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Open Source Software Subsidies and Network Compatibility in a Mixed Duopoly

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Abstract

For many applications, open source software (OSS) can offer a high-quality alternative to proprietary software (e.g. Linux, Apache, Android,...). But even if OSS is usually free of charge, its installation and use require some skills. Should the government intervene to promote the diffusion of OSS and provide some learning or financial support to potential adopters? This paper examines whether public subsidies towards open source software is socially desirable and how the extent of compatibility between open source software and proprietary software can influence the amount of subsidies. We consider a mixed duopoly model in which a proprietary software (PS) company competes with an open source software (OSS) community. Users are heterogeneous in their ability to use OSS, and their utility depends on the number of users who have adopted the same software or a compatible software (existence of network externalities). Four situations are distinguished: full compatibility between OSS and PS, full incompatibility, and one-way compatibility (either only OSS or PS is compatible). We show that if the government only takes care of consumer surplus, public subsidies are welfare-enhancing. But the optimal level of subsidies is larger with full compatibility and PS compatibility than full incompatibility and OSS compatibility. These results suggest that government policy towards OSS must be conditional to the degree of compatibility between PS and OSS.

JEL classification: L11, L15, L17, L38
Keywords: Open source software, Public subsidy, Network compatibility

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1 Introduction

Over the last decade, several governments around the world have actively encouraged the adoption of open source software[1] (OSS), mainly through training programs and direct procurement of OSS (for public administrations and schools). There are several reasons to promote OSS (Varian and Shapiro, 2003; Benkler, 2002; Smith, 2002). First, OSS is available free of charge and is perceived as more secured, reliable and customizable than proprietary software (PS) (e.g. Linux, Apache, Gimp, Sendmail, Android, etc.). Secondly, it may level the playing field in the software industry, and correct market failure arising from the demand and supply features in this industry (switching cost, network effect, ...). The typical example is Linux, an alternative to Windows in the market for operating systems, that has less bugs and is more frequently updated than Windows (Raghunathan et al., 2005). Despite its superior quality, the diffusion of Linux is limited to a population of expert users. Typically, the vast majority of users prefer to buy proprietary software that tends to be more user-friendly and offers technical help and support.[2]

This article aims to examine whether public policy in favor of OSS can be efficient and how they impact users and proprietary software companies. Here we will focus on public subsidies directed to users to reduce their cost of OSS adoption (i.e. the cost of installing and using OSS): (i) What is the welfare impact of public subsidies for OSS? (ii) How the extent of compatibility between PS and OSS can influence the amount of subsidies?

Open source literature exhibits mixed results concerning the welfare impact of public policy in favor of open source software. Using a model of spatial competition between open source and proprietary software, Schmidt and Schnitzer (2002) show that such a policy reduces the software company's incentives to improve the quality of its product and has detrimental effects on welfare. Comino and Manenti (2003) consider a market in which some users ignore the existence and/or characteristics of open source software. They find that OSS subsidies always reduce social welfare. By contrast, Mustonen (2003) obtains that public efforts to provide better information on open source alternative are welfare enhancing.

Our paper revisits this question in a new setting. First we suppose that customers are heterogeneous in their ability to use OSS. Moreover, customer's utility increases with the number of users who have adopted the same software or a compatible software (i.e. presence of network externalities). Secondly, we assume that the proprietary and open source software are vertically differentiated whereas in Comino and Manenti (2003), the two products have the same quality. We develop a two stage model in which the government first chooses the level of subsidies, then the software company set the price of its product and finally customers choose between the proprietary software and the open source alternative that is released free of charge (mixed duopoly). An important question is whether these two software are compatible or incompatible. Four situations are

[1] OSS is software for which the source code is freely available, and that the license under which is distributed enables each user not only to use the software, but also to copy it, to modify it, and to redistribute the original or modified version to other users.

[2] Network effects can also hinder the entry of higher quality software. Network effects arise both directly from the number of consumers who are using compatible software and indirectly from the provision of complementary services. Such network effects may tip the market in favor of only one software product. This can happen for any product or technology with network externality. For instance, it can explain the dominance of QWERTY keyboard even if it is less performant than the DSKs of August Dvorak (David, 1985).

distinguished: full (two-way) compatibility, full incompatibility, and one-way compatibility (either OSS or PS compatibility). OSS compatibility means that the PS users can unilaterally access the OSS community and derive benefits from it. For instance, they can use programs developed by the OSS community, or read and modify the files sent by OSS users, whereas the OSS users cannot open the files or programs created with the proprietary software. With PS compatibility, only OSS users can derive some utility from the PS users.

We show that if the government only takes care of consumer surplus, public subsidies are welfare-enhancing. But the optimal level subsidies is higher with full compatibility and OSS compatibility than full incompatibility and PS compatibility These results suggest that government policy towards OSS must be conditional to the degree of compatibility between PS and OSS. However if the government is maximizing the total welfare (including the firm's profit), subsidies towards open source are not socially desirable regardless of the regime of software compatibility.

The paper is organized as follows. The next section presents our model. Equilibrium outcomes are displayed in Section 3. In Section 4, we compare the price, profits, market shares and subsidies under the four compatibility regimes. Limitations and possible extensions are discussed in section 5.

2 Setting of the Model

We consider a firm that sells a proprietary software (PS) of quality, V_{PS}, at price, p. But consumers have also the alternative to use an open source software (OSS) developed by an open source community. This software is free and has a level of quality, V_{OS}. We assume that $V_{OS} > V_{PS}$ meaning that the OSS has a superior quality or better performance. We define $\Delta = V_{OS} - V_{PS} > 0$.In the remainder of the paper, we suppose that V_{OS} and V_{PS} are sufficiently large to ensure that the market is fully covered.

We assume that there is a disutility to install and deploy an OSS that negatively depends on users' level of expertise. Users' skills θ are uniformly distributed on $(0, 1)$: for high skilled users, θ is closed to 0 and for low skilled users θ is closed to 1. For a given level of expertise θ, the cost or disutility of installing an OSS is equal to $c\theta$ whereas there is no disutility of deploying a proprietary software (as PS are usually characterized by user-friendly interface and technical support[3]). For simplicity, the mass of users is equal to 1 and users only adopt one software (no multi-homing).

User's utility depends on the (intrinsic) quality of the software, but also on network effects. As Katz and Shapiro (1985) and Shy (2001), we assume that the value of network externalities is γ times the number of users who have adopted the same software or compatible software[4]. As the number of users increases, it becomes easier to share or exchange data and files or get support. As the number of software users is only known after users make their adoption choice, individuals have to form expectations about the

[3] Proprietary software is more user friendly than open source software because open source software is developed by high skilled programmers who are also the potential users of these software. For example, the installation of open source software require downloading source code, linking libraries, setting environment variables for the operating system and compelling the source code. In contrast, most proprietary software requires just a few clicks and technical support is usually available.

[4] Following Farrell and Saloner (1992), the value of network externality γ is supposed to be the same for the two software.

respective number of OSS and PS users. We suppose that each user correctly anticipates the size of each software network (self-fulfilling beliefs).

We distinguish four situations, depending on whether the PS and the OSS are fully (two-way) compatible, partially (one-way) compatible or incompatible:

- Full incompatibility: the value of network externality for PS users is γN_{PS} (with N_{PS} the number of users who have adopted the PS) and the value of network externality for OSS users is γN_{OS} (with N_{OS} the number of users who have adopted the OSS)
- Full compatibility: the value of network externality for both users of OSS and PS is[5] $\gamma(N_{OS} + N_{PS}) = \gamma$
- OSS-compatibility: if OSS is unilaterally compatible, PS users can access the OSS community, but OSS users cannot get any utility from the network of PS users. In this case, the value of network externality for PS users is $\gamma(N_{PS} + N_{OS}) = \gamma$, and the value of network externality for OSS users is γN_{OS}.
- PS-compatibility[6]: if PS is unilaterally compatible, only OSS users can access PS users and the value of network externality for OSS users and PS users is respectively $\gamma(N_{PS} + N_{OS}) = \gamma$, and γN_{PS}.

For the sake of simplicity, let II and CC denote the full incompatibility and full compatibility regimes. Similarly, CI and IC represent the OSS-compatibility and PS-compatibility regimes.

The utility of type θ user under the different compatibility regimes is given by $U_\theta = V_{PS} + \gamma N_{PS} - p$ if the user buys a PS that is OS-incompatible, $U_\theta = V_{OS} + \gamma N_{OS} - c\theta + s$ if the user downloads an OSS that is PS-incompatible, $U_\theta = V_{PS} + \gamma - p$ if the user buys a PS that is OS-compatible and $U_\theta = V_{OS} + \gamma - c\theta + s$ if the user downloads an OS that is PS-compatible.

Assuming that the marginal cost to sell a PS is constant and normalized to zero, the profit of the software firm is given by:

$$\Pi^k = p^k N_{PS}^k \quad \text{with } k = II, CC, CI, IC \tag{1}$$

By definition the OS community has no revenue and no cost (i.e. profit equal to zero)[7].

In this paper, we analyze the impact of subsidies that are directed to OSS users. These subsidies can take the form of technical support or training to reduce the cost of adoption of OSS. Let s be the amount of subsidies per user and $S = sN_{OS}$ be the cost of subsidizing OSS users. What should be the optimal level of subsidies? We suppose that the objective of the government is to maximize consumers' surplus net of the cost of subsidies. In other terms, the government doesn't take into account the profit of the software company. Two reasons can be put forward. First, many software companies are operating abroad and their profits cannot be part of the domestic social surplus[8]. Secondly in matters of market

[5] As the market is fully covered and the total number of users is 1, we have $N_{OS} + N_{PS} = 1$.

[6] This case is less realistic as unilateral compatibility from PS to OSS is seldom observed.

[7] Open source software are developed by open source communities whose members voluntarily contribute (during their working hours or free time).

[8] The main software companies are US companies and this explains why many European and Asian governments want to encourage the adoption of OSS (especially for public administrations and schools).

regulation and competition policy, governments tend to put more weight on consumers' welfare. Thus the government will choose the amount of subsidies that maximizes

$$W_U^k = US^k - S^k \text{ with } k = II, CC, CI, IC \tag{2}$$

In the Appendix, we also present the results when the government maximizes the social welfare that includes the profit of the software firm.

The timing of the model is as follows. In the first stage, the government chooses the amount of subsidies for OSS users. In the second stage, the firm sets the price of its software and then the users choose to adopt either the PS or the OSS. Throughout the paper, we restrict our attention to equilibrium outcomes in which both software products are used. The necessary conditions for the existence of an active duopoly is given by the following assumption.

Assumption 1. $c > \Delta + \gamma$ *and* $\Delta > \gamma$

This assumption holds if the adoption cost of OSS is sufficiently large, and PS and OSS are sufficiently differentiated in quality. Under **Assumption 1,** the software company has a positive market share regardless the regime of compatibility[9].

3 Equilibrium outcomes

We start by solving the second stage of our model in which the proprietary firm sets its price and the users make their adoption decisions under the four compatibility regimes.

3.1 Price and market shares

3.1.1 Full incompatibility

When OSS and PS are fully incompatible, the values of network externalities for users of OSS and PS are respectively γN_{OS}, and γN_{PS}. Let $\widehat{\theta}^{II}$ be the marginal user who is indifferent between adopting PS and OSS. The solution is $\widehat{\theta}^{II} = \frac{(p+s-\gamma+\Delta)}{c-2\gamma}$: users with a type $\theta < \widehat{\theta}^{II}$ (high skilled) will prefer OSS and users with $\theta > \widehat{\theta}$ will adopt PS. It implies that the market share of the OS community is $N_{OS}^{II} = \widehat{\theta}^{II}$ whereas the market share of the firm is $N_{PS}^{II} = 1 - \widehat{\theta}^{II}$. The profit function of the software company is $\Pi^{II} = \frac{p(c-p-s-\Delta-\gamma)}{c-2\gamma}$ and its profit-maximizing price is equal to $p^{II}(s) = \frac{c-s-\gamma-\Delta}{2}$.

We observe that the price decreases with the amount of subsidies. The effect of subsidies is to increase competition between the two types of software and reduce the market power of the software firm. After rearrangement the market shares are $N_{OS}^{II}(s) = \frac{c+s-3\gamma+\Delta}{2(c-2\gamma)}$ and $N_{PS}^{II}(s) = \frac{c-s-\gamma-\Delta}{2(c-2\gamma)}$. Market shares are both positive if $c > 2\gamma$ and $c > s + \gamma + \Delta$.

3.1.2 Full (Two-way) Compatibility

When OSS and PS are fully compatible, the value of network externalities is $\gamma(N_{OS} + N_{PS}) = \gamma$ regardless the choice of software. Let $\widehat{\theta}^{CC} = \frac{(p+s+\Delta)}{c}$ be the user who is indifferent be-

[9]The more stringent condition to have an active duopoly is under a regime of full incompatibility (see Appendix).

tween downloading the OSS and buying the PS. The market shares of the OS community and the firm are respectively given by $N_{OS}^{CC} = \widehat{\theta}^{CC}$ and $N_{PS}^{CC} = 1 - \widehat{\theta}^{CC}$. Given the amount of subsidies, s, the profit-maximizing price is equal to $p^{CC}(s) = \frac{c-s-\Delta}{2}$. Then the equilibrium market shares are $N_{OS}^{CC}(s) = \frac{c+s+\Delta}{2c}$, $N_{PS}^{CC}(s) = \frac{c-s-\Delta}{2c}$. The condition for an active duopoly is $c > \Delta + s$.

3.1.3 OSS compatibility

Now, we consider the case where the PS users can access the community of OSS users, but the reverse is not possible. Then, the values of network externalities are γN_{OS} for an OSS user and $\gamma(N_{OS} + N_{PS}) = \gamma$ for a PS user. Let $\widehat{\theta}^{CI} = \frac{p+s-\gamma+\Delta}{c-\gamma}$.be the user indifferent between PS and OSS. Then the optimal price for the software company is $p^{CI}(s) = \frac{c-s-\Delta}{2}$ and the equilibrium market shares are $N_{OS}^{CI}(s) = \frac{(c+s-2\gamma+\Delta)}{2(c-\gamma)}$ and $N_{PS}^{CI}(s) = \frac{c-s-\Delta}{2(c-\gamma)}$. OSS and PS have positive markets shares if $c > \gamma$ and $c > s + \Delta$.

3.1.4 PS compatibility

The last (but probably less realistic) scenario is a PS compatibility regime in which OSS users can unilaterally access the customer base of the proprietary software. Thus, the value of network externalities are $\gamma(N_{OS} + N_{PS}) = \gamma$ for an OSS user and γN_{OS}. for a PS user. Let $\widehat{\theta}^{IC} = \frac{(p+s+\Delta)}{c-\gamma}$. be the user indifferent between PS and OSS. The profit-maximizing price for the proprietary software is given by $p^{IC}(s) = \frac{c-s-\gamma-\Delta}{2}$ and the equilibrium market shares are $N_{OS}^{IC}(s) = \frac{c+s-\gamma+\Delta}{2(c-\gamma)}$ and $N_{PS}^{IC}(s) = \frac{c-s-\gamma-\Delta}{2(c-\gamma)}$ OSS and PS have positive markets shares if $c > \gamma$ and $c > s + \gamma + \Delta$.

3.2 Optimal subsidies

Now, we turn to the first-stage of the model where the government sets the subsidy per OSS users in order to maximize the consumers' surplus (net of the cost of subsidies). Under the four compatibility regimes, the optimal subsidies are given by[10]

$$s^{CC*} = \frac{c-\Delta}{3} \ ; \ s^{CI*} = \frac{c-\Delta}{3} \ ; \ s^{IC*} = \frac{c-\gamma-\Delta}{3} \ ; \ s^{II*} = \frac{c-\gamma-\Delta}{3}$$

Logically, the subsidy per user increases with the cost of OSS adoption and decreases with the quality advantage of open source product. Network effects influence the optimal level of subsidies only under the incompatibility and PS compatibility regimes. In these two situations, subsidies in favor of OSS are less desirable because converting a PS user to OSS either reduces network externalities for this new OSS user (PS compatibility regime) or reduces network externalities for the remaining PS users (full incompatibility). By contrast, network externalities are unchanged when some PS users switch to the OSS community under full compatibility or OS compatibility. Table 1 displays the equilibrium price, market shares and profits for the optimal level of subsidies.

[10] In each regime of compatibility, the optimal subsidy is unique as $\frac{\partial^2 W_U^k}{\partial s^{k2}} < 0$ for $c > \Delta + \gamma$ and $\Delta > \gamma$.

Table 1: Equilibrium outcomes under the four compatibility regimes

	II	CC	CI	IC
p^*	$\frac{c-\gamma-\Delta}{3}$	$\frac{c-\Delta}{3}$	$\frac{c-\Delta}{3}$	$\frac{c-\gamma-\Delta}{3}$
N_{PS}^*	$\frac{c-\gamma-\Delta}{3(c-2\gamma)}$	$\frac{c-\Delta}{3c}$	$\frac{c-\Delta}{3(c-\gamma)}$	$\frac{c-\gamma-\Delta}{3(c-\gamma)}$
N_{OS}^*	$\frac{2c-5\gamma+\Delta}{3(c-2\gamma)}$	$\frac{2c+\Delta}{3c}$	$\frac{2c-3\gamma+\Delta}{3(c-\gamma)}$	$\frac{2c-2\gamma+\Delta}{3(c-\gamma)}$
Π^*	$\frac{(c-\gamma-\Delta)^2}{9(c-2\gamma)}$	$\frac{(c-\Delta)^2}{9c}$	$\frac{(c-\Delta)^2}{9(c-\gamma)}$	$\frac{(c-\gamma-\Delta)^2}{9(c-\gamma)}$

We observe that firm's profit, price and market share increase with the cost of installing OSS and decrease with the differential in quality regardless of the compatibility regime. The role of network externality differs under the four regimes. It has no impact on market shares and profits under full compatibility,. But it increases firm's profitability under OS-compatibility and decreases profits under full incompatibility and PS-compatibility.

4 Implications for public policy

This section provides a comparative statics analysis of the equilibrium outcomes across the four compatibility regimes. This analysis is made under **Assumption 1**.and gives some insights on the desirability of public efforts to promote OSS.

4.1 Comparison of subsidies, public deficits, market shares, prices and profits

First, we compare the optimal amount of subsidy under the four compatibility regimes:

Proposition 1 *Under Assumption 1, the optimal subsidies per OSS user are characterized by* $s^{CC*} = s^{CI*} > s^{IC*} = s^{II*} > 0$

Proof. See **section 3.2**. ■

The government gives a larger subsidy per user in the cases of two-way compatibility and OSS compatibility. This result is quite intuitive. When the OSS is compatible with the PS, then PS users derive some gains from additional OSS users (through network externality). This is not the case under full incompatibility and PS compatibility. Clearly the government has more incentives to subsidize OSS users in the CC and CI situation, as the return in terms of welfare will be larger.

Proposition 2 compares the software company's price, market share, and profit under the four compatibility situations.

Proposition 2 *Under Assumption 1 and given the optimal subsidies in favor of OSS users, the following hold:*

(i) $p^{CC*}\big|_{s=s^{CC*}} = p^{CI*}\big|_{s=s^{CI*}} > p^{IC*}\big|_{s=s^{IC*}} = p^{II*}\big|_{s=s^{II*}}$;
(ii) $N_{PS}^{II*}\big|_{s=s^{II*}} > N_{PS}^{CI*}\big|_{s=s^{CI*}} > N_{PS}^{CC*}\big|_{s=s^{CC*}} > N_{PS}^{IC*}\big|_{s=s^{IC*}}$;
(ii) $\Pi^{II*}\big|_{s=s^{II*}} > \Pi^{CI*}\big|_{s=s^{CI*}} > \Pi^{CC*}\big|_{s=s^{CC*}} > \Pi^{IC*}\big|_{s=s^{IC*}}$.

Proof. See **Table 1** ■

The price of the proprietary software is higher when PS users can access the open source community and get network externalities from OSS users (CC or CI regime). In this case, consumers are willing to pay more for the proprietary software and the software company can take advantage of it to increase its price. However, the ranking of the four regimes is different for profitability. The firm is better under full incompatibility: it enjoys a larger market share and profit. The second best situation is OSS compatibility in which users are charged a higher price for the PS and receive more subsidies for the OSS compared to the full incompatibility case. The result is a lower market share for the proprietary software and profit than under full incompatibility. The worst situation for the software firm (in terms of market share and profit) is PS compatibility because users get more utilities to adopt OSS. The firm has to be more aggressive in its pricing but it is not sufficient to retain its consumers given the subsidies distributed by the government. The situation of full compatibility is between the OSS and PS compatibility cases. The price of the PS (and the subsidy per OSS user) under full compatibility is the same as under OSS compatibility, but its market share is lower because the firm has no exclusive advantage in terms of network externality under full compatibility. Its product is less attractive than under PS compatibility. It implies that if the proprietary software firm has the possibility to choose the compatibility regime, it is incited to deny access to its services and customer base (by making its software incompatible for OSS users) .

Finally, Proposition 3 compares the fiscal burden of these subsidies.

Proposition 3 *Under Assumption 1, the total cost of subsidizing OSS under the four compatibility regimes satisfy* $S^{CC*} > S^{CI*} > S^{IC*} > S^{II*}$

Proof. See **Appendix.** ■

Subsidizing open source is more costly when PS and OSS are fully compatible because of the large number of beneficiaries. A lot of users switch from PS to OSS as a result of the subsidies policy. The full incompatibility situation is the least costly for the government as the subsidy per user is lower than in the CC and CI regimes and the number of beneficiaries is limited.

4.2 Comparison of Welfare Levels

Finally, we compare social welfare under the four compatibility situations. The computation of the welfare with and without subsidies is detailed in the Appendix.

Proposition 4 *When the government only maximizes the surplus of users, then :*

(*i*) $W^k\big|_{s=s^*} - W^k\big|_{s=0} > 0$ for any $k = II, CI, CC, IC$

(*ii*) $W_U^{CC*}\big|_{s=s^{CC*}} > W_U^{IC*}\big|_{s=s^{IC*}} > W_U^{CI*}\big|_{s=s^{CI*}} > W_U^{II*}\big|_{s=s^{II*}}$

Proof. See **Appendix.** ■

The intervention of the government in favor of OSS is always welfare enhancing. But the welfare with subsidies is larger when the proprietary and OSS are perfectly compatible. In this situation, the welfare impact of subsidies is to stimulate competition and push the firm to reduce its price. It also increases the reach of the high quality software (quality effects) . The worst situation in terms of welfare is full incompatibility. Even if subsidies intensify competition, consumers' surplus is lower as network externalities are only intra-network. The intermediate situations are one-way compatibility regime with PS compatibility outranking OSS compatibility. When the OSS users benefit from unilateral network effects (PS compatibility), consumers are more likely to adopt the OSS that offers superior quality and extended network externality. Subsidies can reinforce the attraction of open source product and the utility of OSS users (through quality and network effects).

In the Appendix, we have examined the situation in which the government takes into account the firm's profit. When the government objective is the sum of consumers' surplus and firm's profit, then public subsidies for OSS users have negative impact on welfare regardless of the compatibility regime and the best policy is laissez-faire or "public neutrality".

5 Concluding remarks

Although the open source literature has extensively studied the issue of competition between open source and proprietary software, less attention has been paid to the role of public policy to promote open source software. The aim of this paper is to study the impact of public subsidies for OSS users in presence of network effects and under different compatibility regimes.

Our main findings are that public subsidies push down the price of proprietary software, increase the market share of the OS software and may stimulate network externality when PS and OSS are partially incompatible (PS one-way compatibility). When only users' welfare is taken into account and the adoption cost of OSS is sufficient high, public subsidies for OSS' users are welfare-enhancing. However, the optimal policy is to provide larger subsidies per user under full compatibility and OSS (one-way) compatibility than under full incompatibility and PS (one-way) compatibility.

We have also examined the optimal policy when the government maximizes the total welfare (including the firm's profit). In this case, subsidizing OSS is not socially desirable regardless of the regime of software compatibility. This result is similar to that obtained by Comino and Manenti (2003) with a model of horizontal differentiation and can be used as an argument in favor of a "technology neutrality" (meaning that a government should never intervene to sponsor a technology, but let the market choose the best technologies.

Our theoretical model has several limitations and possible extensions. First, we consider that the quality of OSS and PS is exogenous. It would be interesting to add a stage in which the OS community and the software company can invest in the quality of their software. Moreover, the choice of compatibility could also be endogenized. Our results suggest that the software company has strong incentives to make its product incompatible with the OSS. Another limitation is that our model is static and does not allow for intertemporal pricing strategies. Instead we could consider two periods and two generations of potential users. In the initial period, the software company could be more aggressive to get a critical mass of users and obtain competitive advantage (through network externality) in the second period. In this dynamic setting, optimal public subsidies could clearly be different over time.

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Appendix

Proof of proposition 3

As $S^{CC*}=\frac{(c-\Delta)(2c+\Delta)}{9c}$, $S^{CI*}=\frac{(c-\Delta)(2c-3\gamma+\Delta)}{9(c-\gamma)}$, $S^{IC*}=\frac{(c-\gamma-\Delta)(2c-2\gamma+\Delta)}{9(c-\gamma)}$ and $S^{II*}=\frac{(c-\gamma-\Delta)(2c-5\gamma+\Delta)}{9(c-2\gamma)}$, it can be shown that $S^{CC*}>S^{CI*}>S^{IC*}>S^{II*}$ since under **Assumption 1,** $S^{CC*}-S^{CI*}=\frac{1}{9c}\gamma\frac{(c-\Delta)^2}{c-\gamma}>0$, $S^{CI*}-S^{IC*}=\frac{1}{9}\frac{\gamma}{c-\gamma}\left(c+2\Delta-2\gamma\right)>0$ and $S^{IC*}-S^{II*}=\frac{1}{9}\frac{\gamma(\Delta-c+\gamma)^2}{c(c-3\gamma)+2\gamma^2}>0$

Proof of proposition 4

When the government only cares about the surplus of users, the values of welfare (excluding firm's profit) under the four compatibility regimes are

$$W_U^{*II} = \frac{(7c\gamma+4cV_{OS}+2cV_{PS}+\Delta^2-5\gamma^2-10\gamma V_{OS}-2\gamma V_{PS}-2c^2)}{6\left(c-2\gamma\right)} \tag{A.1}$$

$$W_U^{*CC} = \frac{(4cV_{OS}+2cV_{PS}+6\gamma c+\Delta^2-2c^2)}{6c} \tag{A.2}$$

$$W_U^{*CI} = \frac{(6c\gamma+4cV_{OS}+2cV_{PS}+\Delta^2-3\gamma^2-6\gamma V_{OS}-2c^2)}{6\left(c-\gamma\right)} \tag{A.3}$$

$$W_U^{*IC} = \frac{(7c\gamma+4cV_{OS}+2cV_{PS}+\Delta^2-5\gamma^2-4\gamma V_{OS}-2\gamma V_{PS}-2c^2)}{6\left(c-\gamma\right)} \tag{A.4}$$

We compare welfare levels across the four compatibility regimes if **Assumption 1** ($c>\Delta+\gamma$ and $\Delta>\gamma$) holds.

We start by comparing CC and CI. $\Delta W_{U(CC,CI)}\equiv W_U^{CC*}-W_U^{IC*}=\frac{1}{6c}\frac{\gamma\left(c^2-\gamma c-\Delta^2\right)}{c-\gamma}$, and the sign of this expression is given by the sign of the numerator. The two roots of this quadratic function are $c^1_{(CC,IC)}=\frac{1}{2}\gamma-\frac{1}{2}\sqrt{4\Delta^2+\gamma^2}$ and $c^2_{(CC,IC)}=\frac{1}{2}\gamma+\frac{1}{2}\sqrt{4\Delta^2+\gamma^2}$, with $c^1_{(CC,IC)}<c^2_{(CC,IC)}$. We can check that $c^2_{(CC,IC)}<(\Delta+\gamma)$ as $\frac{1}{2}\gamma+\frac{1}{2}\sqrt{4\Delta^2+\gamma^2}-(\Delta+\gamma)=\frac{\sqrt{4\Delta^2+\gamma^2}-\gamma-2\Delta}{2}\iff\frac{4\Delta^2+\gamma^2-(\gamma+2\Delta)^2}{2}=-2\Delta\gamma<0$. It implies that $\Delta W_{U(CC,CI)}>0$.

We have $\Delta W_{U(IC,CI)}\equiv W_U^{IC*}-W_U^{CI*}=\frac{1}{6}\frac{\gamma}{c-\gamma}\left(c+2\Delta-2\gamma\right)>0$ since we have assumed that $\Delta>\gamma$

Finally $\Delta W_U|_{(CI,II)}\equiv W_U^{CI*}-W_U^{II*}=\frac{1}{6}\frac{\gamma\left(c^2-3c\gamma-\Delta^2+2\Delta\gamma+\gamma^2\right)}{c^2-3c\gamma+2\gamma^2}$. The denominator is always positive. Moreover the two roots of the numerator are $c^1_{(CI,II)}=\frac{3\gamma-\sqrt{4\Delta^2+5\gamma^2-8\Delta\gamma}}{2}$ and $c^2_{(CI,II)}=\frac{3\gamma+\sqrt{4\Delta^2+5\gamma^2-8\Delta\gamma}}{2}$, with $c^1_{(CI,II)}<c^2_{(CI,II)}.<\Delta+\gamma$.This implies that $\Delta W_U|_{(CI,II)}>0$

Consequently, $W_U^{CC*}>W_U^{IC*}>W_U^{CI*}>W_U^{II*}$ if **Assumption 1** holds.

Without public subsidies ($s = 0$), we have[11].

$$W^{II}\big|_{s=0} = \frac{(2c\Delta - 7\Delta^2 + 10c\gamma - 7\gamma^2 - 2\Delta\gamma - 3c^2 + 4c\,(V_{OS} + V_{PS}) - 12\gamma V_{OS} - 4\gamma V_{PS})}{8\,(c - 2\gamma)} \tag{B.1}$$

$$W^{CC}\big|_{s=0} = \frac{(2c\Delta + 8c\gamma + \Delta^2 - 3c^2 + 4c\,(V_{OS} + V_{PS}))}{8c} \tag{B.2}$$

$$W^{CI}\big|_{s=0} = \frac{(2c\Delta - 7\Delta^2 - 4\gamma^2 + 8c\gamma + 4c\,(V_{OS} + V_{PS}) - 8\gamma V_{OS} - 3c^2)}{8\,(c - \gamma)} \tag{B.3}$$

$$W^{IC}\big|_{s=0} = \frac{(2c\Delta - 7\Delta^2 + 10c\gamma - 7\gamma^2 - 2\Delta\gamma - 3c^2 + 4\,(c - \gamma)\,(V_{OS} + V_{PS}))}{8\,(c - \gamma)} \tag{B.4}$$

and we can easily check that $W^k\big|_{s=s^*} - W^k\big|_{s=0} > 0$ for any $k = II, CI, CC, IC$

Table with equilibrium solutions when the government takes into account both users' surplus and firm's profits

Table 2 displays the total welfare-maximizing subsidies and the equilibrium price, market share and profit

Table 2: Equilibrium solutions when the government maximizes the total welfare (*n.c.* necessary conditions for an active duopoly)

	CC	II	CI	IC
p^*	$c - \Delta$	$c - \gamma - \Delta$	$c - \Delta$	$c - \gamma - \Delta$
N^*_{PS}	$\frac{c-\Delta}{c}$	$\frac{c-\gamma-\Delta}{c-2\gamma}$	$\frac{c-\Delta}{c-\gamma}$	$\frac{c-\gamma-\Delta}{c-\gamma}$
N^*_{OS}	$\frac{\Delta}{c}$	$\frac{\Delta-\gamma}{c-2\gamma}$	$\frac{(\Delta-\gamma)}{c-\gamma}$	$\frac{\Delta}{c-\gamma}$
s^*	$-c + \Delta$	$-c + \gamma + \Delta$	$-c + \Delta$	$-c + \gamma + \Delta$
Π^*	$\frac{(c-\Delta)^2}{c}$	$\frac{(c-\gamma-\Delta)^2}{c-2\gamma}$	$\frac{(c-\Delta)^2}{c-\gamma}$	$\frac{(c-\gamma-\Delta)^2}{c-\gamma}$
W^*	$\frac{\Delta^2+2cV_{PS}+2c\gamma}{2c}$	$\frac{c\gamma+\Delta^2+2cV_{PS}-\gamma^2-2\gamma\Delta}{2(c-2\gamma)}$	$\frac{2c\gamma+\Delta^2+2cV_{PS}-\gamma^2-2\gamma V_{OS}}{2c-2\gamma}$	$\frac{2cV_{PS}+c\gamma+\Delta^2-\gamma^2-2\gamma V_{PS}}{2(c-\gamma)}$
$n.c.$	$c > \Delta$	$c > \Delta + \gamma$ and $\Delta > \gamma$	$c > \Delta$	$c > \Delta + \gamma$

Regardless of the compatibility regime, the government prefers to tax OSS users rather than subsidize them. It implies that public subsidies for OSS users are not desirable when the government maximizes the total welfare (including the firm's profit).

[11] Calculations are available on request