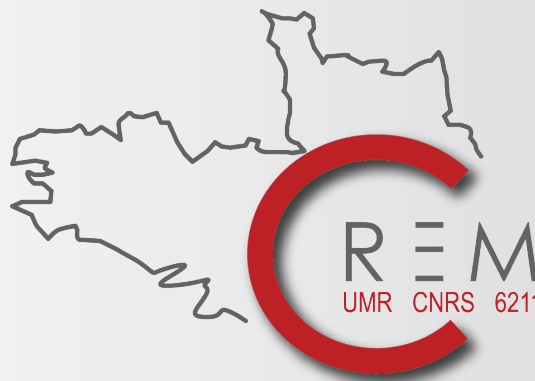




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From fixed to state-dependent duration in public-private contracts

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From fixed to state-dependent duration in public-private contracts

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Abstract

A government delegates a build-operate-transfer project to a private firm. At the contracting stage, the operating cost is unknown. The firm can increase the likelihood of facing a low cost (the good state) by exerting effort when building the infrastructure. Once this is in place, the firm learns the true cost and begins to operate. Under limited commitment, either the firm or the government may renege on the contract. Within this context, we explore how well a contract with a state-dependent duration performs, as compared to the more standard fixed-term contract. Under full commitment, the efficient allocation is decentralized, whether the contractual term is fixed or state-dependent. Under limited commitment, in situations where break-up of the partnership is little costly for the government, the efficient allocation can be decentralized only if it is stipulated that the duration of the contract will be longer in the good state than in the bad state. This result is at odds with the prescription of the literature on "flexible-term" contracts, which recommends a longer contractual length when the operating conditions are unfavourable.

Keywords: Fixed-term contract; state-dependent duration; moral hazard; adverse selection; limited commitment; renegotiation; public-private partnerships

J.E.L. Classification Numbers: D82; H57; H81

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1 Introduction

Public-private partnerships (PPPs henceforth) are characterized by the presence of informational asymmetries between the partners, namely a government and a firm (or a consortium of firms), as well as by the partners' lack of commitment. To prevent the firm from exploiting its informational advantage, the government needs to introduce some risk in the compensation scheme. The assigned compensation is lower the less favourable the operating conditions. In limited-commitment environments, this structure of the compensation scheme has an important consequence. The incentives of the government to renege on the contract are stronger in favourable states, those of the firm in unfavourable states. Moreover, provided the return from the activity accrues to the firm "as time goes by," how strong those incentives are at each instant during the contract execution, is also determined by the residual contractual period. Thus, as we show in Danau and Vinella [5] (DV henceforth), the contractual performance, under limited commitment, depends critically on the choice of the contractual length. In that study, consistent with the usual approach of the literature, and along the common practice in PPPs, the contractual term is taken to be fixed, regardless of the specific operating conditions. However, the fact that the incentives to renege of the firm and those of the government are not equally strong across states of nature, suggests that making the contract duration state-dependent can potentially stimulate the partners to a virtuous behaviour, when a fixed duration fails to do so. So far, the theory of incentives has not touched on the benefits that contracts with a state-dependent duration may deliver, for incentive purposes, in limited-commitment frameworks. The aim of our study is to investigate this issue with regards to PPPs, which are affected by pervasive contract renegotiations.

Reliance on contracts with a state-dependent duration has already been proposed, but not for incentive purposes. Engel, Fischer and Galetovic [7] - [8] (EFG henceforth) argue for that when the goal is to insure a risk-averse firm against the possibility of facing unfavourable operating conditions and obtaining a low return. Basically, the authors' concern is that the contract will be renegotiated in favour of the firm, in that case. To avoid this, they suggest that the duration of the contract be modulated, across possible states, in such a way that the firm obtains the same return in every state. They name the contracts with this characteristic "flexible-term" contracts.

The EFG recipe can be associated to one side of the limited-commitment problem, namely, the inability of the government to enforce the contract with the firm. Thus, renegotiation occurs because the government is "weak" and prone to avoid that the firm incurs financial difficulties, when the project generates a poor cash-flow. In the DV setting, and in

the context of this study, the government may well be "strong" and, yet, it may welcome renegotiating the contract, once the firm has reneged, because break-up of the partnership would be costly. Anticipating a profitable renegotiation, the firm will attempt to come back to the contracting table, regardless of the insurance received *ex ante*. Moreover, a strong government may want to renege, itself, on the contract, especially when it realizes that the firm does benefit from favourable conditions, thus exposing the partnership to break-up. Once it is recognized that both the government and the firm might renege, not necessarily under the same operating conditions, it becomes apparent that it might not be optimal to choose the contractual length, for each state of nature, in such a way that the firm is insured against the risk of facing a low cash-flow.

We develop our analysis in the following analytical framework. Both the government and the firm are risk-neutral. The contract is signed *ex ante*, when the operating marginal cost is unknown. Moral hazard arises at the time when the firm builds the infrastructure. Adverse selection appears as soon as the infrastructure is in place and the firm observes the true cost, which can be either low (the good state) or high (the bad state). Once the state becomes known, the contract may be reneged upon. While information issues arise on the firm's side only, commitment issues concern both partners.

Our first result is drawn looking at a hypothetical full-commitment framework. We find that, in this framework, it is irrelevant whether the contractual term is fixed or contingent on the state. Under both options, it is possible to design a compensation scheme and to set the termination date(s) such that the firm is faced with a desirable amount of risk and, hence, all information issues are solved at no *ex-ante* cost. There is, thus, no loss of generality, for the government, in restricting attention to a fixed-term contract, along the usual practice. This is all the more true that, when opting for a state-dependent duration, an additional complication appears. Starting from a fixed term, given a suitable amount of risk to be transferred to the firm, a change in the contractual length, in one state of nature, must be matched by an adjustment in the per-period compensation accruing to the firm, in that same state, to be calibrated in such a way to preserve the targeted risk transfer.

Our subsequent results are drawn with regards to more realistic situations, in which the partners lack the ability to commit. We assess that, the government being especially prone to renege in the good state, the contract which stipulates the efficient allocation is not honoured, unless the firm is allowed to run the activity for a longer period, in that state, than is otherwise. Therefore, the contractual length is to be differentiated across states of nature. The explanation is as follows. If only the firm were opportunist and keen to renege, as can solely occur in the bad state, in which its compensation is low, then this incentive would be eliminated by instructing the firm to make a sufficiently important investment in the project

up-front. However, provided that also the government is opportunist, and especially so in the good state, in which the compensation it owes to the firm is high, it might not be possible to motivate both partners to behave in a virtuous manner. Instructing the firm to invest massively, in order to incentivize it to remain in the project, might boost the government's appetite to appropriate the private investment, by breaking up the partnership early on. There is, thus, a conflict between mitigating the opportunism of the firm in the bad state and mitigating the opportunism of the government in the good state. Starting from a fixed contractual term, the difficulty is circumvented by extending the term in the good state and shortening it in the bad state. This is feasible as long as the per-period operating profits can be adjusted in such a way that the information issues are still addressed. The duration being longer in the good state, the conflict is eliminated all over the period in which the contract is designed to remain in place only in the good state. This might restore the possibility of incentivizing both partners to honour the contract in situations in which that outcome is beyond reach with a fixed contractual term.

Importantly, the recommendation that the firm be offered to operate for a longer period, when the cash-flow generated by the activity is high, is at odds with the EFG recipe. According to the latter, the firm should rather be allowed to run the activity for a longer period when the cash-flow is low. This divergence rests on that, in EFG, the contractual length is functional to providing insurance to the firm. By contrast, in our study, it is a tool to address the incentive issues, which arise both on the firm's and on the government's side.

Outline The reminder of the paper is organized as follows. In section 2, we describe the model and characterize the efficient allocation under incomplete information. In section 3, we establish the full-commitment benchmark, showing how the efficient allocation is decentralized through the contract between the government and the firm. In section 4, we present the limited-commitment framework. In section 5, we identify and discuss conditions under which the contract which stipulates the efficient allocation is honoured in the limited-commitment framework. Section 6 concludes.

2 Model

A government G delegates to a private firm F the realization of a project. This includes the construction and the management of an infrastructure to provide a good (or service) to the collectivity. The former task takes place at date 0 (the *construction* stage), the latter over the period $(0, T)$ (the *operation* stage). At date T , the contract ends. The infrastructure is transferred to G , which runs the activity thereafter.

To build the infrastructure, F bears a sunk cost of $I > 0$ and exerts effort $a \in \{0, 1\}$. Effort occasions a disutility of $\psi(1) = \psi > \psi(0) = 0$. It is unobservable to both G and third parties and cannot be contracted upon. At each instant $\tau \in (0, T)$, F produces q units of the good, incurring a fixed cost of K and a marginal cost of θ , which is more likely to be low if a positive effort is exerted at the construction stage. In return, F receives a transfer of t from G and collects revenues $p(q)q$ from the market. Consumption of q units of the good yields instantaneous gross surplus $S(q)$, such that $S'(\cdot) > 0$, $S''(\cdot) < 0$, $S(0) = 0$, and the Inada's conditions hold. Customers purchase the output produced at some given τ at price $p(q) \equiv S'(q)$. Once the investment is made, production technology and demand remain constant for the duration of the project.

The timing of events is as follows. The contract between G and F is signed, the investment is made, the effort is exerted, and the disutility is borne, at date 0. At that time, the value of θ is unknown. However, it is common knowledge that θ will be either low (θ_l) or high (θ_h) with probabilities ν_1 and $1 - \nu_1$, respectively, if $a = 1$, and with probabilities ν_0 and $1 - \nu_0$, respectively, if $a = 0$, and such that $\nu_1 > \nu_0$. We let $\Delta\theta = \theta_h - \theta_l$ and $\Delta\nu = \nu_1 - \nu_0$. Once the infrastructure is in place, immediately after date 0, F observes θ_i and begins to produce.

To finance the cost of investment, F uses an amount $M \in [0, E]$ of own funds, where E denotes its resource endowment. It also invests an amount $C \geq 0$ borrowed on the credit market. G makes an up-front transfer of $t_0 \in \mathbb{R}$ to F such that $M + C + t_0 = I$.

2.1 Payoffs under complete information

We now present the payoffs of F and G in the hypothetical event that G observes effort, during construction, as well as the marginal cost of production, once the infrastructure is in place.

Let $d \geq 0$ be the repayment that F makes to the lender L at each instant $\tau \in (0, T)$, in return for the amount of money C received initially. F obtains the instantaneous operating profit $\pi = t + p(q)q - (\theta q + K) - d$. The present value, at date τ , of the stream of profits through date T is given by $\Pi_\tau = \int_\tau^T \pi e^{-r(x-\tau)} dx$, where r is the discount rate. The payoff of F is the net present value of the project:

$$\tilde{\Pi} = \Pi_0 - (M + \psi(a)).$$

The goal of G is to maximize the discounted consumer surplus generated under both private and public management, net of market expenditures and the social cost of transferring resources from taxpayers to the producer. To finance the transfers, G raises distortionary taxes. Each transferred euro requires collecting $1 + \lambda$ euros from taxpayers, where $\lambda > 0$ is

the shadow cost of public funds, constant over time. The discounted return of G, over the period (τ, T) , is $V_\tau = \int_\tau^T w(q)e^{-r(x-\tau)}dx - (1+\lambda)(\Pi_\tau + D_\tau)$, with $w(q) \equiv S(q) + \lambda p(q)q - (1+\lambda)(\theta q + K)$ and where $D_\tau = \int_\tau^T de^{-r(x-\tau)}dx$ is the value of the debt of F at date τ . The credit market is competitive and populated by a large number of lenders, each facing zero outside opportunity. Hence, $D_0 = C$. Accordingly, the discounted return of G from private management is

$$\tilde{V} = \int_0^T w(q)e^{-rx}dx - (1+\lambda)(\Pi_0 + I - M).$$

No additional investment is required to continue the activity after the conclusion of the contract. The production technology is related to the inner characteristics of the infrastructure. Once this is in place, the marginal cost of production remains the same, regardless of who runs the activity. Thus, at date T , the discounted optimized return of G from public management is equal to $\int_T^\infty w(q^*)e^{-r(y-T)}dy$, where q^* is the output level that maximizes $w(\cdot)$. This is defined by the Ramsey-Boiteux condition

$$\frac{p(q^*) - \theta}{p(q^*)} = \frac{\lambda}{1+\lambda} \frac{1}{|\varepsilon(q^*)|}, \quad (1)$$

where $\varepsilon(q) \equiv (dp(q)/dq) q/p(q)$ is the elasticity of demand to price. The payoff of G is

$$W = \tilde{V} + \int_T^\infty w(q^*)e^{-ry}dy.$$

Assuming that effort is desirable,¹ G induces $a = 1$ and recommends the output level q^* . No surplus is given up to F. The payoffs of F and G are, respectively, given by:

$$\begin{aligned} \Pi_0^* &= M + \psi \\ W^* &= \int_0^\infty w(q^*)e^{-ry}dy - (1+\lambda)(I + \psi). \end{aligned}$$

2.2 Contracts

The contract between G and F Henceforth, with a slight abuse of notation, we append the subscript i to all variables that depend on the state of nature. G makes a take-it-or-leave-it offer to F. First, this specifies the triplet (M, C, t_0) . Second, invoking the Revelation Principle to focus on direct revelation mechanisms, the offer includes the menu of allocations $\{(q_l, t_l; T_l); (q_h, t_h; T_h)\}$, where t_i is the transfer to be made at each such instant, and T_i is the duration of the contract, when the cost is θ_i , $i \in \{h, l\}$. From now on, following a

¹Effort is desirable as long as $\mathbb{E}_i[w_i^*] - \tilde{\mathbb{E}}_i[w_i^*] > r\psi$, where $\mathbb{E}_i$ (resp. $\tilde{\mathbb{E}}_i$) is the expectation operator over the two states l and h , corresponding to $a = 1$ (resp. $a = 0$).

standard approach, we refer to the pair of discounted cumulated profits $\{\Pi_{l,0}, \Pi_{h,0}\}$, rather than to the pair of instantaneous transfers $\{t_l, t_h\}$.

The credit contract Consistent with the deal made with G, the contract between F and L states the amount of money C that F will receive from L, to be invested in the project at date 0. The contract also stipulates the instantaneous repayment d_i for each state $i \in \{l, h\}$. Under the assumption that the credit market is competitive, d_i is such that $\mathbb{E}_i [D_{i,0}] = C$.

2.3 Efficient allocation under incomplete information

Under incomplete information, the efficient allocation is defined by the pair of Ramsey-Boiteux quantities $\{q_l^*, q_h^*\}$, together with the pair of profits $\{\Pi_{l,0}^*, \Pi_{h,0}^*\}$ such that, in expectation, F exactly recovers its initial monetary and non-monetary contribution, *i.e.*, $\mathbb{E}_i [\Pi_{i,0}^*] = M + \psi$. At this allocation, G achieves the payoff

$$\mathbb{E}_i [W_i^*] = \int_0^\infty \mathbb{E}_i [w(q_i^*)] e^{-rx} dx - (1 + \lambda) (I + \psi).$$

Our purpose, in the sequel of the study, is to determine conditions under which this allocation is decentralized through the contract between G and F.

3 Full commitment

We begin by considering the benchmark situation in which both G and F commit to their contractual obligations. As usual, G must ensure that the firm is willing to run the project. Taking the best outside opportunity of F to be zero, this requires satisfying the participation constraint:

$$\mathbb{E}_i [\Pi_{i,0}^*] \geq M + \psi. \quad (2)$$

Recall, however, that, for the efficient allocation to be decentralized, (2) must hold as an equality. Moreover, G must prevent F from taking advantage of the information it holds privately. To that end, the following constraints must be satisfied:

$$\Pi_{l,0}^* - \Pi_{h,0}^* \geq \int_0^{T_h} \Delta \theta q_h^* e^{-rx} dx \quad (3a)$$

$$\Pi_{l,0}^* - \Pi_{h,0}^* \leq \int_0^{T_l} \Delta \theta q_l^* e^{-rx} dx \quad (3b)$$

$$\Pi_{l,0}^* - \Pi_{h,0}^* \geq \frac{\psi}{\Delta \nu}. \quad (3c)$$

(3a) (resp. (3b)) is the incentive-compatibility constraint whereby, when the cost is θ_l (resp. θ_h), F is not tempted to choose the quantity-profit pair designed for a cost of θ_h (resp. θ_l). (3c) is the moral-hazard constraint whereby F is not tempted to shirk at the construction stage.

3.1 Incentive conflicts and contractual length

When striving to satisfy (3a) to (3c), G faces two potential conflicts, one between preventing shirking and preventing cost understatement in state h , the other between preventing cost exaggeration in state l and preventing cost understatement in state h . As one can deduce from the formulation of the constraints, to solve these conflicts, at the aim of achieving the efficient outcome, it is necessary that the disutility of effort is not too high, and that the termination dates are properly chosen.

The conflict between moral hazard and adverse selection in the bad state First consider the conflict between preventing shirking and preventing cost understatement in state h . The shorter the contract duration in state l , the smaller the opportunity cost of pretending θ_l when the cost is high and, hence, the stronger the incentive to do so. If T_l is set little, then this incentive cannot be eliminated, unless the compensation granted to F when reporting θ_h is set sufficiently high, relative to the compensation granted when the announcement is θ_l . That is, a sufficiently small profit wedge $\Pi_{l,0}^* - \Pi_{h,0}^*$ must be induced. However, when this strategy is followed, it becomes difficult, for the government, to tackle the moral-hazard problem. A firm that receives a relatively high compensation, when faced with a high operating cost, is little motivated to exert effort in order to increase the likelihood of facing a low cost. Thus, T_l should not be very small. How big T_l should be, exactly, depends also on how important the moral-hazard problem is. When the disutility of effort is so high that the following condition is violated

$$\psi \leq \Delta\nu\Delta\theta\frac{q_l^*}{r}, \quad (4)$$

the efficient outcome is beyond reach, even if T_l is lengthened to infinity. Henceforth, to rule out this possibility, we assume that (4) holds.

The conflict between adverse selection in the good state and in the bad state

Next consider the conflict between preventing cost exaggeration in state l and preventing cost understatement in state h . The longer T_h , the more important the benefit that the firm obtains by pretending θ_h when the cost is low and, hence, the stronger the incentive to do

so. To eliminate this incentive, the compensation granted to F when reporting θ_l , must be sufficiently high, relative to the compensation granted when the announcement is θ_h . That is, a sufficiently wide profit wedge $\Pi_{l,0}^* - \Pi_{h,0}^*$ must be induced. We know, however, that this would trigger a lie in state l , unless T_l is set big enough. Thus, the second incentive conflict is solved only if T_h is not very large, as compared to T_l .

This all leads us to draw our first result. Before stating it, we introduce two useful definitions:

$$\begin{aligned}\underline{T}(z_l) &\equiv \frac{1}{r} \ln \frac{\Delta\nu\Delta\theta z_l}{\Delta\nu\Delta\theta z_l - r\psi}, \text{ for } z_l \in \left[\frac{r\psi}{\Delta\nu\Delta\theta}, q_l^* \right] \\ \tilde{T}_h(z_h, z_l, T_l) &\equiv \frac{1}{r} \ln \frac{z_h}{z_h - z_l(1 - e^{-rT_l})}, \text{ for } z_h \geq z_l(1 - e^{-rT_l}), z_h \in [q_h^*, \infty).\end{aligned}$$

Lemma 1 *The contract between G and F decentralizes the efficient allocation, if and only if T_l and T_h are chosen such that, for some given $z_l \in [\frac{r\psi}{\Delta\nu\Delta\theta}, q_l^*]$ and $z_h \in [q_h^*, \infty)$,*

$$T_l \geq \underline{T}(z_l) \tag{5}$$

and, when either $z_h > z_l$, or $z_h \leq z_l$ and $T_l \leq \frac{1}{r} \ln \frac{z_l}{z_l - z_h}$,

$$T_h = \tilde{T}_h(z_h, z_l, T_l) \leq \tilde{T}_h(q_h^*, q_l^*, T_l). \tag{6}$$

This result generalizes the content of Proposition 1 in DV to the possibility that $T_l \neq T_h$. The novelty resides in the presence of the additional condition in (6). It ensues from the need to solve the second incentive conflict, which does not arise when $T_l = T_h \equiv T$ and, hence, $z_l = z_h \equiv z$. That is, information release, in the two states, is not an issue, as long as the firm cannot pick, with a false cost claim, a more convenient contract duration than the one corresponding to the true cost. According to (6), moving away from the fixed-term approach, the conflict is solved only if T_h is not set too large, relative to T_l . How much, exactly, the two termination dates can be differentiated, depends on how much the per-period compensation, accruing to F in each of the two states, can be adjusted, in order to transfer a suitable amount of risk to F, *i.e.*, to induce a profit wedge satisfying (3a) to (3c). To illustrate this point, we notice that the discounted cumulated profits, assigned to F in the two states, can be formulated as

$$\Pi_{l,0}^* = M + \psi + (1 - \nu_1) \Delta\theta \int_0^{T_j} z_j e^{-rx} dx \tag{7a}$$

$$\Pi_{h,0}^* = M + \psi - \nu_1 \Delta\theta \int_0^{T_j} z_j e^{-rx} dx, \tag{7b}$$

for some $j \in \{l, h\}$. From (7a) and (7b), we see that, in addition to recovering the initial investment $M + \psi$, F receives a "reward" in the good state, whereas it is inflicted a "punishment" in the bad state. In compliance with (6), the profit wedge $\Pi_{l,0}^* - \Pi_{h,0}^* = \Delta\theta \int_0^{T_j} z_j e^{-rx} dx$ must be the same, regardless of which j is exactly picked. Thus, for any $\{z_h, T_h\}$ – pair, chosen to induce the targeted wedge, as z_l is reduced, within the feasible set, F receives a lower per-period reward, which must then be compensated by an extension of the contractual length in state l . As a consequence, following to a decrease in z_l , (5) is tightened. Conversely, for some $\{z_l, T_l\}$ – pair, chosen to induce the desired wedge, as z_h is increased, F is assigned a lower per-period punishment, which must then be matched by a reduction in the contractual length in state h . It is, thus, explained why a raise in z_h triggers a shrink in T_h , as from (6). For instance, when $z_h > z_l$, the difference between the per-period reward and the per-period punishment is so little that a suitable amount of risk is transferred to the firm only if the termination dates are ranked as $T_l > T_h$.

In definitive, when shifting from a fixed to a state-dependent duration, there is no change in the contractual performance, provided that, in each state of nature, the per-period compensation is adjusted in such a way that the firm is prevented from exaggerating the cost, when it is low, without being induced to understate it, when it is high. We can, thus, state the following result.

Proposition 1 (*Irrelevance result*) *Under full commitment, $\exists T_l > 0, T_h > 0$, for which the contract between G and F decentralizes the efficient allocation, regardless of whether $T_l = T_h$ or $T_l \neq T_h$.*

Under full commitment, there is no loss of generality, for the government, in focusing on a fixed-term contract, along the usual practice. However, as will become apparent soon, this does not need to be the case in environments where the government and the firm lack the ability to commit.

Before making further progress with the analysis, it is worth mentioning that, as from DV, with a fixed duration, the financial structure of the project matters only under limited commitment. Not surprisingly, this result holds with a state-dependent duration as well. Because of this, when exploring the usefulness of conditioning the duration on the state, we will also be concerned with the choice of a suitable mix of financing sources.

4 Limited commitment

Consider now the situation where G and F sign the contract which stipulates the efficient allocation (henceforth, the contract, for the sake of brevity), F borrows money from L

accordingly, but neither G nor F is able to commit. In this section, we first describe what would happen, if some party were to renege on the contract. We then suggest a way to discourage the partners from seeking a new negotiation. This will enable us, in the subsequent section, to assess when and how, under limited commitment, the contract is enforced.

Before presenting the returns that each partner would obtain, in the event of contract renege, we need to remark that the partners' inability to commit challenges not only the execution of the contract between G and F, but also that of the contract between F and L. F may stop reimbursing L at some point during the development of the project. In turn, this involves that F would be unable to raise funds in the first place, provided that it is desirable to do so. A very natural possibility is that G provides governmental guarantees to L, in order to induce the latter to lend money to F. As argued in DV, in a framework where the government does not commit, this can be done by relying on some authoritative third party, say, an Investment Insurance Agency, the World Bank, or a multilateral development bank. However, as further stressed in DV, for debt finance to play a role in the renegotiation game between G and F, such guarantees should be *conditional*. That is, it should be contractually stipulated that the guarantees will come into force only if the relationship between G and F remains in place. Moreover, the exact amount of the guarantee should depend on whether the relationship continues under the initial contract, or a new agreement is reached.

4.1 Contractual renege

Suppose that, in some commonly-known state $i \in \{l, h\}$, at some date $\tau \in (0, T_i)$, either F or G reneges on the contract, and they return to the contracting table.

Replacement of the firm and break-up of the partnership If renegotiation fails, then F is relieved of the activity and replaced with another firm F'. F no longer receives any compensation from date τ to date T_i . Moreover, F has no reason to make further payments to L. As the guarantee does not come into force, L foregoes the part of the loan that remains unpaid. G appropriates the resources of F and L that are locked in the project, and continues to benefit from the productive activity undertaken by F'. Nonetheless, it bears a *replacement cost*, hereafter denoted R_{δ_i} , where $\delta_i = T_i - \tau$ is the residual contractual period. Essentially, this cost is a loss of reputation/credibility, associated with the fact that replacement implies expropriation of the investment sunk by the firm and, indirectly, by the lender. We assume that $R_{\delta_i} > 0, \forall \delta_i \in (0, T_i)$, with $\lim_{\delta_i \rightarrow 0} R_{\delta_i} = \varepsilon > 0$, and $R_0 = 0$. Moreover, R_{δ_i} is continuously differentiable on $(0, T_i)$ and $R'_{\delta_i} \equiv (dR/d\delta_i) > 0, \forall \delta_i \in (0, T_i)$. That is, the cost faced by the government is more important, the earlier the break-up occurs, relative to the termination

date stipulated in the contract. In our framework, this is matched by the circumstance that both the firm and the lender recover the initial contribution "as time goes by" and, for each of them, there is more to recover the higher that contribution. Therefore, the earlier F is replaced, the bigger the quota of the total private investment $(M + C)$ that G appropriates and, consequently, the more important the loss of reputation/credibility that it incurs. Appending the superscript rp to indicate the *replacement* scenario, the payoff of F and the discounted return of G at date τ are given, respectively, by

$$\Pi_{i,\tau}^{rp} = 0 \quad (8a)$$

$$V_{i,\tau}^{rp} = w_i^* \frac{1 - e^{-r\delta_i}}{r} - R_{\delta_i}. \quad (8b)$$

Renegotiation Suppose now that renegotiation succeeds. With probability $\alpha \in [0, 1]$, G makes a take-it-or-leave-it offer to F; with probability $1 - \alpha$, F makes a take-it-or-leave-it offer to G. The party that takes the initiative optimally makes an offer, under which the recipient is indifferent between renegotiation and replacement. If G makes the offer, F gets the same payoff as under replacement. If F makes the offer, it extracts what G would lose in the replacement scenario, net of the guarantee $D_{i,\tau}^{rn}$, which G promised to L in the event of renegotiation at τ . In turn, whoever makes the offer, G obtains the largest benefit from consumption of the good. This is, then, diminished by the social cost of the surplus that is given up to F when it makes the offer, plus the guarantee provided to L. Appending the superscript rn to indicate the *renegotiation* regime, the payoff of F and the discounted return of G at date τ are given, respectively, by

$$\Pi_{i,\tau}^{rn} = (1 - \alpha) \left(\frac{R_{\delta_i}}{1 + \lambda} - D_{i,\tau}^{rn} \right) \quad (9a)$$

$$V_{i,\tau}^{rn} = w_i^* \frac{1 - e^{-r\delta_i}}{r} - (1 + \lambda) \left[(1 - \alpha) \left(\frac{R_{\delta_i}}{1 + \lambda} - D_{i,\tau}^{rn} \right) + D_{i,\tau}^{rn} \right]. \quad (9b)$$

These expressions are obtained under the implicit assumption that the contract is renegotiated at date τ , and not further renegotiated beyond that date.

4.2 Additional constraints

In case of contractual renege, the partners' payoffs would be distorted away from the levels which are efficient *ex ante*. To avoid this outcome, when designing the contract, G must prevent any attempt to renege. This requires satisfying a few additional constraints. Let $\Pi_{i,\tau}^*$ be the value of the *residual* profit of F, over the period (τ, T_i) , and $V_{i,\tau}^*$ the value of

the *residual* return of G, over that same period, in the realized state i , when the contract is honoured. The constraints, which add up to the programme of G, are given by:

$$\Pi_{i,\tau}^* \geq \max \{ \Pi_{i,\tau}^{rn}, \Pi_{i,\tau}^{rp} \}, \quad \forall \tau \in (0, T_i) \quad (10)$$

$$V_{i,\tau}^* \geq \max \{ V_{i,\tau}^{rn}, V_{i,\tau}^{rp} \}, \quad \forall \tau \in (0, T_i) \quad (11)$$

$$\Pi_{i,\tau}^{rn} \geq e^{-r(\tau'-\tau)} \Pi_{i,\tau'}^{rn}, \quad \forall \tau, \tau' \in (0, T_i), \quad \tau' \geq \tau \quad (12)$$

$$V_{i,\tau}^{rn} \geq e^{-r(\tau'-\tau)} V_{i,\tau'}^{rn}, \quad \forall \tau, \tau' \in (0, T_i), \quad \tau' \geq \tau. \quad (13)$$

(10) and (11) are the constraints whereby, respectively, F and G are tempted neither to reach a new agreement nor to break up the partnership, at date τ , rather than to honour the contract. (12) and (13) are the constraints whereby, respectively, F and G are not tempted to renegotiate at date $\tau' > \tau$, provided that the contract was already renegotiated at τ .²

4.3 Out-of-equilibrium guarantees: a tool to prevent renegotiation

The expressions in (9a) and (9b) evidence that G can make a strategic use of the guarantees, which would come into force if the contract were renegotiated. That is, G can play them as a tool to weaken the incentives of the two parties to renege. This is possible because implementation of the contract involves weaker requirements when replacement, rather than renegotiation, is to be prevented. The next lemma states how G should use the guarantees to that end.

Lemma 2 $\max \{ \Pi_{i,\tau}^{rn}, \Pi_{i,\tau}^{rp} \} = \Pi_{i,\tau}^{rp}$ and $\max \{ V_{i,\tau}^{rn}, V_{i,\tau}^{rp} \} = V_{i,\tau}^{rp}$ if and only if:

$$D_{i,\tau}^{rn} \geq \frac{R_{\delta_i}}{1 + \lambda}. \quad (14)$$

Consider now the possibility of repeated renegotiation. Denote $\widehat{D}_{i,\tau/\tau'}^{rn}$ the value at $\tau' > \tau$ of the guarantee that G provides to L at date 0, anticipating the possibility of the contract being renegotiated at τ . In fact, as the next lemma states, repeated renegotiation is not an issue, if the associated guarantees are properly chosen.

Lemma 3 *Suppose that the contract is renegotiated in state $i \in \{l, h\}$ at date $\tau \in (0, T_i)$.*

²It should also be ensured that F has no incentive to cheat at the outset of the operation phase, anticipating that the contract will be renegotiated at a later stage. In Appendix B.2, we prove that this incentive arises neither in state l nor in state h , as long as (3a) and (3b) are satisfied, together with, respectively, (10) and (11). Showing that F is not tempted to shirk at the construction stage, anticipating contractual renege, is immediate and, hence, omitted.

Then, no further renegotiation occurs at date $\tau' > \tau$, if and only if:

$$D_{i,\tau'}^{rn} - \widehat{D}_{i,\tau/\tau'}^{rn} \geq \frac{1}{1+\lambda} \max \left\{ \left(R_{\delta'_i} - R_{\delta} \frac{1 - e^{-r\delta'_i}}{1 - e^{-r\delta_i}} \right); \left(R_{\delta_i} \frac{1 - e^{-r\delta'_i}}{1 - e^{-r\delta_i}} - R_{\delta'_i} \right) \frac{1 - \alpha}{\alpha} \right\}. \quad (15)$$

Henceforth, we take the guarantees for the renegotiated contract to be set such that (14) and (15) are satisfied. Then, the only new constraints in the programme of G, under limited commitment, are (10) and (11). They are equivalent to, respectively, $\Pi_{i,\tau}^* \geq \Pi_{i,\tau}^{rp}$ and $V_{i,\tau}^* \geq V_{i,\tau}^{rp}$.

5 Implementation under limited commitment

Our purpose, in this section, is to establish conditions under which the contract is honoured in the limited-commitment framework. This involves satisfying (10) and (11), without distorting the allocation away from efficiency. To that end, we first identify a new incentive conflict, which must be tackled when, in addition to the firm attempting to exploit its informational advantage early on in the relationship, both the firm and the government may behave opportunistically during the operation phase.

5.1 A new incentive conflict

Recall, from the full-commitment analysis, that the government faces two potential conflicts, one between preventing shirking and preventing cost understatement in state h , the other between preventing cost exaggeration in state l and preventing cost understatement in state h . As from Lemma 1, while the former requires that T_l be large enough, the latter requires that T_h be not too large, relative to T_l . How tight these requirements are, relates to how z_j , $j \in \{l, h\}$, is chosen to design the per-period reward and punishment for the firm and, ultimately, to induce a suitable profit wedge $\Pi_{l,0}^* - \Pi_{h,0}^*$. In addition, under limited commitment, a new incentive conflict appears between preventing F from renegeing on the contract in state h , in which it is "punished," and preventing G from renegeing in state l , in which F is "rewarded." To see this, consider that the constraint $\Pi_{h,\tau}^* \geq \Pi_{h,\tau}^{rp}$ is more relaxed the higher the contribution M made by the firm. If G instructs F to invest much up-front, then it is compelled to assign a higher compensation to F, during the operation phase, in order to secure its participation. This reinforces the motivation of F to honour the contract in the bad state. On the other hand, the constraint $V_{l,\tau}^* \geq V_{l,\tau}^{rp}$ is more relaxed, the lower the value of M . If G instructs F to invest little up-front, then it can afford to grant a lower compensation to the firm, during the operation phase. This reinforces the motivation of G

to honour the contract in the good state. A size of the firm's contribution can be found, for which neither F nor G is tempted to renege on the contract, only if, at each date τ during the operation phase, the wedge between the *residual* values that the profits take, at each such date, is sufficiently low. Of course, this is related to how the initial wedge $\Pi_{l,0}^* - \Pi_{h,0}^*$ is set. In environments where the parties are unable to commit, this leads to an important contractual complication. At the time when the contract is drawn up, the compensation scheme must be designed and, in particular, the termination dates must be picked, taking into account not only the incentive conflicts due to the firm's informational advantage, but also the incentive conflict related to the firm's and the government's *ex-post* opportunism, respectively, in state h and in state l .

5.2 Implementability conditions

We now present the exact conditions under which it is possible to reconcile all the relevant incentive conflicts and have the contract honoured under limited commitment.

Lemma 4 *Under limited commitment, the contract is implementable, if and only if:*

$$E \geq \nu_0 \frac{\psi}{\Delta\nu} \quad (16)$$

$$C > 0, \quad (17)$$

and, additionally, $\exists T_l > 0, T_h > 0$ fulfilling Lemma 1 together with:

$$R_{\delta_l} \geq (1 + \lambda) (\Pi_{l,0}^* - \Pi_{h,0}^*) \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}}, \quad \forall \delta_l \in (0, T_l). \quad (18)$$

Lemma 4 generalizes the content of Proposition 2 in DV, which is stated with regards to fixed-term contracts, to situations in which the contractual length is conditioned on the realized cost. More precisely, (16) and (17) are just the same as for a fixed-term contract. They involve that both own funds of the firm and external funds must be available to run the project. On the one hand, requiring the firm to invest reinforces its willingness to preserve the relationship. On the other, without external funds, it would be impossible to use the conditional guarantees to make renegotiation unappealing. What really generalizes results to the case of a state-dependent duration is (18). As for a fixed-term contract, this condition reflects the difficulty, illustrated above, to get rid of the opportunism that each of the two partners may exhibit during the operation phase, while still tackling the incentives of F to shirk and cheat early on in the relationship with G. What changes, when the contractual length can be differentiated across states of nature, is illustrated hereafter.

5.3 Contractual length and the benefits of state-dependence

Take first $T_l = T_h \equiv T$. (18) specifies as

$$R_{T-\tau} \geq (1 + \lambda) (\Pi_{l,\tau}^* - \Pi_{h,\tau}^*), \quad \forall \tau \in (0, T). \quad (19)$$

We know that, to eliminate the incentive to shirk, together with the incentive to lie in state l , a sufficiently large wedge must be induced between profits $(\Pi_{l,0}^* - \Pi_{h,0}^*)$. Consequently, a certain wedge is determined between the residual profits at each date τ during the operation phase $(\Pi_{l,\tau}^* - \Pi_{h,\tau}^*)$. In turn, this affects the incentives to renege that one or the other partner may exhibit. If, at some date τ , the residual profit becomes too low in state h , then F may prefer to abandon the partnership. If it becomes too high in state l , relative to the replacement cost $R_{T-\tau}$, then G may prefer to replace F and appropriate the private investment. To be able to eliminate the incentives to renege of both F and G, the residual profit wedge must be little, relative to the replacement cost, at each instant during the operation phase. Thus, on the one hand, the wedge $\Pi_{l,0}^* - \Pi_{h,0}^*$ is to be large enough to address the information issues, which arise on the firm's side before operation begins. On the other, the wedge $\Pi_{l,\tau}^* - \Pi_{h,\tau}^*$ is to be small enough, all along the operation phase, to address the implementation issues, which arise both on the firm's side and on the government's side. It might, of course, be difficult to reconcile these two goals. However, when T_l is not bound to be equal to T_h , in which case (18) is reformulated as

$$R_{T_l-\tau} \geq (1 + \lambda) (\Pi_{l,\tau}^* - \Pi_{h,\tau}^*) - (1 + \lambda) \Pi_{h,0}^* \left(\frac{1 - e^{-r(T_l-\tau)}}{1 - e^{-rT_l}} - \frac{1 - e^{-r(T_h-\tau)}}{1 - e^{-rT_h}} \right), \quad \forall \tau \in (0, T_l), \quad (20)$$

one can see that an "adjustment" term, namely $-(1 + \lambda) \Pi_{h,0}^* \left(\frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} - \frac{1 - e^{-r\delta_h}}{1 - e^{-rT_h}} \right)$, appears. For $T_l > T_h$, this term is negative. Thus, by ranking the termination dates in that way, (18) is made weaker than (19). The reason is intuitive. As, in state h , the per-period profit accruing to F is sufficiently large, relative to that in state l , involving that the contract can have a shorter duration, the incentive of F to renege is no longer an issue over the extra period (T_h, T_l) . Hence, all over that period, there is no longer any need to reconcile conflicting incentives to renege. This explains why, in the next two results, implementability of the contract calls for $T_l > T_h$.

Proposition 2 *Assume that*

$$\psi < \Delta \nu \Delta \theta \frac{q_h^*}{r}. \quad (21)$$

If

$$(1 + \lambda) \frac{\psi}{\Delta\nu} r e^{-rx} \leq R'_x < (1 + \lambda) \Delta\theta q_h^* e^{-rx}, \quad \forall x \in (0, \infty) \quad (22)$$

$$R_{T(q_h^*)} < (1 + \lambda) \frac{\psi}{\Delta\nu}, \quad (23)$$

then, the contract is implementable only if $T_l > T_h$.

From Corollary 1 in DV, we know that, if the contractual length is fixed, then, a situation in which the right-hand inequality in (22) holds, together with (23), represents a case in which the replacement cost, faced by the government in state l , is too low to eliminate all the incentive conflicts, which challenge the efficiency of the partnership. This is explained as follows. For any given τ , a raise in the duration induces an increase not only in the replacement cost, but also in the residual profit wedge. Indeed, F receives a higher per-period compensation in state l , than in state h , for a longer lapse of time. The more slowly R_δ increases, as compared to the wedge $\Pi_{l,\tau}^* - \Pi_{h,\tau}^*$, the more difficult it is to find a duration for which the partners' incentives to renege can be reconciled. This difficulty is, however, lessened when $T_l > T_h$ because, as we said, over the period (T_h, T_l) , the firm's incentives to renege are not an issue, given that, at that time, the contractual relationship will no longer be in place in state h . To see this formally, observe that, for any given $\tau \in (0, T_l)$, as T_l is enlarged and T_h remains constant (or it shrinks), the wedge $\Pi_{l,\tau}^* - \Pi_{h,\tau}^*$ is raised, as if the duration was fixed, whereas the "adjustment" term $-(1 + \lambda) \Pi_{h,0}^* \left(\frac{1 - e^{-r(T_l - \tau)}}{1 - e^{-rT_l}} - \frac{1 - e^{-r(T_h - \tau)}}{1 - e^{-rT_h}} \right)$ becomes more negative. Overall, for $T_l > T_h$, the contract is enforceable, even when the replacement cost increases more slowly than is required with a fixed term.

Let us now come back to (21) and explain why the previous result, together with the one that follows, arises in situations where the disutility of effort is small. First consider the conflict between preventing shirking and preventing cheating in state h . Recall that, while the former requires widening the wedge $\Pi_{l,0}^* - \Pi_{h,0}^*$, the latter pushes in the opposite direction. When the disutility of effort is small, moral hazard is of little concern, and the wedge can be downsized. Once this is done, the conflict is handily solved by raising T_l enough, in compliance with Lemma 1, and decreasing, accordingly, the per-period reward in state l . One is then left with securing truthtelling in the good state, which boils down to choosing T_h low enough, and increasing, accordingly, the per-period punishment in state h . The two incentive conflicts, related to the firm's informational advantage, are thus disentangled and tackled separately. This possibility is only available as long as the contractual length is state-dependent. While it delivers no specific benefit under full commitment, it is very helpful under limited commitment, as it makes it easier to solve the conflict between the

opportunism of F and that of G.

Proposition 3 *Assume that (21) holds. If*

$$R'_x < (1 + \lambda) \frac{\psi}{\Delta\nu} r e^{-rx}, \quad \forall x \in (0, \infty) \quad (24)$$

$$R_{T(q_h^*)} < (1 + \lambda) \frac{\psi}{\Delta\nu} \leq \lim_{x \rightarrow \infty} R_x, \quad (25)$$

then, the contract is implementable, only if $T_l > T_h$. If

$$\lim_{x \rightarrow \infty} R_x < (1 + \lambda) \frac{\psi}{\Delta\nu}. \quad (26)$$

then, the contract is not implementable.

Under (24) and (25), the replacement cost is, again, too small for the contract to be implementable with a fixed duration. Also in this situation, a state-dependent duration may be helpful. As we stressed, by unbinding the termination dates in the two states, it is possible to lengthen the duration in state l , without triggering a lie in that same state. Thus, by doing so, G can still afford to make the contract implementable. Obviously, the contractual term in state l can be set, at most, infinitely long. This explains why that strategy works as long as the replacement cost becomes sufficiently large as x tends to infinity (as from (25)). When this requirement is not met, involving that (26) holds, there is no way to have the contract honoured.

5.4 Contractual length and financial structure

We identified and discussed conditions under which the contract is enforceable in environments where the parties are unable to commit. Provided that those conditions hold, actual enforcement of the contract still requires setting the termination dates T_l and T_h , and, then, calibrating the private contributions M and C (and, consequently, the public transfer t_0), in a proper manner. The next two corollaries conclude the analysis, illustrating how exactly these two tasks should be accomplished.

Before stating the first corollary, it is useful to introduce two definitions:

$$\begin{aligned} \tilde{T}(z_l, E) &\equiv \frac{1}{r} \ln \frac{\nu_1 \Delta \theta z_l}{\nu_1 \Delta \theta z_l - r(E + \psi)} < \infty \\ \bar{T}(z_l) &\equiv \frac{1}{r} \ln \frac{(1 + \lambda) \Delta \theta z_l}{(1 + \lambda) \Delta \theta z_l - r R_{\bar{T}(z_l)}}. \end{aligned}$$

Corollary 1 Assume that (21) holds. First suppose that (22) and (23) are satisfied. Then, the optimal value of T_l is such that, for some $z_l \in [\frac{r\psi}{\Delta\nu\Delta\theta}, q_h^*)$:

$$\begin{aligned} T_l &\in [\underline{T}(z_l), \tilde{T}(z_l, E)] \quad \text{when } E \in \left[\frac{\nu_0}{\Delta\nu}\psi, \nu_1 \frac{\Delta\theta z_l}{r} - \psi \right) \\ T_l &\in [\underline{T}(z_l), \infty) \quad \text{when } E \geq \nu_1 \frac{\Delta\theta z_l}{r} - \psi. \end{aligned}$$

Next suppose that (24) and (25) are satisfied. Then, the optimal value of T_l is such that, for some $z_l \in [\frac{r\psi}{\Delta\nu\Delta\theta}, q_h^*)$:

$$\begin{aligned} T_l &\in \left[\underline{T}(z_l), \min \left\{ \tilde{T}(z_l, E); \bar{T}(z_l) \right\} \right] \quad \text{when } E \in \left[\frac{\nu_0}{\Delta\nu}\psi, \nu_1 \frac{\Delta\theta z_l}{r} - \psi \right) \\ T_l &\in [\underline{T}(z_l), \bar{T}(z_l)] \quad \text{when } E \geq \nu_1 \frac{\Delta\theta z_l}{r} - \psi. \end{aligned}$$

In either case, T_h is optimally chosen in compliance with (6).

The content of this corollary is akin to that of Corollary 1 in DV, except that the conditions reported here concern the contract duration in state l , specifically. First, in the situations represented in Proposition 2 and 3, T_l cannot be raised above $\tilde{T}(z_l, E)$, unless F is sufficiently wealthy, i.e., $E \in [\frac{\nu_0}{\Delta\nu}\psi, \nu_1 \frac{\Delta\theta z_l}{r} - \psi)$. Otherwise, it would be impossible to require so large a monetary contribution from the firm that F would then be prone to honour the contract through the termination date. Second, when the replacement cost is as little as in the situations represented in Proposition 3, a new upper bound appears to T_l , namely $\bar{T}(z_l)$. If the duration of the contract were raised above this threshold, in state l , then, the replacement cost would become low, relative to the profit wedge. It would, then, be impossible to impose discipline on the two partners for the stipulated period.

Corollary 2 Assume that $\exists T_l > 0, T_h > 0$, fulfilling Lemma 1 and (18). Further assume that (16) and (17) are satisfied. Then, the contract is implemented by choosing M and C such that:

$$\begin{aligned} \nu_1(\Pi_{l,0}^* - \Pi_{h,0}^*) - \psi &\leq M \leq \min \left\{ \frac{R\delta_h}{1+\lambda} \frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + \nu_1(\Pi_{l,0}^* - \Pi_{h,0}^*) - \psi; \right. \\ &\quad \left. \frac{R\delta_l}{1+\lambda} \frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} - (1-\nu_1)(\Pi_{l,0}^* - \Pi_{h,0}^*) - \psi \right\}, \quad (27) \\ &\quad \forall \delta_h \in (0, T_h), \delta_l \in (0, T_l), \end{aligned}$$

and

$$C \leq \frac{\mathbb{E}_i[R_{T_i}]}{1+\lambda} - (M + \psi). \quad (28)$$

Also the message delivered by Corollary 2 is the same with a state-dependent duration as with a fixed duration (compare Corollary 2 in DV). First, to motivate both F and G to honour the contract, the amount of funds invested by the firm in the project should be neither too small nor too large, for the reasons previously explained. Second, F should be instructed to take a loan but not be encouraged to rely heavily on external financing. While the presence of debt paves the way for a desirable use of conditional guarantees, too large a value of C (just as too large a value of M) would trigger expropriation by the government. This mirrors the circumstance that the incentive of G to terminate the relationship is driven by the total private contribution that it would appropriate in so doing. Consequently, as from (28), the higher the monetary contribution of F, set in accordance with (27), the lower the admissible loan.

When the duration is contingent on the true cost, there is, nonetheless, a peculiarity to the choice of the financial structure of the project. That is, the appropriate values of M and C are related to the size of the replacement cost in the two different states. Take first (27). If it were $T_l = T_h$, then, clearly, G would be more opportunist in the good state. The resulting upper bound to M would be $\frac{R_{\delta_l}}{1+\lambda} \frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} - (1 - \nu_1) (\Pi_{l,0}^* - \Pi_{h,0}^*) - \psi$. With $T_l > T_h$, given that $R'_x > 0$, G faces a larger replacement cost in state l , all over the residual period (τ, T_l) , than in state h , all over the residual period (τ, T_h) . Thus, as a consequence to setting a longer duration in state l , the temptation of G to break up the partnership is lessened, in that state, relative to the other state. When the replacement-cost function is steep enough to satisfy the first inequality in (22), this effect is so strong that M is to be adjusted to mitigate the opportunism of G in the bad state, rather than in the good one, despite that the compensation to F is smaller when the production cost is high. The relevant upper bound to M is, then, $\frac{R_{\delta_h}}{1+\lambda} \frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + \nu_1 (\Pi_{l,0}^* - \Pi_{h,0}^*) - \psi$. For analogous reasons, the maximum feasible loan is calibrated on the expected value of the replacement cost over the two states, as (28) shows.

6 Conclusion

There is one essential lesson, on public-private partnerships in infrastructure projects, to be drawn from our analysis. What causes contracts with a state-dependent duration to perform better than fixed-term contracts is the concomitant inability of *both* the firm and the government to commit to contractual obligations. Moreover, when each of the two partners needs to be incentivized to honour the contract, it is necessary to move away from the flexible-term paradigm. Rather than letting the firm manage the activity for a longer period when it faces unfavourable conditions, the contract should be lengthened when conditions

are favourable.³

To the best of our knowledge, in incentive theory, the possibility of using a state-dependent contractual length as an instrument to prevent opportunistic behaviour has not been explored hitherto. In the existing studies, the contractual length is assumed to be either exogenous or fixed across states of nature. Under those circumstances, information issues are addressed by differentiating enough the cash-flow accruing to the agent across states of nature. Our results highlight that, when commitment issues add up to information issues and, all the more, when they concern both contractual parties, a more effective strategy consists in differentiating the termination dates, while making the cash-flows to the firm more similar across states.

In line with the findings in DV, it is confirmed that, in environments where either partner is to be motivated to abide by the contract, there exists an essential link between the duration of the latter and the financial structure of the project. Specifically, the duration is useful to reconcile the relevant incentive conflicts, a task which might be accomplished more easily by adopting the state-dependent approach. Once this is done to that end, the different kinds of funds are functional to addressing the various specific incentive issues. Thus, the decision to follow a certain strategy in the determination of the contract duration affects finely the choice of a suitable financial structure. In particular, as one can infer from the comment to Corollary 2, it might be necessary to downsize the firm's investment, as compared to the case of a fixed-term contract, in order to mitigate the government's opportunism.

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³In this study, as in EFG, uncertainty about the operating conditions does not increase with the time horizon. From Danau [4], we know that, in situations where uncertainty does increase with the time horizon, the flexible-term contract (in which $T_h > T_l$) has a side effect. For some given level of the reservation utility, to be granted to the firm, the expected duration is longer than the duration of the fixed-term contract, yielding that same utility to the firm. That is, the partnership would last for a longer period, in expectation. This circumstance may not appear to be desirable, as the PPP would come closer to pure privatisation. The state-dependent approach here investigated, which leads to $T_l > T_h$, would not trigger that effect.

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A Full commitment

A.1 Proof of Lemma 1

Conditions under which (3a) and (3b) are satisfied when (2) is saturated

From (3a) and (3b), $\exists \varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ such that

$$\Pi_{l,0}^* = \Pi_{h,0}^* + \int_0^{T_h} \Delta\theta(q_h^* + \varepsilon_1) e^{-rx} dx \quad (29a)$$

$$\Pi_{h,0}^* = \Pi_{l,0}^* - \int_0^{T_l} \Delta\theta(q_l^* - \varepsilon_2) e^{-rx} dx. \quad (29b)$$

Denote $z_h = q_h^* + \varepsilon_1$ and $z_l = q_l^* - \varepsilon_2$. Saturate (2) and replace into (29a) and (29b) to obtain (7a) and (7b). Replacing (7a) into (29b), we further obtain

$$\Pi_{h,0}^*(z_j, T_j) = M + \psi - \int_0^{T_l} \Delta\theta z_l e^{-rx} dx + (1 - \nu_1) \int_0^{T_j} \Delta\theta z_j e^{-rx} dx. \quad (30)$$

For $j = h$, from (7b) and (30), we get

$$z_h (1 - e^{-rT_h}) = z_l (1 - e^{-rT_l}). \quad (31)$$

With $T_h > 0$, we have $z_l (1 - e^{-rT_l}) \in (0, z_h]$. Hence, (31) is rewritten as

$$T_h = \frac{1}{r} \ln \frac{z_h}{z_h - z_l (1 - e^{-rT_l})} \equiv \tilde{T}_h(z_h, z_l, T_l), \quad (32)$$

where z_h , z_l , and T_l are such that $z_h \geq z_l (1 - e^{-rT_l})$. When $z_l < z_h$, this is satisfied for all values of T_l . Otherwise, it is equivalent to $T_l \leq \frac{1}{r} \ln \frac{z_l}{z_l - z_h}$.

The conditions $\varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ are equivalent, respectively, to $z_h \geq q_h^*$ and $z_l \leq q_l^*$.

Condition under which (3c) is satisfied when $\Pi_{i,0} = \Pi_{i,0}^*$, $\forall i \in \{l, h\}$, and (6) holds

Replacing (7a) and (7b) into (3c), we find that (3c) holds if and only if

$$\Delta\theta z_j \frac{1 - e^{-rT_j}}{r} \geq \frac{\psi}{\Delta\nu}. \quad (33)$$

When $\psi > \Delta\nu\Delta\theta z_j/r$, $\nexists T_j > 0$ such that (33) holds. Take $\psi \leq \Delta\nu\Delta\theta z_j/r$. Then, (33) is equivalent to

$$T_j \geq \frac{1}{r} \ln \frac{\Delta\nu\Delta\theta z_j}{\Delta\nu\Delta\theta z_j - r\psi} \equiv \underline{T}(z_j). \quad (34)$$

Because $\underline{T}(z_h) \leq \tilde{T}_h(z_h, z_l, T_l)$ is equivalent to $T_l \geq \underline{T}(z_l)$, and provided that (32) is satisfied, it follows that $T_h \geq \underline{T}(z_h)$ when $T_l \geq \underline{T}(z_l)$. Using $\underline{T}'(z_j) < 0$ and $z_l \leq q_l^*$, $T_l \geq \underline{T}(z_l)$ is rewritten as $T_l \geq \underline{T}(q_l^*)$. Under (4), the assumed condition $\psi \leq \Delta\nu\Delta\theta z_j/r$ holds for $z_j = q_l^*$.

B Limited commitment

B.1 The renegotiation game

Suppose that, at the outset of the operation phase, F observes θ_i , $i \in \{l, h\}$, and reports it correctly to G. Further suppose that, at date $\tau \in (0, T_i)$, some party reneges on the contract.

B.1.1 Replacement

If F is replaced, then its instantaneous profit is $\pi_i^{rp} = 0, \forall x \in (\tau, T_i)$. Thus, the payoff of F at τ is (8a). At each $x \in (\tau, T_i)$, G assigns to the new firm F' the production $q_i^{rp} = q_i^*$ and the transfer $t_i^{rp} = \theta_i q_i^* + K - p(q_i^*) q_i^*$ so that, at τ , the payoff of F' is zero and the optimised discounted return of G through date T_i is (8b).

B.1.2 Renegotiation

Suppose that G makes the offer to F: at each $x \in (\tau, T_i)$, F will produce q_i^G and receive t_i^G such that its payoff at τ is $\Pi_{i,\tau}^G = \Pi_{i,\tau}^{rp} = 0$. This requires setting $t_i^G = \theta_i q_i^G + K - p(q_i^G) q_i^G + d_{i,\tau}^{rn}$ (t_i^G includes the amount $d_{i,\tau}^{rn}$ destined to L; alternatively, F receives $t_i^G - d_{i,\tau}^{rn}$ and L $d_{i,\tau}^{rn}$). The net return of G from the renegotiated contract is

$$V_{i,\tau}^G = \int_{\tau}^{T_i} w(q_i^G) e^{-r(x-\tau)} dx - (1 + \lambda) (\Pi_{i,\tau}^G + D_{i,\tau}^{rn}).$$

Replacing $\Pi_{i,\tau}^G$ into $V_{i,\tau}^G$ and then maximizing with respect q_i^G , we see that G chooses $q_i^G = q_i^*$ so that, under the contract renegotiated at τ , its optimised discounted return is $V_{i,\tau}^G = \varpi_i^* - (1 + \lambda) D_{i,\tau}^{rn}$.

Next suppose that F makes the offer to G: at each $x \in (\tau, T_i)$, F offers to produce q_i^F and to receive t_i^F such that G is left with the same discounted return as in the replacement situation: $V_{\tau}^F = V_{\tau}^{rp} = \int_{\tau}^{T_i} w_i^* e^{-r(x-\tau)} dx - R_{\delta_i}$. This requires setting $t_i^F = (S(q_i^F) - p(q_i^F) q_i^F - w_i^* + r R_{\delta_i} / (1 - e^{-r(T_i-\tau)})) / (1 + \lambda)$, together with $q_i^F = q_i^*$. The optimised payoff of F is then $\Pi_{i,\tau}^F = R_{\delta_i} / (1 + \lambda) - D_{i,\tau}^{rn}$. (9a) and (9b) are computed, respectively, as $\Pi_{i,\tau}^{rn} = \alpha \Pi_{i,\tau}^{rp} + (1 - \alpha) \Pi_{i,\tau}^F$ and $V_{i,\tau}^{rn} = \alpha V_{i,\tau}^G + (1 - \alpha) V_{i,\tau}^{rp}$.

B.2 Removing the incentives of F to cheat anticipating renege

We identify conditions under which F has no incentive to lie on θ_i , anticipating that some party will renege at some date $\tau \in (0, T_i)$.

Let $\Pi_{i,\tau}^{RN}$ denote the payoff that F would obtain in state i , discounted at time τ , if it were to cheat at the outset of the operation phase and renegotiation were to occur at τ . Also let $\pi_{i,x}^*$ the instantaneous profit in state $i \in \{l, h\}$ at instant $x \in (\tau, T_i)$. F has no incentive to lie if and only if

$$\Pi_{l,0}^* (z_j, T_j) \geq \int_0^{\tau} (\pi_{h,x}^* + \Delta \theta q_h^*) e^{-rx} dx + \max \{0; \Pi_{l,\tau}^{RN}\} \quad (35a)$$

$$\Pi_{h,0}^* (z_j, T_j) \geq \int_0^{\tau} (\pi_{l,x}^* - \Delta \theta q_l^*) e^{-rx} dx + \max \{0; \Pi_{h,\tau}^{RN}\}. \quad (35b)$$

We hereafter show that (35a) is satisfied. If F reports h at date zero, in state l , and the contract is renegotiated at some $\tau \in (0, T_h)$, the profit of F at each instant during the period (τ, T_h) is:

$$\pi_{l,\tau}^{RN} = t_h^{rn} + p(q_h^*) q_h^* - (\theta_l q_h^* + K) - d_{h,\tau}^{rn}, \quad (36)$$

where $t_h^{rn} = \alpha t_h^G + (1 - \alpha) t_h^F$ is the expected transfer which results from renegotiating at τ ,

given the report h . Replacing t_h^G and t_h^F , from Appendix B.1, into (36), the latter is rewritten as $\pi_{l,\tau}^{RN} = (1 - \alpha) (rR_{\delta_h} / (1 - e^{-r\delta_h}) (1 + \lambda) - d_{h,\tau}^{rn}) + \Delta\theta_h q_h^*$. Hence:

$$\Pi_{l,\tau}^{RN} = \int_{\tau}^{T_h} \pi_{l,\tau}^{RN} e^{-r(x-\tau)} d\tau = \Pi_{h,\tau}^{rn} + \int_{\tau}^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} d\tau.$$

(35a) becomes

$$\begin{aligned} \Pi_{l,0}^* (z_j, T_j) \geq & \Pi_{h,0}^* (z_j, T_j) + \int_0^{T_h} \Delta\theta q_h^* e^{-rx} dx \\ & + e^{-r\tau} \left[\max \left\{ 0; \Pi_{h,\tau}^{rn} + \int_{\tau}^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx \right\} \right. \\ & \left. - \left(\Pi_{h,0}^* (z_j, T_j) + \int_{\tau}^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx \right) \right]. \end{aligned} \quad (37)$$

This is implied by (3a) and (10). Hence, (35a) holds.

Symmetrically, (35b) is implied by (3b) and (10). Hence, it is satisfied.

B.3 No incentive to renegotiate the contractual length

First suppose that in state i G makes the offer and proposes to terminate the contract at some date $T_i^G > \tau$, $T_i^G \neq T$. If $T_i^G > T_i$, then G proposes the quantity-transfer pair $(q_i^*, t_i^{G,1})$ for all $x \in [\tau, T)$ and the quantity-transfer pair $(q_i^*, t_i^{G,2})$ for all $x \in [T_i, T_i^G]$. $t_i^{G,1}$ and $t_i^{G,2}$ are set such that the instantaneous profits are zero ($\pi_{i,\tau}^{G,1} = \pi_{i,\tau}^{G,2} = \pi_i^{rp} = 0$), so that the payoff of F at τ is zero as well. Using $q_i = q_i^*$ in the expressions of the instantaneous profits $\pi_{i,\tau}^{G,1}$ and $\pi_{i,\tau}^{G,2}$, and denoting $t_i = t_i^{G,1}$ in $\pi_{i,\tau}^{G,1}$ and $t_i = t_i^{G,2}$ in $\pi_{i,\tau}^{G,2}$, we get $t_i^{G,1} = \theta_i q_i^* + K - p(q_i^*) q_i^* + d_{i,\tau}^{rn}$ for all $x \in [\tau, T_i)$, and $t_i^{G,2} = \theta_i q_i^* + K - p(q_i^*) q_i^*$ for all $x \in [T_i, T_i^G]$. With these expressions of $t_i^{G,1}$ and $t_i^{G,2}$, we can write the discounted return of G at τ as

$$\widehat{V}_{i,\tau}^G = \int_{\tau}^{T_i^G} w_i^* e^{-r(x-\tau)} dx - (1 + \lambda) D_{i,\tau}^{rn}.$$

If $T_i^G < T_i$, then, G proposes the pair $(q_i^*, t_i^{G,1})$ for all $x \in [\tau, T_i^G]$ so that the payoff of F at τ is zero. The discounted return of G at τ is $\widehat{V}_{i,\tau}^G$. Hence, the payoff of G at τ is given by

$$W_{i,\tau} = \widehat{V}_{i,\tau}^G + \int_{T_i^G}^{+\infty} w_i^* e^{-r(y-T_i^G)} dy = \int_{\tau}^{+\infty} w_i^* e^{-r(x-\tau)} dx - (1 + \lambda) D_{i,\tau}^{rn},$$

which is independent of T_i^F .

Next suppose that F makes the offer and proposes to terminate the contract at some date $T_i^F > \tau$, $T_i^F \neq T_i$. Then, the proposal includes the quantity-transfer pair $(q_i^*, t_i^{F,1})$ for all $x \in [\tau, T_i^G]$. For the discounted return of G at τ to be equal to (8b), it must be

$t_i^{F,1} = (S(q_i^*) - p(q_i^*)q_i^* - w_i^* + rR_{\delta_i}/(1 - e^{-r(T_i^F - \tau)}))/(1 + \lambda)$. The payoff of F at τ is independent of T_i^F as it is given by

$$\Pi_{i,\tau}^F = \frac{R_{\delta}}{1 + \lambda} - D_{i,\tau}^{rn}.$$

B.4 Proof of Lemma 4

Using $\Pi_{i,\tau}^*(z_j, T_j) = \Pi_{i,0}^*(z_j, T_j) \frac{1 - e^{-r\delta_i}}{1 - e^{-rT_i}}$ together with (7a) and (7b), we obtain

$$\Pi_{l,\tau}^*(z_j, T_j) = \left(M + \psi + (1 - \nu_1) \Delta\theta z_j \frac{1 - e^{-rT_j}}{r} \right) \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} \quad (38a)$$

$$\Pi_{h,\tau}^*(z_j, T_j) = \left(M + \psi - \nu_1 \Delta\theta z_j \frac{1 - e^{-rT_j}}{r} \right) \frac{1 - e^{-r\delta_h}}{1 - e^{-rT_h}}. \quad (38b)$$

As $V_{i,\tau}^* = \int_{\tau}^{T_i} w(q_i^*)e^{-r(x-\tau)}dx - (1 + \lambda)(\Pi_{i,\tau}^* + D_{i,\tau})$, the expected return of G, at date τ , from private management, is written, in state l and in state h , as

$$\begin{aligned} V_{l,\tau}^*(z_j, T_j) &= w_l^* \frac{1 - e^{-r\delta_l}}{r} - (1 + \lambda) \left[\left(M + \psi + (1 - \nu_1) \Delta\theta z_j \frac{1 - e^{-rT_j}}{r} \right) \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} + D_{l,\tau} \right] \\ V_{h,\tau}^*(z_j, T_j) &= w_h^* \frac{1 - e^{-r\delta_h}}{r} - (1 + \lambda) \left[\left(M + \psi - \nu_1 \Delta\theta z_j \frac{1 - e^{-rT_j}}{r} \right) \frac{1 - e^{-r\delta_h}}{1 - e^{-rT_h}} + D_{h,\tau} \right]. \end{aligned}$$

B.4.1 Derivation of (16)

Using $E \geq M$ and (40), $\Delta\theta z_j \frac{1 - e^{-rT_j}}{r} \leq \frac{E + \psi}{\nu_1}$. Then, because it is necessary that $\frac{\psi}{\Delta\nu} \leq \Delta\theta z_j \frac{1 - e^{-rT_j}}{r}$ (see Proof of Lemma 1), we obtain (16).

B.4.2 Derivation of (17)

Suppose that $C = 0$ and that the efficient outcome is effected. Then, $D_{i,\tau}^{rn} = 0$ for all $i \in \{l, h\}$ and $\tau \in (0, T_i)$. Moreover, $D_{i,\tau'}^{rn} = \widehat{D}_{i,\tau/\tau'}^{rn} = 0$ for all $\tau, \tau' \in (0, T)$, $\tau' > \tau$. Hence, (14) cannot be satisfied. Nor can one have $D_{i,\tau'}^{rn} - \widehat{D}_{i,\tau/\tau'}^{rn} > 0$, as is required for (15) to be met. This contradicts the hypothesis that the efficient outcome is effected with $C = 0$. Hence, (17) must hold.

B.4.3 Derivation of (18)

Take $D_{i,\tau}^{rn}$ such that (14) holds $\forall i \in \{l, h\}$, $\tau \in (0, T_i)$. Then, $\max\{\Pi_{i,\tau}^{rn}, \Pi_{i,\tau}^{rp}\} = \Pi_{i,\tau}^{rp}$ and $\max\{V_{i,\tau}^{rn}, V_{i,\tau}^{rp}\} = V_{i,\tau}^{rp}$. (10) and (11) are rewritten, respectively, as $\Pi_{i,\tau}^*(z_j, T_j) \geq \Pi_{i,\tau}^{rp}$ and $V_{i,\tau}^*(z_j, T_j) \geq V_{i,\tau}^{rp}$.

When $i = l$, it is obvious that $\Pi_{i,\tau}^*(z_j, T_j) \geq \Pi_{i,\tau}^{rp}$. When $i = h$, this is the case, if and only if:

$$M \geq \nu_1 \Delta\theta z_j \frac{1 - e^{-rT_j}}{r} - \psi. \quad (40)$$

Moreover, when $i = l$, we have $V_{i,\tau}^*(z_j, T_j) \geq V_{i,\tau}^{rp}$, together with $D_{l,\tau} \geq 0$, if and only if:

$$M \leq \frac{R_{\delta_l}}{1+\lambda} \frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} - (1-\nu_1) \Delta\theta z_j \frac{1-e^{-rT_j}}{r} - \psi \quad (41a)$$

$$D_{l,\tau} \leq \frac{R_{\delta_l}}{1+\lambda} - \left(M + \psi + (1-\nu_1) \Delta\theta z_j \frac{1-e^{-rT_j}}{r} \right) \frac{1-e^{-r\delta_l}}{1-e^{-rT_l}}. \quad (41b)$$

When $i = h$, we have $V_{i,\tau}^*(z_j, T_j) \geq V_{i,\tau}^{rp}$, together with $D_{h,\tau} \geq 0$, if and only if:

$$M \leq \frac{R_{\delta_h}}{1+\lambda} \frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + \nu_1(\Pi_{l,0}^* - \Pi_{h,0}^*) - \psi \quad (42a)$$

$$D_{h,\tau} \leq \frac{R_{\delta_h}}{1+\lambda} - \left(M + \psi - \nu_1 \Delta\theta z_j \frac{1-e^{-r\delta_j}}{r} \right) \frac{1-e^{-r\delta_h}}{1-e^{-rT_h}}. \quad (42b)$$

We see that $\exists M \geq 0$ such that (40) and (42a) hold jointly. Moreover, $\exists M \geq 0$ such that (40) and (41a) hold jointly if and only if

$$R_{\delta_l} \geq (1+\lambda) \Delta\theta z_j \frac{1-e^{-rT_j}}{r} \frac{1-e^{-r\delta_l}}{1-e^{-rT_l}}. \quad (43)$$

Because $\Pi_{l,0}^* - \Pi_{h,0}^* = \Delta\theta z_j \frac{1-e^{-rT_j}}{r}$, $\forall j \in \{l, h\}$, (43) is rewritten as (18).

B.5 Proof of Proposition 2 and Proposition 3

B.5.1 $T_l = T_h \equiv T$

From (31), when $T_l = T_h$, it is $z_l = z_h \equiv z$. As $z_h \geq q_h^*$ and $z_l \leq q_l^*$ (proof of Lemma 1), $z \in [q_h^*, q_l^*]$. Denote $\delta \equiv T - \tau$. (18) becomes

$$R_\delta \geq (1+\lambda) \Delta\theta z \frac{1-e^{-r\delta}}{r}. \quad (44)$$

If

$$R'_\delta \geq (1+\lambda) \Delta\theta z e^{-r\delta}, \forall \delta \in (0, \infty), \quad (45)$$

then, (44) is satisfied $\forall T > 0$, provided that it is satisfied as $T \rightarrow 0$ and, hence, as $\delta \rightarrow 0$. This is true, by assumption. If $\nexists \delta \in (0, \infty)$ for which (45) holds, then, $\exists T$ for which (44) is satisfied, if and only if

$$R_{\underline{T}(z)} \geq (1+\lambda) \frac{\psi}{\Delta\nu}. \quad (46)$$

Each of the two conditions (45) and (46) is violated $\forall z \in [q_h^*, q_l^*]$ if and only if it is violated for $z = q_h^*$. Thus, the contract is not enforceable when $R'_x \leq (1+\lambda) \Delta\theta q_h^* e^{-r\delta}$, $\forall x \in (0, \infty)$, and (23) is satisfied.

B.5.2 $T_l \neq T_h$

Take $j = l$. From the proof of Lemma 1, it is necessary that $z_l > \underline{z}$, where $\underline{z} = \frac{r\psi}{\Delta\nu\Delta\theta}$. If, for some given $z_l > \underline{z}$, it is:

$$R'_{\delta_l} \geq (1 + \lambda) \Delta\theta z_l e^{-r\delta_l}, \quad \forall \delta_l \in (0, \infty), \quad (47)$$

then, (18) holds $\forall T_l > 0$. If (47) is violated $\forall \delta_l \in (0, \infty)$, then, $\exists T_l \geq \underline{T}(z_l)$ for which (18) holds, if and only if:

$$R_{\underline{T}(z_l)} \geq (1 + \lambda) \frac{\psi}{\Delta\nu}. \quad (48)$$

Both (47) and (48) are weakest when $z_l \rightarrow \underline{z}$, in which case $\underline{T}(z_l) \rightarrow \infty$. Then, (47) is rewritten as $R'_x \geq (1 + \lambda) \Delta\theta \frac{r\psi}{\Delta\nu\Delta\theta} e^{-rx}$, $\forall x \in (0, \infty)$. (48) is rewritten as $\lim_{x \rightarrow \infty} R_x \geq (1 + \lambda) \frac{\psi}{\Delta\nu}$.

B.6 Proof of Corollary 1

By assumption, $\lim_{\delta \rightarrow 0} R_\delta > 0$ and finite. Hence, (18) holds as $\delta \rightarrow 0$.

First suppose that (22) holds. Then, as δ_l is raised, (18) is relaxed. Then, given that it is satisfied as $\delta_l \rightarrow 0$, it is for all $\delta_l \in (0, T_l)$, $T_l \in [\underline{T}(z), \infty)$.

Next suppose that (24) holds. Then, for any given $T_l \geq \underline{T}(z_l)$, (18) is tightest as $\delta_l \rightarrow T_l$. Then, replacing $\delta_l \rightarrow T_l$, (18) holds if and only if $T_l \leq \bar{T}(z_l)$. As (5) is necessary (Lemma 1), it must be the case that the interval $[\underline{T}(z_l), \bar{T}(z_l)]$ exists and that $T_l \in [\underline{T}(z_l), \bar{T}(z_l)]$.

The remaining condition is that $T_l \leq \tilde{T}(z_l, E)$ when $E < (\nu_1 \Delta\theta z_l / r) - \psi$. This is derived from condition $\Delta\theta z_l \frac{1 - e^{-rT_l}}{r} \leq \frac{E + \psi}{\nu_1}$ in the proof of Lemma 4.

B.7 Proof of Corollary 2

Using (40), (41a), and (42a), we obtain (27). Using the definition of $\mathbb{E}_i[D_{i,\tau}]$ in (41b) and (42b), we further get

$$\mathbb{E}_i[D_{i,\tau}] \leq \frac{\mathbb{E}_i[R_{\delta_i}]}{1 + \lambda} - (M + \psi) \mathbb{E}_i \left[\frac{1 - e^{-r\delta_i}}{1 - e^{-rT_i}} \right].$$

Then, recalling that $\mathbb{E}_i[D_{i,0}] = C$, this condition collapses onto (28).