*i.e.*, the sum of the surplus of the legal users CS_B and legal revenues pm_B) while supporting the cost of this policy ($\varsigma(\phi)$). The objective of the monitoring authority is hence to maximize $H_{PU} = pm_B + CS_B - \varsigma(\phi)$ with respect to ϕ . As opposed to the case of the private enforcement setting, the profit of the legal seller is simply equal to her revenues ($\pi_{PU} = pm_B$).

The game consists of three successive steps. In Step 1, the monitoring level is defined by the legal seller (private enforcement setting) or by the monitoring authority (public enforcement setting). In Step 2, the legal seller selects her pricing strategy p . In Step 3, users adopt from one of the three channels. This defines a sequential game with perfect information which we solve by backward induction.

3.3 Optimal User strategy and consumption outcomes

We consider the last stage of the game so the users' strategies are conditional to p and ϕ . Given these values, users maximize their utility by selecting among channels (B, M or A). For each user i , this choice depends on her expertise e_i as follows ($i \in [0, 1]$):

User i strictly prefers B to M if $U_{B,i} > U_{M,i} \Leftrightarrow e_i < (v - p + \phi f)/v \equiv \tilde{e}$.

User i strictly prefers B to A if $U_{B,i} > U_{A,i} \Leftrightarrow e_i < (v - p)/v\alpha_A \equiv \hat{e}$.

User i strictly prefers A to M if $U_{A,i} > U_{M,i} \Leftrightarrow e_i < \phi f/v(1 - \alpha_A) \equiv \bar{e}$.

From the previous restrictions on parameters, we can easily demonstrate that $\hat{e} \geq 0$, $\tilde{e} \geq 0$ and $\bar{e} \geq 0$. We deduce from that observation that the no-file consumption strategy (which would yield a null payoff) is never a relevant option. However, depending on the price on the legal market (p), on the enforcement exerted (ϕ) and on the efficiency of alternative channels (α_A), we need to consider several parameter configurations to describe all potential consumption outcomes. Excluding inconsistent outcomes (see Appendix 7.1), we identify four qualitative types of consumption outcomes.

Lemma. *Adoption outcomes. For given price p and monitoring level ϕ , four types of outcomes could prevail:*

1. *B-A-M Outcome (hereafter BAM). In this case, considering increasing values of e_i , users select Strategy B ($i \in [0, \hat{e}]$), then A ($i \in [\hat{e}, \bar{e}]$) and then M ($i \in [\bar{e}, 1]$). This outcome occurs only if $\phi < p/f$ and $(v - p)/(v - p + \phi f) \leq \alpha_A < (v - \phi f)/v$.*
2. *B-A Outcome (hereafter BA). In this case, considering increasing values of e_i , users select Strategy B ($i \in [0, \hat{e}]$), then A ($i \in [\hat{e}, 1]$). This outcome occurs if (i) $\phi \leq p/f$ and $\alpha_A > (v - \phi f)/v$ or if (ii) $\phi > p/f$ and $\alpha_A > (v - p)/v$.*
3. *B-M Outcome (hereafter BM). In this case, considering increasing values of e_i , users select Strategy B ($i \in [0, \tilde{e}]$) then M ($i \in [\tilde{e}, 1]$). This outcome occurs when $\phi < p/f$ and $\alpha_A < (v - p)/(v - p + \phi f)$.*
4. *B Outcome (hereafter B). In this case, all users select the legal channel only. The*

existence conditions are detailed in Appendix 7.1.

See proof in Appendix 7.1 ■

As defined by this lemma, these adoption outcomes are conditional on parameters α_A and ϕ . To illustrate this, Figure 1 maps these outcomes according to these two parameters.

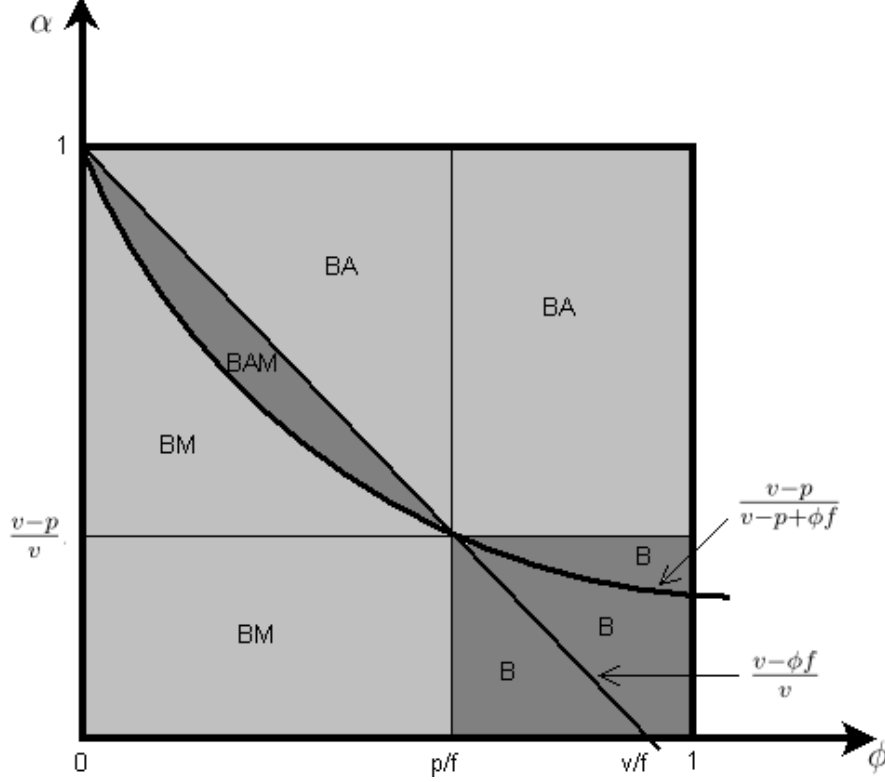


Figure 1: Potential Consumption outcomes as a function of α_A and ϕ

As Figure 1 suggests, when channel A is relatively efficient ($\alpha_A > \frac{v-p}{v}$), the legal seller and the monitoring authority always face illegal behaviors whatever the monitoring strategy. However, the type of illegal behavior is affected by the monitoring strategy. To illustrate this, consider Figure 1 in the case $\alpha_A > \frac{v-p}{v}$. For very low monitoring levels (*i.e.*, ϕ close to 0), we should expect an outcome in which some users buy legally while others use channel M to get access to digital content. To some extent, this case provides a broad description of the situation prior to a demand-side anti-piracy policy being implemented on a large scale. All else being equal, increasing monitoring levels marginally induces users to switch from channel A to channel B. However, beyond some point, any further increase in monitoring leads some users to switch to channel A (BAM outcome). Also, in the case of an increase in the monitoring level, users gradually switch from the illegal monitored channel to the illegal non-monitored channel up to a point where illegal users stop using channel M and use only channel A (BA outcome). This summarizes some of the main issues related to the model: designing an appropriate policy implementation (ϕ) depends on elements that monitoring cannot directly govern namely the efficiency (α_A) of the outside illegal option (*i.e.* illegal non-monitored channel), and the legal seller's pricing

reaction (p). First, the existence of an outside illegal option is crucial for designing an optimal monitoring policy. If it is neglected, inappropriate monitoring levels may push users towards undesired file consumption strategies. Second, since the legal seller plays second, she may react strategically to an increase in monitoring levels by increasing or lowering her price. This generates strategic interdependency between the design of both monitoring and price levels. In addition, the nature of this interdependency is strongly affected by the enforcement setting (public *vs.* private), since the monitoring strategy is defined by the legal seller in the private enforcement setting and by the monitoring authority in the public enforcement setting.

4 Results

Solving backwards, we present the results of the public and private enforcement settings. When presenting the two equilibrium outcomes, we pay attention to the impact of the efficiency of alternative channels. Our motivation is that supply-side policies targeted to alternative channels are designed to impact on their efficiency. Hence, looking at this parameter enables us to consider the interaction between some demand-side and supply-side anti-piracy policies. Thus, we focus on the most interesting case where channels A, B and M can coexist (B-A-M outcome) and we present the results for high values for cost parameter a (*i.e.*, $a > \frac{4f^2}{v}$).

4.1 Public enforcement setting

In the public enforcement setting, the monitoring authority is in charge of the monitoring activity. Recall that the monitoring authority maximizes the whole of the surplus generated by the legal market (minus monitoring costs). This includes the legal seller's profit but also the legal user surplus. Hence, the interests of the monitoring authority and those of the legal seller are not *ex ante* aligned.

Proposition 1. *In the public enforcement setting, the monitoring authority enforces anti-piracy policy with optimal monitoring level $\phi_{PU}^* = \left(\frac{f}{a(1-\alpha_A)}\right)$ and the legal seller charges optimal price $p_{PU}^* = \left(v - \left(\frac{\alpha_A}{a}\right) \left(\frac{f}{1-\alpha_A}\right)^2\right)$ (proof in Appendix) ■*

Corollary. *In the public enforcement setting, illegal non-monitored channel A is deterred from entering the market at equilibrium. Market shares are $m_{B,PU}^* = \frac{f^2}{av(1-\alpha_A)^2}$, $m_{A,PU}^* = 0$ and $m_{M,PU}^* = 1 - m_{B,PU}^*$. The optimal profit of the legal seller amounts to $\pi_{PU}^* = \frac{f^2}{av} \left(\frac{1}{1-\alpha_A}\right)^2 \left(v - \frac{\alpha_A f^2}{a} \left(\frac{1}{1-\alpha_A}\right)^2\right)$.*

Proposition 1 and its corollary characterize the equilibrium in the public enforcement setting. First note that the illegal non-monitored channel A ends up being deterred from the market at equilibrium. This may appear somewhat counter-intuitive since the monitoring authority does not drive any *direct* action against it. What is interesting is that

even if channel A cannot be directly monitored, the strategies of the legal seller and the monitoring authority indirectly influence the use of channel A. At the same time, our results show that when the monitoring authority enforces its optimal level ϕ_{PU}^* in the public enforcement setting, the illegal monitored channel M continues to operate. The mechanisms at stake can be illustrated as follows. Consider an equilibrium outcome as described by proposition 1. Starting from this outcome, suppose an increase in the level of efficiency of channel A. If this occurs and if the legal seller and the monitoring authority do not adjust their strategies, channel A will be reintroduced. The market shares of channels B and M will then decrease and the profit of the legal seller and the surplus of B-users will decrease. These two effects have a negative impact on the objective of the monitoring authority negatively and induce the monitoring authority to increase its monitoring level. However, since monitoring channel A is costly, the monitoring authority is not able to set a monitoring level that would lead to deter channel A. The legal seller also has to contribute some effort so that channel A is deterred, which decreases the seller's price level until the market is again shared by channels B and M. The illegal non-monitored channel A plays the role of a potential incumbent whose efficiency strongly affects the formulation of the optimal price and monitoring strategies.

Proposition 2. *In the public enforcement setting, the efficiency of the illegal non-monitored channel A (α_A) has (i) a negative impact on price and (ii) a positive impact on monitoring level. (proof in Appendix) ■*

Corollary. *In the public enforcement setting, the efficiency of the illegal non-monitored channel A (α_A) has (i) a negative impact on the market share of channel M, and (ii) a positive impact on the market share of channel B.*

We define legal welfare as the sum of the seller's profit and the legal users' surplus minus monitoring costs. We derive the following corollary from proposition 2.

Corollary. *Consider the public enforcement setting. (i) The efficiency of channel A has a positive (resp. negative) impact on the surplus of the users of channel B (resp. M). (ii) The efficiency of channel A has a positive effect on profit if $0 < \alpha_{A,1} < \alpha_A$, negative otherwise (with $\alpha_{A,1} \equiv \frac{1}{4av} \left(3f^2 + 4av - f\sqrt{9f^2 + 32av} \right)$, $0 < \alpha_{A,1} < 1$). (iii) The efficiency of channel A has a positive effect on legal welfare.*

Comparative statics on α_A reveal how the legal seller is able to benefit from the efficiency of the non-monitored illegal channel in this setting. Although the legal seller needs to lower her price, she benefits from a higher sales volume. Because of public enforcement, this is achieved through a higher enforcement level that comes at no cost to her. Taking the twin effect of $\alpha_A > \alpha_{A,1}$ on both p_{PU}^* and $m_{B,PU}^*$ into account, we find mixed results for the impact of α_A on the profit of the legal seller: a greater efficiency of channel A is detrimental to the legal seller's profit only if channel A is relatively efficient, and beneficial otherwise. Indeed, if channel A is highly efficient ($\alpha_A > \alpha_{A,1}$), the legal seller needs to consent to relatively large price cuts to deter use of channel A. For these high values, the negative price effect associated with an increase in α_A outweighs the positive volume effect. Instead, consider lower values of α_A in the public enforcement setting. In this case, the negative price effect is more than compensated for the positive volume

effect which drives a profit increase.

Considering the surplus of legal buyers, we can show that this surplus positively depends on α_A . A straightforward explanation is that an increase in α_A both lowers the price and increases the market share of B. Conversely, the surplus of the users of the illegal monitored channel negatively depends on α_A because of the ensuing decrease in the market share of M on the one hand, and higher enforcement level on the other hand (recall that higher levels of enforcement induce higher detection risks for illegal users). Interestingly, legal welfare always increases with α_A meaning that, for highly efficient illegal non-monitored channels, the positive effect on the legal users' surplus overcomes the negative effect on profit. For relatively high values for α_A , an increase of the efficiency of channel A leads to an increase of the size of the legal market and generates a net surplus transfer from the legal seller to her users.

These mechanisms are illustrated in Panel (a) in Figure 2. Consider first the case of a poorly efficient channel A ($\alpha_A < \alpha_{A,1}$, *Ia* area). Initially, a marginal increase in α_A causes increases in both the legal users' surplus and legal seller's profit. Despite higher control costs, this pushes legal welfare upwards. However, at some point ($\alpha_A > \alpha_{A,1}$, *Ila* area), the positive volume effect is offset by the negative price effect and further increases in α_A are detrimental to profit. However, these increases in α_A benefit legal users through greater access to the legal supply at a cheaper price. Hence, the negative effect on profit is outweighed by a positive effect on user surplus, and the aggregate effect pushes legal welfare upwards again. This situation could be likened to as a net transfer from both the control authority and the legal seller to the legal users. From a (legal) welfare viewpoint, these transfers are welfare-improving.

The effects of supply-side anti-piracy policies. In our model, supply-side anti-piracy policies on alternative channels translate into negative variations of α_A (consisting of harming the efficiency of alternative networks). As suggested by the previous analysis, it is clear that the effect of these policies may be counterproductive. This is because these policies may not fully integrate their effect on the price and monitoring strategies of other illegal channels. Whatever the initial efficiency level of alternative channels (α_A), legal welfare is always harmed by such supply-side policies in the public enforcement setting. However, for less efficient channels ($\alpha_A < \alpha_{A,1}$), these policies may have a positive effect on profit despite their negative effects on legal users' surplus. Such effects cannot be predicted by a framework that does not explicitly consider alternative non-monitored illegal channels as an outside option for users.⁹

4.2 Private enforcement setting

In the private enforcement setting, the legal seller is directly responsible for the monitoring activity. The objective of the legal seller is now defined by the profit she can derive

⁹Note that only the conclusion on profit would change if we considered non-infinitesimal variations in α_A instead of infinitesimal changes.

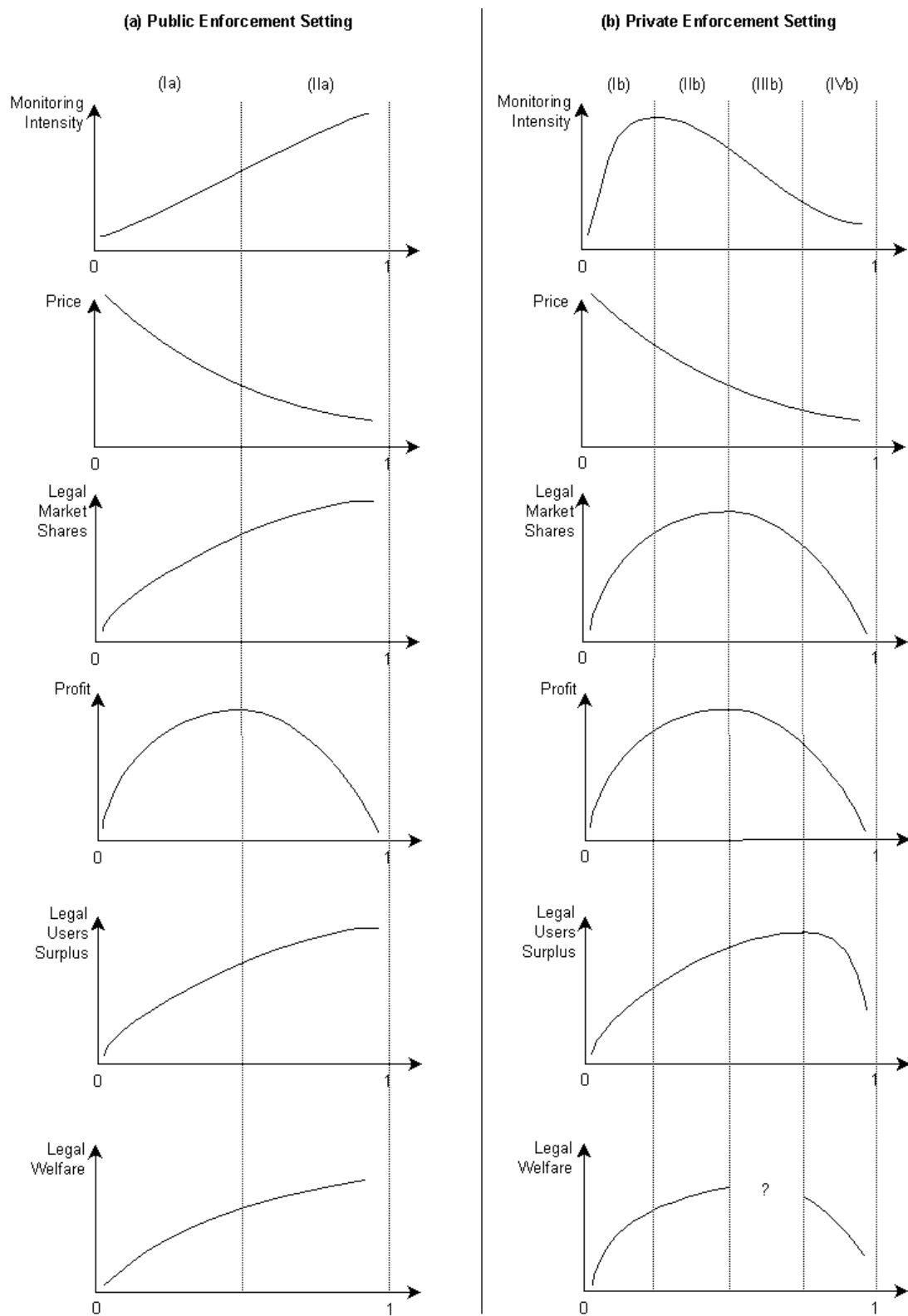


Figure 2: Comparative statics on α_A in the public (panel *a*) *vs.* private (panel *b*) enforcement setting

from legal sales minus the monitoring costs she has to carry¹⁰. Note that, as opposed to the public enforcement setting, the interests of the legal users are not taken into account in the objective-maximization scheme of the legal seller, since the legal seller strictly aligns to her own interests when formulating her monitoring strategy.

Proposition 3. *In the private enforcement setting, the legal seller enforces anti-piracy policy at the optimal monitoring level $\phi_{PR}^* = \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2}$ and charges the optimal price $p_{PR}^* = v \left(\frac{\alpha_A f^2 + av(1-\alpha_A)^2}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right)$. (proof in Appendix) ■*

Corollary. *In the private enforcement setting, illegal non-monitored channel A is deterred from entering the market at equilibrium. Market shares are $m_{B,PR}^* = \frac{f^2}{av(1-\alpha_A)^2 + 2f^2\alpha_A}$, $m_{A,PR}^* = 0$ and $m_{M,PR}^* = 1 - \frac{f^2}{av(1-\alpha_A)^2 + 2f^2\alpha_A}$. The optimal profit of the legal seller amounts to $\pi_{PR}^* = \frac{vf^2}{2(2f^2\alpha_A + av(1-\alpha_A)^2)}$.*

Similar to the public enforcement setting, both price and monitoring strategies are defined so that channel A is deterred from the market, whatever channel A's efficiency. The intuitions underlying this result are the same as those for the case of the public enforcement setting. However, the ability of the legal seller to benefit from the efficiency of channel A in the private enforcement setting differs from what we observed in the public enforcement setting because of different monitoring strategies.

Proposition 4. *In the private enforcement setting, the efficiency of the illegal non-monitored channel has (i) a negative effect on price for any value of $\alpha_A \in [0, 1]$ and (ii) a positive effect on monitoring level if $0 < \alpha_A < \alpha_{A,2}$, negative otherwise (with $\alpha_{A,2} \equiv 1 - \sqrt{\frac{2f^2}{av}}$, $0 < \alpha_{A,2} < 1$). (proof in Appendix) ■*

Corollary. *Consider the private enforcement setting. The efficiency of the non-monitored illegal channel A positively impacts on the market share of channel B and negatively impacts on the market share of channel M if $0 < \alpha_A < \alpha_{A,3}$ (with $\alpha_{A,3} \equiv \frac{av-f^2}{av}$, $0 < \alpha_{A,3} < 1$). Otherwise, it negatively impacts on the market share of channel B and positively impacts on the market share of M.*

Defining again legal welfare as the sum of the seller's profit and of the legal users' surplus, we derive the following corollary from proposition 4 and the previous corollary.

Corollary. *Consider the private enforcement setting. (i) The efficiency of the non-monitored illegal channel A has a positive (resp. negative) impact on profit if $0 < \alpha_A < \alpha_{A,3}$ (resp. $\alpha_{A,3} < \alpha_A < 1$). (ii) The efficiency of channel A has a positive (resp. negative) impact on the surplus of the users of channel B if $0 < \alpha_A < \alpha_{A,4}$ (resp. $\alpha_{A,4} < \alpha_A < 1$).*

¹⁰We chose not to include the revenues derived from the fines/penalties collected from detected infringers for three reasons. First, little is known about the recipients of these fines according to the legal frameworks (copyrights holders, recording associations such as RIAA, public recipients, etc). Second, the penalty incurred may take non monetary forms (e.g. reduced speed Internet connection) that harm only the users without benefiting any other agent. Third, the magnitude of these fines is generally negligible compared to the actual detection costs, and control mechanisms are generally implemented to dissuade rather than as a source of side-revenue.

$\alpha_A < 1$), with $\alpha_{A,4} \equiv \frac{1}{3av} \left(av - f^2 + \sqrt{f^4 - 2avf^2 + 4a^2v^2} \right)$ ($0 < \alpha_{A,4} < 1$). (iii) The efficiency of channel A has a positive impact on legal welfare if $0 < \alpha_A < \widetilde{\alpha}_A$, negative otherwise, with $0 < \widetilde{\alpha}_A < 1$.

Similarly to the public enforcement setting, the optimal price p_{PR}^* charged by the legal seller negatively depends on α_A in the private enforcement setting. However, here the impact of α_A on the optimal monitoring level ϕ_{PR}^* is no longer monotonic. Improving the efficiency of alternative channels (*i.e.*, increasing values for α_A) has a positive impact on monitoring level ϕ_{PR}^* only when channel A is weakly efficient. In the opposite situation (channel A is highly efficient), a legal seller facing a more efficient channel A chooses to *relax* her monitoring effort while continuing to decrease her price. Again, when α_A is relatively high ($\alpha_A > \alpha_{A,3}$), the legal seller has to agree to quite high price cuts which at some point, are detrimental to her. Recall that, in the public enforcement setting, these price cuts had a positive effect on legal users and provided some incentives for the monitoring authority to increase its monitoring effort. This is not the case here since the surplus of legal users is excluded from the objective function of the legal seller. Thus, when α_A is relatively high ($\alpha_A > \alpha_{A,3}$), increasing monitoring effort is not only costly but it also does not provide any additional benefit here from the legal seller's point of view.

All the effects of α_A are summarized and depicted in Panel (b) in Figure 2. Consider first that channel A is weakly efficient ($\alpha_A < \alpha_{A,2}$, *Ib* area). If the efficiency of channel A increases marginally, the legal seller has an incentive to increase her enforcement level and to simultaneously decrease price. These two changes are profit-increasing and also benefit legal users (through a lower price and a higher legal market share). However, because stronger enforcement is costly for the legal seller, the legal seller has to substitute control for price at some point ($\alpha_{A,2} < \alpha_A < \alpha_{A,3}$, *IIb* area). The impact on both profit and legal market share is positive up to some point. Because the alternative channel A is more efficient, price cuts need to be larger and larger. As $\alpha_{A,2} < \alpha_A < \alpha_{A,3}$, it becomes impossible for the legal seller to protect her market share and profit starts declining (*IIIb* area). Here, the effect on legal welfare is ambiguous in the general case. On the one hand, profit (including monitoring costs) is decreasing while on the other hand, legal users' surplus is still increasing: while less and less numerous, legal users initially (*IIIb* area) benefit from reduced prices which increases their surplus. Yet, this positive price effect on legal users vanishes and the negative volume effect is dominant as $\alpha_A > \alpha_{A,4}$ (*IVb* area). Hence, legal welfare is decreasing.

The effects of supply-side anti-piracy policies. As previously, supply-side anti-piracy policies on alternative channels translate into negative variations of α_A (consisting of harming the efficiency of alternative channels). As suggested by the previous analysis on the impact of α_A , one can see that the effect of these policies is far more ambiguous (compared to the public enforcement setting). For highly efficient alternative channels (*IVb* area), there is a rationale for such supply-side anti-piracy policies. However, there is less support for such policies in the *IIIb* area where legal users' surplus and legal welfare may move in opposite directions. In the *Ib* and *IIb* areas, we obtain the results of the private enforcement setting: supply-side anti-piracy policies may harm both legal users

and legal seller.¹¹

4.3 Equilibrium comparison of the public *vs.* private enforcement settings

Here we compare the equilibrium outcomes of public and private enforcement settings.

Proposition 5. *The legal seller achieves a higher profit level in the public enforcement setting compared to that achieved in the private enforcement setting when the efficiency of channels A is relatively low (i.e., $0 < \alpha_A < \widehat{\alpha}_A$, with $0 < \widehat{\alpha}_A < 1$). (proof in Appendix)* ■

Proposition 5 introduces an apparent paradox. It may sound paradoxical that in the public enforcement setting (where the monitoring intensity is not directly ruled by the legal seller) it is possible to generate a higher level of profit compared to the private enforcement setting (where the legal seller has complete latitude to define the monitoring strategy). Proposition 6 helps explain this paradox.

Proposition 6. *Whatever v , a , f and α_A , in the private enforcement setting, as compared to the public enforcement setting, (i) the legal seller charges higher price, (ii) lower monitoring level is implemented and (iii) the market share of the legal channel is lower. (proof in Appendix)* ■

In the public enforcement setting, the monitoring authority exerts a higher monitoring effort towards illegal channels compared to the level exerted by the legal seller in the private enforcement setting. This is because the surplus of the legal users is included in the legal seller's objective function and this surplus increases with higher monitoring levels. Put differently, the legal seller benefits from a positive externality that pushes the monitoring authority to provide a higher monitoring level while incurring no direct cost for this control. Because the control authority also cares about the legal users' surplus, monitoring intensity is higher than implemented by a private actor (the legal seller). In some senses, the legal seller internalizes monitoring costs in the private enforcement setting. Hence, here the monitoring strategy is defined as a balance only between the legal seller's revenues and the monitoring costs she invests. This provides less incentive to control in the private setting. In turn, higher monitoring effort in the public enforcement setting allows the legal seller to relax her price while at the same time increasing her market share. Switching from the private to the public enforcement one is profit-enhancing only if the alternative channels is weakly efficient (i.e., $0 < \alpha_A < \widehat{\alpha}_A$).

In the case of a highly efficient alternative channel, this no longer holds because the control authority tends to control more strictly than the legal seller would. Remember that the rationale for this higher level of monitoring is that it is costly but generates a welfare-improving transfer to legal users. Yet, this transfer is detrimental to the legal seller. Hence, in the case of private enforcement, the rationale for such high levels of

¹¹Note again that these deductions apply for infinitesimal variations in α_A . Because of non-monotonic relationships here, non-infinitesimal changes in α_A may lead to less predictable impacts.

monitoring disappears since the legal seller will not choose a monitoring level that is detrimental to her.

Proposition 7. *Whatever the values for v , a , f and α_A are, the level of the surplus of legal users is higher in the public enforcement setting than that in the private setting. The same applies to legal welfare. (proof in Appendix) ■*

Considering legal welfare, proposition 7 shows clearly the superiority of the public over the private enforcement setting. Indeed, in the public enforcement setting, the legal seller charges a lower price whereas the monitoring authority chooses a higher monitoring level compared to that chosen by the legal seller in the private enforcement setting. The market share of the legal seller is higher in the public enforcement setting and the lower price level thus leads to higher levels of surplus for legal users. Although the legal seller's profit may in some cases be lower in the public enforcement setting than that in the private enforcement setting, this loss is less than the benefit generated by legal users when switching from the private to the public enforcement setting.

Taken together, propositions 5 and 7 highlight potential coordination issues between the legal seller and some social planner when the efficiency of channel α_A is high. Indeed, when $\widehat{\alpha}_A < \alpha_A < 1$, the legal seller is better off operating in the private enforcement setting whereas the social planner prefers the public enforcement setting to the private one. However, when the efficiency of channel α_A is relatively low (*i.e.*, $0 < \alpha_A < \widehat{\alpha}_A$), both legal seller and social planner prefer the public enforcement setting to the private enforcement one.

5 Discussion and Conclusion

Should some rights be publicly or privately enforced? What happens if some illegal behaviors can *never* be monitored? This article proposes a theoretical framework to understand these two issues in the case of digital piracy. In contrast to previous papers which consider that illegal distribution channels can always be monitored (although imperfectly and at some cost), we consider here that, due to their diversity, characteristics, and frequent replacement, anti-piracy policies cannot fully monitor all illegal channels (illegal streaming, offline "hand-to-hand" file copies, etc.) and so the design of public policy about copyright infringements must take account of the presence of this outside option. We build a theoretical model to analyze the interplay between anti-piracy policies, the legal sellers' pricing, and user's legal and illegal behaviors. The first original feature or our approach lies in its contrasting a private and a public enforcement setting in this framework. The second original contribution is considering an outside illegal option for file consumption.

We considered a situation characterized by relatively high monitoring costs (based on anecdotal evidence) and by the initial use of three channels. We show that the optimal strategies of legal sellers and the monitoring authority always lead to deter the non-monitored illegal distribution channel out. However, although this happens at equi-

librium, this non-monitored illegal distribution channel plays the role of a potential incumbent and affects the actual price charged by the legal seller. We analyzed two resulting issues.

The role of supply-side anti-piracy policies. Legal authorities are aware of the potential use of alternative channels and some supply-side anti-piracy policies targeting these channels have been implemented to degrade their efficiency and discourage users from exploiting them. We show that these policies can lead to unexpected and sometimes counter-productive effects. When copyrights are publicly enforced, we showed that these policies always harm legal welfare (sum of the legal seller's revenues and the legal users' surplus minus monitoring costs), because the legal seller has some incentive to increase the price. These measures are not neutral *vis-à-vis* the distribution of welfare. While legal users are always harmed by these measures, the legal seller may sometimes benefit from them since the illegal alternative channel is highly efficient initially. When copyrights are privately enforced, the impact of supply side policies depends on the efficiency of the non-monitored illegal channels before these policies are implemented. As this efficiency is rather low, the results are similar in the private enforcement setting. However, for more efficient alternative channels, the conclusion is either undetermined or reversed. Thus, our findings do not provide full support for supply-side anti-piracy measures targeted towards those channels that are not currently monitored by demand-side anti-piracy policies. It allows pricing and monitoring strategies to be endogenized and the joint effect of these strategies may have effects opposite to that the initially expected effects (discouraging piracy, increasing the size of the legal market).

Should monitoring be publicly or privately enforced? As long as legal welfare is used as the relevant criterion to address this issue, our results clearly points to the superiority of the public enforcement setting. However, public enforcement results in higher monitoring levels compared to private enforcement. The justification for higher control levels is that they enable the legal seller to decrease price (compared to private enforcement) and this also benefits legal users. Are legal sellers better off in this situation? On the one hand, legal sellers save on monitoring costs in the public enforcement setting. On the other hand, they benefit from a larger legal market. If sellers have to choose between public and private enforcement, based on these arguments, they should plead for public enforcement. However, since their ability to raise price in case of public enforcement is lower, public enforcement will be favorable to legal sellers only if alternative illegal channels are weakly efficient. In the opposite case, there is a conflict of interests between the social planner (which here prefers private to public enforcement) and legal sellers (that would here prefer private to public enforcement). Since any issue related to digital piracy is frequently surrounded by high levels of lobbying from both sides, our paper predicts situations where some consensus may be reached compared to other where conflicts may arise. Note however, that the choice between private and public enforcement is not neutral towards public finance, and public enforcement generates higher legal welfare is higher (compared to the private enforcement case), but also generate a non-financed public deficit. While this consideration is beyond the scope of the present study, it may interfere in the choice between public and private enforcement in a context characterized by scarce public resources.

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7 Appendix

7.1 Adoption outcomes

Proof of Lemma (adoption outcomes). The game is solved by backward induction. We first focus on the last step of the game, thus considering given levels for ϕ and p , as well as a given set of parameters (α_A, f, v) . $\tilde{e}$ represents the location of the user indifferent between B and M , whereas $\hat{e}$ (resp. $\bar{e}$) represents that of the indifferent user between B and A (resp. A and M). From the specifications of the utility functions, we find that $\tilde{e} = (v - p + \phi f)/v$, $\hat{e} = \frac{v-p}{\alpha_A v}$ and $\bar{e} = \frac{\phi f}{v(1-\alpha_A)}$. To identify the adoption outcomes, we have to define the relative rankings of $\tilde{e}$, $\hat{e}$ and $\bar{e}$ on the unit line. By doing so, we can deduce the user’s optimal choice as a function of her location e_i for each area on this line. Under our assumptions, we show that

only two rankings between $\tilde{e}$, $\hat{e}$ and $\bar{e}$ are possible, namely $\hat{e} \leq \tilde{e} \leq \bar{e}$ and $\bar{e} \leq \tilde{e} \leq \hat{e}$. Moreover, for the market to be both fully-served and shared by B , A and M , we show that $\tilde{e}$, $\hat{e}$, $\bar{e}$, 0 and 1 have to be ranked so that $0 < \hat{e} \leq \tilde{e} < \bar{e} < 1$ or $0 < \bar{e} < \tilde{e} \leq \hat{e} < 1$. Both rankings apply when ϕ , p , α_A , f and v are defined as follows: $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha_A < \frac{v-\phi f}{v}$. This expresses the two conditions which have to be simultaneously fulfilled for the three channels to cover and share the market.

7.2 Public enforcement setting

Proof of proposition 1. Consider the market to be fully-covered and shared by the three channels, *i.e.*, $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha_A < \frac{v-\phi f}{v}$. In the second stage in the game, the legal seller maximizes her profit w.r.t p . The profit function is written as $\pi(p, \phi) = p\hat{e} = \left(\frac{1}{\alpha_A v}\right)(-p^2 + pv)$. It is straightforward to see that $\pi(p, \phi)$ is an inverted U -shaped function of $p \in \mathbb{R}^+$ which reaches out its optimal state at $p = \frac{v}{2}$, with $\pi(0, \phi) = \pi(v, \phi) = 0$. Remember that conditions $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha_A < \frac{v-\phi f}{v}$ need to be considered when identifying the optimal pricing strategy of the legal seller. These two conditions can be expressed as $\phi f < p$ and $v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f < p$. From here, we have to consider two possible cases: $\phi f < v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f$ and $\phi f > v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f$.

When $\phi f < p$ and $\phi f < v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f$, the two conditions can be rewritten and combined so that $v - \left(\frac{1}{1-\alpha_A}\right)\phi f < p < v$. We again have to account of two subcases. In subcase 1a, we have $v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f < \frac{v}{2}$, which can be rewritten as $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha_A}{\alpha_A}\right) < \phi$. From the shape of function $\pi(p, \phi)$ for $p \in \mathbb{R}$, it is straightforward to see that the optimal pricing strategy of the legal seller is $p^* = \frac{v}{2}$. In subcase 1b, we have $v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f > \frac{v}{2}$, which can be rewritten as $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha_A}{\alpha_A}\right) > \phi$. In this subcase, the optimal pricing strategy of the legal seller can expressed as $p^*(\phi) = v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f$. We proceed in a similar way when $\phi f < p$ and $\phi f > v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f$. The combination of these two conditions leads to $v - \left(\frac{\alpha_A}{1-\alpha_A}\right)\phi f < \phi f < p < v$. Two subcases have to be considered. In subcase 1c, we have $\phi f < \frac{v}{2}$, which can be reexpressed as $\phi < \frac{1}{2}\frac{v}{f}$. Again, from the shape of function $\pi(p, \phi)$ for $p \in \mathbb{R}$, the optimal pricing strategy of the legal seller is given by $p^* = \frac{v}{2}$. In subcase 1d, ϕ is defined so that $\phi f > \frac{v}{2}$, which can be rewritten as $\phi > \frac{v}{2f}$. This latter subcase leads to $p^*(\phi) = \phi f$. We then move to the first stage in the game. Here, the monitoring authority maximizes its objective function w.r.t. monitoring intensity ϕ . This function is defined as the sum of legal consumers' and the seller's surpluses minus the cost invested by the monitoring authority to monitor channel M . Formally, the objective function of the monitoring authority is given by $H(p, \phi) = \pi(p, \phi) + CS_B(p, \phi) - \varsigma(\phi)$, with $\varsigma(\phi) = \frac{a}{2}\phi^2$. Here, as we come from the second stage of the game, p is given as an expression of ϕ . To identify the optimal strategy (p^*, ϕ^*) of the legal seller and the monitoring authority, we successively deal with subcases 1a, 1b, 1c and 1d. Subcase 1a refers to the subcase in which $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha_A}{\alpha_A}\right) < \phi < \frac{v}{f}(1-\alpha_A)$. It results from the latter condition that α_A is here defined so that $\frac{1}{2} < \alpha_A < 1$. By substituting $p^* = \frac{v}{2}$ into the objective of the monitoring authority, we find $H(\phi) = \frac{v}{2\alpha_A} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints of subcase 1a, we show that ϕ

has to be defined so that $0 < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \phi < \frac{v}{f} (1 - \alpha_A) < \frac{1}{2} \frac{v}{f}$. The monitoring authority therefore sets its optimal monitoring strategy ϕ^* which maximizes function $H(\phi) = \frac{v}{2\alpha_A} - \frac{a}{2}\phi^2$ on $\frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \phi < \frac{v}{f} (1 - \alpha_A)$. This leads it to define $\phi^* = \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$. The optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2} \frac{v(1-\alpha_A)}{\alpha_A f} \right)$ defines the solution of the public enforcement setting in subcase 1a. Here, we obtain $\pi^* = \frac{v}{4\alpha_A}$ and $H^* = \frac{v}{2\alpha_A} - \frac{a}{2} \left(\frac{1}{2} \frac{v(1-\alpha_A)}{\alpha_A f} \right)^2$. Subcase 1b refers to the subcase in which $\phi < \min \left\{ \frac{v}{f} (1 - \alpha_A), \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) \right\}$. By substituting $p^*(\phi) = v - \left(\frac{\alpha_A}{1-\alpha_A} \right) \phi f$ into the objective function of the monitoring authority, we find $H(\phi) = \frac{f}{1-\alpha_A} \phi - \frac{a}{2} \phi^2$. One can show that $H(\phi) = \frac{f}{1-\alpha_A} \phi - \frac{a}{2} \phi^2$ is an inverted U -shaped function of $\phi \geq 0$ which reaches its optimal state at $\phi = \frac{f}{a(1-\alpha_A)}$, with $H(0) = H\left(\frac{2f}{a(1-\alpha_A)}\right) = 0$. Let us first consider subcase 1b1 in which $0 < \phi < \frac{v}{f} (1 - \alpha) < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$. From the latter expression of ϕ , α_A is here defined so that $0 < \alpha_A < \frac{1}{2}$. The monitoring authority therefore sets its optimal monitoring strategy ϕ^* which maximizes function $H(\phi) = \frac{f}{1-\alpha_A} \phi - \frac{a}{2} \phi^2$ on $0 < \phi < \frac{v}{c} (1 - \alpha) < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$. The monitoring authority sets $\phi^* = \frac{v}{f} (1 - \alpha_A)$ when $\frac{v}{f} (1 - \alpha_A) < \frac{f}{a} \left(\frac{1}{1-\alpha_A} \right)$ and $\phi^* = \frac{f}{a} \left(\frac{1}{1-\alpha_A} \right)$ when $\frac{v}{f} (1 - \alpha_A) > \frac{f}{a} \left(\frac{1}{1-\alpha_A} \right)$. As such, the optimal strategy $(p^*, \phi^*) = \left(v(1 - \alpha_A), \frac{v}{f} (1 - \alpha_A) \right)$ (resp. $(p^*, \phi^*) = \left(v - \alpha_A \left(\frac{1}{1-\alpha_A} \right)^2 \left(\frac{f^2}{a} \right), \frac{f}{a} \left(\frac{1}{1-\alpha_A} \right) \right)$) defines the solution of the public enforcement setting in subcase 1b1 when $0 < a < \frac{f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$ (resp. $a > \frac{f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$). From here, we obtain $\pi^* = v(1 - \alpha_A)$ and $H^* = v - \frac{a}{2} \left(\frac{v}{f} (1 - \alpha_A) \right)^2$ when $0 < a < \frac{f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$, whereas we have $\pi^* = \frac{f^2}{av} \left(\frac{1}{1-\alpha_A} \right)^2 \left(v - \frac{\alpha_A f^2}{a} \left(\frac{1}{1-\alpha_A} \right)^2 \right)$ and $H^* = \frac{f^2}{a} \left(\frac{1}{1-\alpha_A} \right)^2 - \frac{a}{2} \left(\frac{f}{a} \left(\frac{1}{1-\alpha_A} \right) \right)^2$ when $a > \frac{f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$. Let us next consider subcase 1b2 in which $0 < \phi < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \frac{v}{c} (1 - \alpha_A)$. From the latter expression of ϕ , α_A is now defined such that $\frac{1}{2} < \alpha_A < 1$. The monitoring authority therefore sets its optimal monitoring strategy ϕ^* which maximizes function $H(\phi) = \frac{f}{1-\alpha_A} \phi - \frac{a}{2} \phi^2$ on $0 < \phi < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \frac{v}{c} (1 - \alpha_A)$. Recall that $H(\phi) = \frac{f}{1-\alpha_A} \phi - \frac{a}{2} \phi^2$ is an inverted U -shaped function of $\phi \geq 0$ which reaches out its optimal state at $\phi = \frac{f}{a(1-\alpha_A)}$, with $H(0) = H\left(\frac{2f}{a(1-\alpha_A)}\right) = 0$. The monitoring authority sets $\phi^* = \frac{v}{2f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$ when $0 < a < \frac{2\alpha_A f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$ and $\phi^* = \frac{f}{a} \left(\frac{1}{1-\alpha_A} \right)$ when $a > \frac{2\alpha_A f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$. $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{2f} \left(\frac{1-\alpha_A}{\alpha_A} \right) \right)$ (resp. $(p^*, \phi^*) = \left(v - \alpha_A \left(\frac{1}{1-\alpha_A} \right)^2 \left(\frac{f^2}{a} \right), \frac{f}{a} \left(\frac{1}{1-\alpha_A} \right) \right)$) defines the solution of the public enforcement setting in subcase 1b2 when $0 < a < \frac{2\alpha_A f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$ (resp. $a > \frac{2\alpha_A f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$). From here, we obtain $\pi^* = \frac{v}{4\alpha_A}$ and $H^* = \frac{v}{2\alpha_A} - \frac{a}{2} \left(\frac{v}{2f} \left(\frac{1-\alpha_A}{\alpha_A} \right) \right)^2$ when $0 < a < \frac{2\alpha_A f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$, whereas we have $\pi^* = \frac{f^2}{av} \left(\frac{1}{1-\alpha_A} \right)^2 \left(v - \frac{\alpha_A f^2}{a} \left(\frac{1}{1-\alpha_A} \right)^2 \right)$ and $H^* = \frac{f^2}{a} \left(\frac{1}{1-\alpha_A} \right)^2 - \frac{a}{2} \left(\frac{f}{a} \left(\frac{1}{1-\alpha_A} \right) \right)^2$ when $a > \frac{2\alpha_A f^2}{v} \left(\frac{1}{1-\alpha_A} \right)^2$. Subcase 1c refers to the subcase in which $\frac{v}{f} (1 - \alpha_A) < \phi < \frac{1}{2} \frac{v}{f}$. It results from the latter condition that α_A is here defined so that $\frac{1}{2} < \alpha_A < 1$. By substituting $p^* = \frac{v}{2}$ into the objective function of the monitoring authority, we find $H(\phi) = \frac{v}{2\alpha_A} - \frac{a}{2} \phi^2$, which is a decreasing function of $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints

of subcase 1c, we show that ϕ has to be defined such that $\phi = \frac{v}{f}(1 - \alpha_A)$ and $\frac{1}{2} < \alpha_A < 1$. This defines the optimal monitoring strategy of the monitoring authority. As such, the optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1 - \alpha_A)\right)$ defines the solution for the public enforcement setting in subcase 1c. From here, we obtain $\pi^* = \frac{v}{4\alpha_A}$ and $H^* = \frac{v}{2\alpha_A} - \frac{a}{2} \left(\frac{v}{f}(1 - \alpha_A)\right)^2$. Subcase 1d refers to the subcase in which $\phi > \max\left\{\frac{v}{f}(1 - \alpha_A), \frac{1}{2}\frac{v}{f}\right\}$. By substituting $p^* = \phi f$ into the objective function of the monitoring authority, we find $H(\phi) = \frac{v - \phi f}{\alpha_A} - \frac{a}{2}\phi^2$, which is a decreasing function of $\phi \geq 0$. When combining expression $p^* = \phi f$ with all the constraints of subcase 1d, we show that ϕ has to be defined such that $\phi = \frac{v}{f}(1 - \alpha_A)$ and $0 < \alpha_A < \frac{1}{2}$. This defines the optimal monitoring strategy of the monitoring authority. As such, the optimal strategy $(p^*, \phi^*) = \left(v(1 - \alpha_A), \frac{v}{f}(1 - \alpha_A)\right)$ defines the solution for the public enforcement setting in subcase 1d. From here, we obtain $\pi^* = v(1 - \alpha_A)$ and $H^* = v - \frac{a}{2} \left(\frac{v}{f}(1 - \alpha_A)\right)^2$. Putting all these results together we identify four candidate solutions for the public enforcement setting, regardless of the nature of the subcases. These depend on α_A and a : (i) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2}\frac{v(1 - \alpha_A)}{\alpha_A f}\right)$ when $\frac{1}{2} < \alpha_A < 1$, (ii) $(p^*, \phi^*) = \left(v(1 - \alpha_A), \frac{v}{f}(1 - \alpha_A)\right)$ when $0 < \alpha_A < \frac{1}{2}$, (iii) $(p^*, \phi^*) = \left(v - \alpha_A \left(\frac{1}{1 - \alpha_A}\right)^2 \left(\frac{f^2}{a}\right), \frac{f}{a} \left(\frac{1}{1 - \alpha_A}\right)\right)$ when $0 < \alpha_A < \frac{1}{2}$ and $a > \frac{f^2}{v} \left(\frac{1}{1 - \alpha_A}\right)^2$ or when $\frac{1}{2} < \alpha_A < 1$ and $a > \frac{2\alpha_A f^2}{v} \left(\frac{1}{1 - \alpha_A}\right)^2$, and (iv) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1 - \alpha_A)\right)$ when $\frac{1}{2} < \alpha_A < 1$. As the monitoring authority plays first, it also indirectly shapes the strategy of the legal seller. Indeed, the monitoring authority selects its strategy so that the outcome of the public enforcement setting provides the best level H^* for her objective function $H(p, \phi)$. Let us thus focus on levels of H for the four candidate solutions of the public enforcement setting, *i.e.*, (i) $H_1 = \frac{v}{2\alpha_A} - \frac{a}{2} \left(\frac{1}{2}\frac{v(1 - \alpha_A)}{\alpha_A f}\right)^2$, (ii) $H_2 = v - \frac{a}{2} \left(\frac{v}{f}(1 - \alpha_A)\right)^2$, (iii) $H_3 = \frac{f^2}{a} \left(\frac{1}{1 - \alpha_A}\right)^2 - \frac{a}{2} \left(\frac{f}{a} \left(\frac{1}{1 - \alpha_A}\right)\right)^2$ and (iv) $H_4 = \frac{v}{2\alpha_A} - \frac{a}{2} \left(\frac{v}{f}(1 - \alpha_A)\right)^2$. We can show that $H_3 > H_2$ and that $H_3 > H_1 > H_4$. Recall that we have assumed cost parameter a to be high (*i.e.*, $a > \frac{4f^2}{v}$), the solution to the public enforcement setting is therefore given by $(p_{PU}^*, \phi_{PU}^*) = \left(v - \alpha_A \left(\frac{1}{1 - \alpha_A}\right)^2 \left(\frac{f^2}{a}\right), \frac{f}{a} \left(\frac{1}{1 - \alpha_A}\right)\right)$. The profit of the legal seller is $\pi_{PU}^* = \frac{f^2}{av} \left(\frac{1}{1 - \alpha_A}\right)^2 \left(v - \frac{\alpha_A f^2}{a} \left(\frac{1}{1 - \alpha_A}\right)^2\right)$ and the surplus of her customers is $CS_{B,PU}^* = \frac{\alpha_A f^4}{a^2 v (1 - \alpha_A)^4}$.

Proof of proposition 2. In the public enforcement setting, optimal values for price and monitoring level are given by $(p_{PU}^*, \phi_{PU}^*) = \left(v - \alpha_A \left(\frac{1}{1 - \alpha_A}\right)^2 \left(\frac{f^2}{a}\right), \frac{f}{a} \left(\frac{1}{1 - \alpha_A}\right)\right)$. The sign of $\frac{\partial p_{PU}^*}{\partial \alpha_A}$ (resp. $\frac{\partial \phi_{PU}^*}{\partial \alpha_A}$) shows that a higher level of efficiency for the illegal non-monitored channel affects the optimal value for price (resp. monitoring level). $\frac{\partial p_{PU}^*}{\partial \alpha_A} = \frac{\partial \left(v - \alpha_A \left(\frac{1}{1 - \alpha_A}\right)^2 \left(\frac{f^2}{a}\right)\right)}{\partial \alpha_A} = -\frac{f^2(1 + \alpha_A)}{a(1 - \alpha_A)^3} < 0$. In similar way, $\frac{\partial \phi_{PU}^*}{\partial \alpha_A} = \frac{\partial \left(\frac{f}{a} \left(\frac{1}{1 - \alpha_A}\right)\right)}{\partial \alpha_A} = \frac{f}{a(1 - \alpha_A)^2} > 0$.

7.3 Private enforcement setting

Proof of proposition 3. Consider again that the market is fully-covered and fully-shared by the three channels, *i.e.*, $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha_A < \frac{v-\phi f}{v}$. In the second stage of the private enforcement setting, the legal seller still maximizes their profit function w.r.t p . However note that she now also has to account of monitoring costs in her profit-maximizing program. Since here monitoring costs do not here depend on p , the results are the same as thos for the public enforcement setting. Indeed, we eventually have to consider subcases 1a, 1b, 1c and 1d, which were previously identified in the proof of proposition 1. We now move to the first stage of the private enforcement game. Recall here that the legal seller is also in charge of monitoring channel M. Here, she maximizes her objective function w.r.t. monitoring level ϕ . This function is defined here by the difference between her revenues and the cost she has to invest to control channel M. Here, as we come from the second stage of the game, p is given as an expression of ϕ . To identify the optimal strategy (p^*, ϕ^*) of the legal seller, we successively deal with subcases 1a, 1b, 1c and 1d. Subcase 1a refers to the subcase in which $\frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \phi < \frac{v}{f} (1 - \alpha_A)$. It results from the latter condition that α_A is defined here such that $\frac{1}{2} < \alpha_A < 1$. By substituting $p^* = \frac{v}{2}$ into the objective function of the legal seller, we find $\pi(\phi) = \frac{v}{4\alpha_A} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints of subcase 1a, we show that ϕ has to be defined so that $0 < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \phi < \frac{v}{f} (1 - \alpha_A) < \frac{1}{2} \frac{v}{f}$. The legal seller therefore sets her optimal monitoring strategy ϕ^* which maximizes function $\pi(\phi) = \frac{v}{2\alpha_A} - \frac{a}{2}\phi^2$ on $\frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \phi < \frac{v}{f} (1 - \alpha_A)$. This leads her to define $\phi^* = \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$. The optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) \right)$ defines the solution of the public enforcement setting in subcase 1a. Here, we obtain $\pi^* = \frac{v}{4\alpha_A} - \frac{a}{2} \left(\frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) \right)^2$. Subcase 1b refers to the subcase in which $\phi < \min \left\{ \frac{v}{f} (1 - \alpha_A), \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) \right\}$. By substituting $p^*(\phi) = v - \left(\frac{\alpha_A}{1-\alpha_A} \right) \phi f$ into the objective function of the legal seller, we find $\pi(\phi) = \frac{\phi f}{1-\alpha_A} - \frac{\phi^2 f^2 \alpha_A}{v(1-\alpha_A)^2} - \frac{a}{2}\phi^2$. We can show that $\pi(\phi) = \frac{\phi f}{1-\alpha_A} - \frac{\phi^2 f^2 \alpha_A}{v(1-\alpha_A)^2} - \frac{a}{2}\phi^2$ is an inverted U-shaped function of $\phi \geq 0$ which reaches its optimal state at $\phi = \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2}$, with $\pi(0) = \pi \left(\frac{2fv(1-\alpha_A)^2}{2\alpha_A f^2 - av(1-\alpha_A)^2} \right) = 0$. When combining expression $p^* = v - \left(\frac{\alpha_A}{1-\alpha_A} \right) \phi f$ with all the constraints of subcase 1b, we show that ϕ has to be defined such that $0 < \phi < \frac{v}{f} (1 - \alpha_A)$, $0 < \phi < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$ and $0 < \phi < \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$. Note that $\frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$ and that $\frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) > \frac{v}{f} (1 - \alpha_A)$. Consequently, we can show that ϕ has to be defined here such that $0 < \phi < \frac{v}{f} (1 - \alpha_A) < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$ (subcase 1b1') or $0 < \phi < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) < \frac{v}{f} (1 - \alpha_A) < \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$ (subcase 1b2'). Let us first consider subcase 1b1'. From the expression of ϕ , α_A is here defined such that $0 < \alpha_A < \frac{1}{2}$. Since ϕ is defined such that $0 < \phi < \frac{v}{f} (1 - \alpha_A)$, we show that the legal seller sets $\phi^* = \frac{v}{f} (1 - \alpha_A)$ when $\frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} > \frac{v}{f} (1 - \alpha_A)$ and $\phi^* = \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2}$ when $\frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} < \frac{v}{f} (1 - \alpha_A)$. The optimal strategy $(p^*, \phi^*) = \left(v(1 - \alpha_A), \frac{v}{f} (1 - \alpha_A) \right)$ (resp. $(p^*, \phi^*) = \left(v \left(\frac{\alpha_A f^2 + av(1-\alpha_A)^2}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right), \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right)$) defines the solution for the public enforcement setting in subcase 1b1' when $0 < a < \frac{f^2(1-2\alpha_A)}{v(1-\alpha_A)^2}$ (resp. $a > \frac{f^2(1-2\alpha_A)}{v(1-\alpha_A)^2}$). From this, we obtain $\pi^* = v(1 - \alpha_A) - \frac{a}{2} \left(\frac{v}{f} (1 - \alpha_A) \right)^2$ when $0 < a < \frac{f^2(1-2\alpha_A)}{v(1-\alpha_A)^2}$,

whereas we have $\pi^* = \frac{vf^2}{2(2f^2\alpha_A + av(1-\alpha_A)^2)}$ when $a > \frac{f^2(1-2\alpha_A)}{v(1-\alpha_A)^2}$. Let us also consider subcase 1b2'. From the expression of ϕ , α_A is now defined such that $\frac{1}{2} < \alpha_A < 1$. Since ϕ is defined such that $0 < \phi < \frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right)$ and as it can be shown that $\frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) > \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2}$ for any value of a ($a > 0$), we find that the legal seller sets $\phi^* = \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2}$. The optimal strategy $(p^*, \phi^*) = \left(v \left(\frac{\alpha_A f^2 + av(1-\alpha_A)^2}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right), \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right)$ defines the solution of the private enforcement setting in subcase 1b2'. From here, we obtain $\pi^* = \frac{vf^2}{2(2f^2\alpha_A + av(1-\alpha_A)^2)}$.

Subcase 1c refers to the subcase in which $\frac{v}{f}(1-\alpha_A) < \phi < \frac{1}{2} \frac{v}{f}$. It results from the latter condition that α_A is here defined so that $\frac{1}{2} < \alpha_A < 1$. By substituting $p^* = \frac{v}{2}$ into the objective function of the legal seller, we find $\pi(\phi) = \frac{v}{4\alpha_A} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints of subcase 1c, we can show that ϕ has to be defined so that $\phi = \frac{v}{f}(1-\alpha_A)$ and $\frac{1}{2} < \alpha_A < 1$. This defines the optimal monitoring strategy of the legal seller. As such, the optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1-\alpha_A) \right)$ defines the solution for the private enforcement setting in subcase 1c. From here, we obtain $\pi^* = \frac{v}{4\alpha_A} - \frac{a}{2} \left(\frac{v}{f}(1-\alpha_A) \right)^2$.

Subcase 1d refers to the subcase in which $\phi > \max \left\{ \frac{v}{f}(1-\alpha_A), \frac{1}{2} \frac{v}{f} \right\}$. By substituting $p^* = \phi f$ into the objective function of the legal seller, we find $\pi(\phi) = \left(\frac{f}{\alpha_A} \right) \phi - \left(\frac{2f^2 + av\alpha_A}{2v\alpha_A} \right) \phi^2$. We can show that $\pi(\phi) = \left(\frac{f}{\alpha_A} \right) \phi - \left(\frac{2f^2 + av\alpha_A}{2v\alpha_A} \right) \phi^2$ is an inverted U -shaped function of $\phi \geq 0$ which reaches its optimal state at $\phi = \frac{fv}{2f^2 + av\alpha_A}$, with $\pi(0) = \pi \left(\frac{2fv}{2f^2 + av\alpha_A} \right) = 0$. When combining expression $p^* = \phi f$ with all the constraints of subcase 1d, we eventually show that ϕ has to be defined such that $\phi = \frac{v}{f}(1-\alpha_A)$ and $0 < \alpha_A < \frac{1}{2}$. This defines the optimal monitoring strategy of the legal seller. As such, the optimal strategy $(p^*, \phi^*) = \left(v(1-\alpha_A), \frac{v}{f}(1-\alpha_A) \right)$ defines the solution for the private enforcement setting in subcase 1d. From this, we obtain $\pi^* = v(1-\alpha_A) - \frac{a}{2} \left(\frac{v}{f}(1-\alpha_A) \right)^2$.

Putting all these results together we identify four candidate solutions of the private enforcement setting, regardless of the nature of the subcases we have taken into account, which depend on α_A and a : (i) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2} \frac{v(1-\alpha_A)}{\alpha_A f} \right)$ when $\frac{1}{2} < \alpha_A < 1$, (ii) $(p^*, \phi^*) = \left(v(1-\alpha_A), \frac{v}{f}(1-\alpha_A) \right)$ when $0 < \alpha_A < \frac{1}{2}$, (iii) $(p^*, \phi^*) = \left(v \left(\frac{\alpha_A f^2 + av(1-\alpha_A)^2}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right), \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right)$ when $0 < \alpha_A < \frac{1}{2}$ and $a > \frac{f^2(1-2\alpha_A)}{v(1-\alpha_A)^2}$ or when $\frac{1}{2} < \alpha_A < 1$, and (iv) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1-\alpha_A) \right)$ when $\frac{1}{2} < \alpha_A < 1$. As opposed to the public enforcement setting, the legal seller plays at both first and second stages of the private enforcement setting game. The legal seller selects her strategy so that the outcome of the private enforcement setting provides the best level π^* for her objective function $\pi(p, \phi)$. Let us focus on levels of π for the four candidate solutions of the private enforcement setting, *i.e.*, (i) $\pi'_1 = \frac{v}{4\alpha_A} - \frac{a}{2} \left(\frac{1}{2} \frac{v}{f} \left(\frac{1-\alpha_A}{\alpha_A} \right) \right)^2$, (ii) $\pi'_2 = v(1-\alpha_A) - \frac{a}{2} \left(\frac{v}{f}(1-\alpha_A) \right)^2$, (iii) $\pi'_3 = \frac{vf^2}{2(2f^2\alpha_A + av(1-\alpha_A)^2)}$ and (iv) $\pi'_4 = \frac{v}{4\alpha_A} - \frac{a}{2} \left(\frac{v}{f}(1-\alpha_A) \right)^2$. We can show that $\pi'_3 > \pi'_2$ and that $\pi'_3 > \pi'_1 > \pi'_4$. Recall that we have assumed cost parameter a to be high (*i.e.*, $a > \frac{4f^2}{v}$), the solution for the private enforcement setting is therefore given by $(p_{PR}^*, \phi_{PR}^*) = \left(v \left(\frac{\alpha_A f^2 + av(1-\alpha_A)^2}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right), \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right)$. The profit of the legal seller amounts is $\pi^* = \frac{vf^2}{2(2f^2\alpha_A + av(1-\alpha_A)^2)}$ and the surplus of her users is $CS_{B,PR}^* = \frac{v\alpha_A f^4}{(av(1-\alpha_A)^2 + 2\alpha_A f^2)^2}$.

Proof of proposition 4. In the private enforcement setting, optimal values of price and monitoring level are given by $(p_{PR}^*, \phi_{PR}^*) = \left(v \left(\frac{\alpha_A f^2 + av(1-\alpha_A)^2}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right), \frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right)$. As in the proof of proposition 2, the sign of $\frac{\partial p_{PR}^*}{\partial \alpha_A}$ (resp. $\frac{\partial \phi_{PR}^*}{\partial \alpha_A}$) shows how a higher level of efficiency for the illegal non-monitored channel affects the optimal value for price (resp. monitoring level). $\frac{\partial p_{PR}^*}{\partial \alpha_A} = \frac{\partial \left(v \left(\frac{\alpha_A f^2 + av(1-\alpha_A)^2}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right) \right)}{\partial \alpha_A} = \frac{-af^2v^2(1-\alpha_A)(1+\alpha_A)}{(2f^2\alpha_A + av\alpha_A^2 - 2av\alpha_A + av)^2} < 0$. Similarly, $\frac{\partial \phi_{PR}^*}{\partial \alpha_A} = \frac{\partial \left(\frac{fv(1-\alpha_A)}{2\alpha_A f^2 + av(1-\alpha_A)^2} \right)}{\partial \alpha_A} = \frac{-fv(2f^2 - av\alpha_A^2 + 2av\alpha_A - av)}{(2f^2\alpha_A + av\alpha_A^2 - 2av\alpha_A + av)^2}$. The sign of $-(2f^2 - av\alpha_A^2 + 2av\alpha_A - av)$ gives that of $\frac{\partial \phi_{PR}^*}{\partial \alpha_A}$. Define $h(\alpha_A) = av\alpha_A^2 - 2av\alpha_A + av - 2f^2$, where h is a U-shaped function w.r.t. α_A . $h(\alpha_A) = 0$ allows two solutions, namely $\alpha_{A,1} = 1 - \sqrt{\frac{2f^2}{av}}$ and $\alpha_{A,2} = 1 + \sqrt{\frac{2f^2}{av}}$. Note that $\alpha_{A,2} > 1 > \alpha_{A,1}$. It is also possible to show that $\alpha_{A,1} > 0$. Indeed, $\alpha_{A,1} > 0$ if and only if $1 > \sqrt{\frac{2f^2}{av}}$, i.e., $a > \frac{2f^2}{v}$, which holds sincere here as we here focus on values for a defined such that $a > \frac{4f^2}{v}$. Consequently, the sign of $\frac{\partial \phi_{PR}^*}{\partial \alpha_A}$ is positive if $0 < \alpha_A < 1 - \sqrt{\frac{2f^2}{av}}$, negative otherwise.

7.4 Equilibrium comparison

Proof of proposition 5. Define $\Delta\pi^* = \pi_{PR}^* - \pi_{PU}^*$. Expliciting $\Delta\pi^*$, we find $\Delta\pi^* = \frac{-k(\alpha_A)}{2a^2v(\alpha_A-1)^4(2f^2\alpha_A + av(1-\alpha_A)^2)}$, with $k(\alpha_A) \equiv a^2v^2\alpha_A^4 - 4a^2v^2\alpha_A^3 + 6a^2v^2\alpha_A^2 - 4a^2v^2\alpha_A + a^2v^2 + 2af^2v\alpha_A^3 - 4af^2v\alpha_A^2 + 2af^2v\alpha_A - 4f^4\alpha_A^2$. The sign of $\Delta\pi^*$ is that of $-k(\alpha_A)$. Full numerical analyses show that there exists a value $\widehat{\alpha_A}$ ($0 < \widehat{\alpha_A} < 1$) such that k takes positive values if $0 < \alpha_A < \widehat{\alpha_A}$, negative values otherwise. The sign of $\Delta\pi^*$ is therefore negative for values of α_A defined such that $0 < \alpha_A < \widehat{\alpha_A}$, and positive otherwise.

Proof of proposition 6. Define $\Delta p^* = p_{PR}^* - p_{PU}^*$, $\Delta\phi^* = \phi_{PR}^* - \phi_{PU}^*$ and $\Delta m_B^* = m_{B,PR}^* - m_{B,PU}^*$. Explaining Δp^* , we find $\Delta p^* = \frac{2\alpha_A^2 f^4}{a(1-\alpha_A)^2(2f^2\alpha_A + av\alpha_A^2 - 2av\alpha_A + av)}$, which sign is that of $2f^2\alpha_A + av\alpha_A^2 - 2av\alpha_A + av$. As some manipulations yield $2f^2\alpha_A + av\alpha_A^2 - 2av\alpha_A + av = 2f^2\alpha_A + av(1-\alpha_A)^2 > 0$, we show that $p_{PR}^* > p_{PU}^*$. We proceed in the same way to analyze the sign of $\Delta\phi^*$. We eventually show that $\Delta\phi^* = \frac{2\alpha_A f^3}{-a(1-\alpha_A)(2f^2\alpha_A + av\alpha_A^2 - 2av\alpha_A + av)} < 0$, that is, $\phi_{PR}^* < \phi_{PU}^*$. Dealing with the market share of the legal seller, simple calculations lead to $\Delta m_B^* = -\frac{2f^4\alpha_A}{av(1-\alpha_A)^2(2f^2\alpha_A + av\alpha_A^2 - 2av\alpha_A + av)} = -\frac{2f^4\alpha_A}{av(1-\alpha_A)^2(2f^2\alpha_A + av(1-\alpha_A)^2)} < 0$, that is, $m_{B,PR}^* < m_{B,PU}^*$. This defines the impact of the regime enforcement setting on optimal pricing and monitoring strategies, as well as on the ensuing market share of the legal seller.

Proof of proposition 7. Define $\Delta CS_B^* = CS_{B,PR}^* - CS_{B,PU}^*$. Explaining ΔCS_B^* , we find $\Delta CS_B^* = \frac{-4\alpha_A^2 f^6(f^2\alpha_A + av(\alpha_A-1)^2)}{a^2v(1-\alpha_A)^4(2f^2\alpha_A + av(1-\alpha_A)^2)^2} < 0$ for any value of v, a, f and α_A . Define now $\Delta LW^* = LW_{PR}^* - LW_{PU}^*$. Explaining ΔLW^* , we find that $\Delta LW^* = -\frac{2f^6\alpha_A^2}{a(1-\alpha_A)^2(2f^2\alpha_A + av(1-\alpha_A)^2)^2}$. This expression is negative for any value of v, a, f and α_A .