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Source-Dependence of Utility and Loss Aversion: A Critical Test of Ambiguity Models

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Source-dependence of utility and loss aversion:

A critical test of ambiguity models

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Abstract:

This paper tests whether utility is the same for risk and for uncertainty. This test is critical for models that capture ambiguity aversion through a difference in event weighting between risk and uncertainty, like the multiple priors models and prospect theory. We present a new method to measure utility and loss aversion under uncertainty without the need to introduce simplifying parametric assumptions. Our method extends Wakker and Deneffe's (1996) trade-off method by allowing for standard sequences that include gains, losses, and the reference point. It provides an efficient way to measure loss aversion and a useful tool for practical applications of ambiguity models. We could not reject the hypothesis that utility and loss aversion were the same for risk and uncertainty, suggesting that utility primarily reflects attitudes towards outcomes. Utility was S-shaped, concave for gains and convex for losses and there was substantial loss aversion. Our findings support models that explain ambiguity aversion through a difference in event weighting and suggest that descriptive ambiguity models should allow for reference-dependence of utility.

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Keywords: prospect theory; loss aversion; utility for gains and losses; probability distortion; decision analysis; risk aversion

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1. Introduction

An extensive amount of empirical work, originating from Ellsberg's (1961) famous thought experiment, shows that people are not neutral towards ambiguity, as assumed by subjective expected utility. New models have been proposed to explain these ambiguity attitudes. Broadly speaking, these ambiguity models can be subdivided into two classes. The first class models ambiguity aversion through a difference in utility between risk (known probabilities) and uncertainty (unknown probabilities). The best-known model of this class is the smooth ambiguity model of Klibanoff et al. (2005). Other models that belong to this class were proposed by Nau (2006), Chew et al. (2008), Seo (2009), and Neilson (2010). The second class of models assumes that utility does not depend on the source of uncertainty and is the same for risk and uncertainty. Instead, ambiguity aversion is modeled through a difference in event weighting. This class includes the multiple priors models (Gilboa and Schmeidler 1989, Jaffray 1989, Ghirardato et al. 2004) and modifications thereof (Gajdos et al. 2008, Maccheroni et al. 2006), vector expected utility (Siniscalchi 2009), Choquet expected utility (Gilboa 1987, Schmeidler 1989), and prospect theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992).

This paper tests whether utility is source-independent and the same for risk and uncertainty. We assume a general utility model, previously suggested by Miyamoto (1988), Luce (1991), and Ghirardato and Marinacci (2001), that includes most of the ambiguity models of the second class as special cases, and generalize it to include sign-dependence to also cover prospect theory. We tested the central condition underlying this model and obtained support for it. We measured utility for gains and for losses and also measured loss aversion. Previous evidence suggests that the distinction between gains and losses is relevant because ambiguity attitudes differ between gains and losses (e.g. Cohen et al. 1987, Hogarth and Kunreuther 1989, Abdellaoui et al. 2005, Du and Budescu 2005) and loss aversion is crucial in explaining attitudes towards both risk (Rabin 2000) and ambiguity (Roca et al. 2006).

Measuring loss aversion is complex, in particular if event weighting may be different for gains and losses. Previous measurements of loss aversion sidestepped this problem by introducing simplifying assumptions. We introduce a new method to measure loss aversion that imposes no simplifying assumptions and requires no complete measurement of utility. It can easily be applied, which may encourage the use of ambiguity models in decision analysis. Our method extends the trade-off method of Wakker and Deneffe (1996) by allowing standard sequences (sequences of outcomes for which the utility difference between successive elements is constant) to pass through the reference point. Our method also simplifies the axiomatization of ambiguity models as there is a close connection between measurements of utility using the trade-off method and preference conditions (Köbberling and Wakker 2003).

Our experimental data contain two messages. First, they provide support for models that explain ambiguity aversion through a difference in event weighting. We could not reject the hypothesis that utility and loss aversion were the same for risk and uncertainty. This suggests that utility is source-independent and primarily reflects attitudes towards outcomes.

The second message is that descriptive ambiguity models should allow for reference-dependence of utility. We obtained clear evidence that utility differed for gains and losses and there was sizeable loss aversion. Most ambiguity models do not allow for reference-dependence and assume that ambiguity attitudes are the same for gains and losses. This assumption may be adequate for normative purposes, but, as our data clearly show, does not match behavior.

2. Background

2.1. Binary prospect theory

Consider a decision maker who has to make a choice in the face of uncertainty. Uncertainty is modeled through a *state space* S. Exactly one of the states will obtain, but the decision maker does not know which one. Subsets E of S are called *events* and E^c denotes the complement of E.

Acts map states to outcomes. Outcomes are money amounts and more money is preferred to less. In our measurements, we will only use two-outcome acts $x_E y$, signifying that the decision maker obtains € x if event E occurs and € y otherwise. If probabilities are known, we will write $x_p y$ for the act that pays € x with probability p and € y with probability $1 - p$. We will refer to $x_E y$ as an *uncertain act* (meaning that probabilities are unknown) and to $x_p y$ as a *risky act* (meaning that probabilities are known).

We use conventional notation to express the preference of the decision maker, letting $\succ$, $\succcurlyeq$, and $\backsim$ represent strict preference, weak preference, and indifference. Preferences are defined relative to a reference point x_0. *Gains* are outcomes strictly preferred to x_0 and *losses* are outcomes strictly less preferred than x_0. An act is *mixed* if it involves both a gain and a loss. For mixed acts the notation $x_E y$ signifies that x is a gain and y is a loss. A *gain act* involves no losses (i.e. both x and y are nonnegative) and a *loss act* involves no gains. For gain and loss acts the notation $x_E y$ signifies that the absolute value of x exceeds the absolute value of y, i.e. if x and y are gains then $x \geq y$ and if x and y are losses then $x \leq y$.

Under *binary prospect theory (PT)* the decision maker's preferences over mixed acts $x_E y$ are evaluated by:

$$W^+(E)U(x) + W^-(E^c)U(y), \tag{1a}$$

and preferences over gain or loss acts by:

$$W^i(E)U(x) + \left(1 - W^i(E)\right)U(y), \tag{1b}$$

where $i = +$ for gains and $i = -$ for losses. U is a strictly increasing, real-valued *utility function* that satisfies $U(x_0) = 0$. The utility function is a ratio scale and we can choose the utility of one outcome other than the reference point. U is an overall utility function that includes loss aversion. In empirical applications U is often decomposed in a basic utility function, capturing the decision maker's attitudes towards final outcomes, and a loss aversion coefficient λ capturing attitudes towards gains and losses (Sugden 2003, Köbberling and Wakker 2005, Köszegi and Rabin 2006). Our method does not require this decomposition.

The *event weighting functions* $W^i, i = +, -$, assign a number $W^i(E)$ to each event E such that

(i) $W^i(\emptyset) = 0$

(ii) $W^i(S) = 1$

(iii) W^i is *monotonic*: $E \supset F$ implies $W^i(E) \geq W^i(F)$.

The event weighting functions W^i depend on the sign of the outcomes and may be different for gains and losses. They need not be additive. For gains, binary PT contains most transitive ambiguity models as special cases, as was pointed out by Miyamoto (1988), Luce (1991), and Ghirardato and Marinacci (2001). The ambiguity models only differ when the number of outcomes is at least three. Equations (1a) and (1b) represent the extension of these models to include sign-dependence.

Binary PT evaluates mixed risky acts $x_p y$ as

$$w^+(p)U(x) + w^-(1-p)U(y) \tag{2a}$$

and gain and loss risky acts $x_p y$ as

$$w^i(p)U(x) + \left(1 - w^i(p)\right)U(y), i = +, -. \tag{2b}$$

w^i is a strictly increasing *probability weighting function* that satisfies $w^i(0) = 0$ and $w^i(1) = 1$ and again may differ between gains and losses. Hence, in the evaluation of risky acts the event weighting functions W^i are replaced by probability weighting functions w^i. Binary PT assumes that utility is the same for risk and uncertainty. Ambiguity aversion is modeled through a difference between W^i and w^i.

2.2. Previous evidence

Tversky and Kahneman (1992) assumed that utility differs between gains and losses and is S-shaped, concave for gains and convex for losses. In addition, they assumed that utility is steeper for losses than for gains, reflecting loss aversion. Nearly all the empirical evidence on utility comes from decision under risk. There is much evidence that utility for gains is indeed concave (Wakker 2010). For losses the evidence is more equivocal. While most studies found convex utility, some have also found linear or concave utility (e.g. Bruhin et al. 2010). The utility for losses was usually closer to linear than the utility for gains.

Empirical evidence on utility under uncertainty is scarce. Abdellaoui et al. (2005) measured utility under uncertainty and confirmed that it was concave for gains and slightly convex for losses. Their parametric estimates were close to those previously obtained under risk, but they did not directly measure utility under risk. Abdellaoui et al. (2011) and Vieider et al. (2013) measured utility under risk and under uncertainty for small stakes and under parametric assumptions about utility. They found that utility was linear both for risk and for uncertainty. This finding might be due to the small stakes used in these studies: for small stakes utility is usually close to linear (Wakker 2010).

Nearly all empirical measurements of loss aversion made simplifying assumptions about utility and probability weighting, typically assuming linear utility and either ignoring probability weighting (Pennings and Smidts 2003, Booij and van de Kuilen 2009, Baltussen et al. 2012)[1] or assuming equal weighting for gains and losses (Gaechter et al. 2007). The exception is Abdellaoui et al. (2007) who imposed no simplifying assumptions on either probability weighting or utility. However, they measured loss aversion in decision under risk only and their method is not applicable in decision under uncertainty.

Most studies found loss aversion coefficients around 2, meaning that losses weight approximately twice as much as absolutely commensurate gains (Booij et al. 2010). A difficulty in comparing the results of these studies is that they not only made different parametric assumptions, but also adopted different definitions of loss aversion.

[1]Booij and van de Kuilen (2009) tested for the robustness of their findings by using probability weights estimated in other studies.

Finally, even though binary PT is consistent with much of the empirical data that has been collected on decision under risk and uncertainty and includes many ambiguity models as special cases, there is some evidence challenging it. For example, Starmer and Sugden (1993) and Birnbaum (2008) reported event-splitting effects that violate binary PT and Birnbaum and Bahra (2007) and Wu and Markle (2008) obtained violations of binary PT for mixed acts. We, therefore, included a test of the main condition underlying binary PT in our experiment. This test is explained below.

3. Measurement method

Our method for measuring utility and loss aversion consists of three stages and is summarized in Table 1. In the first stage, a gain and a loss are elicited that connect utility for gains (measured in the second stage) with utility for losses (measured in the third stage). The measurements in the second and in the third stage employ the trade-off method of Wakker and Deneffe (1996). Within each domain, we determine a *standard sequence* of outcomes such that the utility difference between successive elements of the sequence is constant. The trade-off method is commonly used in decision theory (Wakker 2010), but thus far it could only be used to measure utility for gains and utility for losses separately. It could not be used to measure loss aversion, which requires that the utility for gains and the utility for losses can be compared. Our method allows measuring utility for gains and utility for losses jointly and, consequently, it permits the measurement of loss aversion. In all the derivations presented below we imposed no parametric assumptions on utility and the weighting functions W^i and $w^i, i = +, -$. Hence, our method is parameter-free.

Table 1: Three-stage procedure to measure utility

The third column shows the quantity that was assessed in each of the three stages of the procedure. The fourth column shows the indifference that was elicited. The fifth column shows the stimuli used in the experiment. ℓ_{alt} and $k_{L_{alt}}$ were used to test for consistency (see Section 4 for explanation).

		Assessed quantity	Indifference	Choice variables
Stage 1		L	$G_E L \sim x_0$	$G =$ €2000 $E =$ color of a ball drawn from an unknown Ellsberg urn, $p =$ ½
		x_1^+	$x_1^+ \sim G_E x_0$	
		x_1^-	$x_1^- \sim L_{E^c} x_0$	
Stage 2	Step 1	$\mathcal{L}$	$x_1^+ {}_E \mathcal{L} \sim \ell_{E^c} x_0$	$\ell = -€300$; $k_G = 6$ $\ell_{alt} = €0$; $k_{G_{alt}} = 3$
	Step 2 to k_G	x_j^+	$x_j^+ {}_E \mathcal{L} \sim x_{j-1}^+ {}_E \ell$	
Stage 3	Step 1	$\mathcal{G}$	$\mathcal{G}_E x_1^- \sim \mathscr{g}_E x_0$	$\mathscr{g} = €300$; $k_L = 6$
	Step 2 to k_L	x_j^-	$\mathcal{G}_E x_j^- \sim \mathscr{g}_E x_{j-1}^-$	

3.1 First stage: elicitation of the gauge outcomes

We start by selecting an event E that will be kept constant throughout the first stage and a gain G. Then we elicit the loss L for which $G_E L \sim x_0$. It follows from equation (1a) that:

$$W^+(E)U(G) + W^-(E^c)U(L) = U(x_0) = 0. \tag{3}$$

We next elicit certainty equivalents x_1^+ and x_1^- such that $x_1^+ \sim G_E x_0$ and $x_1^- \sim L_{E^c} x_0$. The indifference $x_1^+ \sim G_E x_0$ implies that

$$U(x_1^+) = W^+(E)U(G). \tag{4}$$

The indifference $x_1^- \sim L_{E^c} x_0$ implies that

$$U(x_1^-) = W^-(E^c)U(L). \tag{5}$$

Combining Eqs. (3)– (5) gives

$$U(x_1^+) = -U(x_1^-). \tag{6}$$

Equation (6) defines the first elements x_1^+ and x_1^- of the standard sequences for gains and losses that we will construct in the second and third stages.

For choice under risk, the elicitation of x_1^+ and x_1^- is similar except that the event E is replaced by a known probability p, and that the weights $W^+(E)$ and $W^-(E^c)$ are replaced by $w^+(p)$ and $w^-(1-p)$, respectively.

3.2 Second and third stage: elicitation of utility for gains and losses

In the second stage, we elicit a standard sequence of gains. Let ℓ be a prespecified loss. We first elicit the loss $\mathcal{L}$ such that the decision maker is indifferent between the acts $x_1^+{}_E\mathcal{L}$ and $\ell_{E^c} x_0$, where x_1^+ is the gain that was elicited in the first stage. We could take an event E' different from the event E used in the first stage, but, for simplicity, we used in our experiment the same event in all three stages. The indifference $x_1^+{}_E\mathcal{L} \sim \ell_{E^c} x_0$ implies that

$$W^+(E)U(x_1^+) + W^-(E^c)U(\mathcal{L}) = W^-(E^c)U(\ell). \tag{7}$$

Rearranging Eq. (7) and using $U(x_0) = 0$ gives,

$$U(x_1^+) - U(x_0) = \frac{W^-(E^c)}{W^+(E)}\big(U(\ell) - U(\mathcal{L})\big). \tag{8}$$

Next, we elicit the gain x_2^+ such that $x_2^+{}_E\mathcal{L} \sim x_1^+{}_E\ell$. From this indifference we obtain after rearranging

$$U(x_2^+) - U(x_1^+) = \frac{W^-(E^c)}{W^+(E)}\big(U(\ell) - U(\mathcal{L})\big). \tag{9}$$

Combining Eqs. (8) and (9) gives :

$$U(x_2^+) - U(x_1^+) = U(x_1^+) - U(x_0). \tag{10}$$

We proceed by eliciting a series of indifferences $x_j^+{}_E\mathcal{L} \sim x_{j-1}^+{}_E\ell, j = 2, \dots, k_G$, to obtain the sequence $\{x_0, x_1^+, x_2^+, \dots, x_{k_G}^+\}$. It is easy to see that for all j, $U(x_j^+) - U(x_{j-1}^+) = U(x_1^+) - U(x_0)$. For decision under risk, we apply the above procedure with the event E replaced by a probability p.

The standard sequence of losses is constructed similarly. We select a gain g and an event E and elicit the gain $\mathcal{G}$ such that $\mathcal{G}_E x_1^- \sim g_E x_0$.[2] We then proceed to elicit a standard sequence $\{x_0, x_1^-, x_2^-, \dots, x_{k_L}^-\}$ by eliciting a series of indifferences $\mathcal{G}_E x_j^- \sim g_E x_{j-1}^-$, $j = 2, \dots, k_L$. For risk, we replace the event E by a probability p.

By combining the second and the third stages we have elicited a sequence $\{x_{k_L}^-, \dots, x_1^-, x_0, x_1^+, \dots, x_{k_G}^+\}$ that runs from the domain of losses through the reference point to the domain of gains and for which the utility difference between successive elements is constant. We can scale utility by selecting the utility of an arbitrary element. In the analyses reported below, we set $U(x_{k_G}^+) = 1$ from which it follows that $U(x_j^+) = j/k_G$ for $j = 1, \dots, k_G$, and $U(x_j^-) = -j/k_G$, for $j = 1, \dots, k_L$.

4. Experiment

4.1 Experimental set-up

Subjects were 75 economics students of the Erasmus School of Economics, Rotterdam (29 female, mean age of 20.7 years). Each subject was paid a flat fee of €10 for participation in the experiment. Before conducting the actual experiment, the experimental protocol was tested in several pilot sessions.

The experiment was run on computers. Subjects answered the questions individually in sessions of

[2] Again, we could have selected an event E'' different from the events used in the first two stages, but we used the same event in our experiment.

at most two subjects. They first received instructions about the tasks and then completed five training questions. Subjects were told that there were no right or wrong answers and that they should go through the experiment at their own pace. They were instructed to approach the experimenter if they needed any advice concerning the experiment. A session lasted 40 minutes on average.

The order in which utility under risk and uncertainty were measured was randomized between sessions. When a subject had completed the first part of the experiment, the experimenter would approach her to explain the next part. Within the risk and uncertainty elicitations, the second and third stage were also randomized; some subjects started with the elicitation of the gain sequence, others with the elicitation of the loss sequence. The first stage always had to come first because it served as an input for the other two stages.

We used sizeable monetary amounts because we were interested in studying both utility curvature and loss aversion. Utility is approximately linear over small intervals (Wakker and Deneffe 1996) and we feared that it would be hard to detect differences between utility under risk and uncertainty for small stakes. Given that substantial losses were involved, all choices were hypothetical. It is impossible to find subjects willing to participate in an experiment where they can lose substantial amounts of money. We will provide a more detailed discussion of the use of incentives in the Discussion Section.

We did not directly ask subjects for their indifference values, but, instead, used a series of binary choice questions to zoom in at them. Examples of such a zooming-in process can be found in Table A1 in the appendix. We applied a choice-based elicitation procedure as previous research suggests that it leads to more reliable results than directly asking for indifference values (Bostic et al. 1990, Noussair et al. 2004).

4.2 Details

To perform the elicitation described in Section 3, we had to specify a number of parameters, which are depicted in the final column of Table 1. We made the common assumption that the reference point x_0 was equal to 0. In the risk condition, the outcome of an act was determined by drawing a ball from an urn containing five red balls and five black balls. Subjects could state which color they preferred to bet on with the chance of winning always equal to 50 percent. In the uncertainty condition, the outcome of an act was determined by drawing a ball from an urn containing ten balls, which were either red or black in unknown proportions. Again, subjects could select the color they preferred to bet on.

Both for gains and for losses, we elicited six points of the utility function under both risk and uncertainty. Next to these elicitations, we performed a second smaller sequence in the domain of gains, varying the gauge amount ℓ. By definition ℓ needs to be smaller or equal to x_0. In the main elicitation we set $\ell = -€300$. Asking the question whether the elicited amounts would depend on the value of ℓ, we also elicited x_2^+ and x_3^+ using an alternative gauge amount $\ell_{alt} = €\,0$. Under binary PT the elicitations of x_2^+ and x_3^+ should not depend on the selected value of ℓ. This second elicitation was meant to test sign-comonotonic trade-off consistency (Köbberling and Wakker 2003), the central condition underlying binary PT.

Figures A1-A3 in the appendix show the displays used under uncertainty. The screens under risk were similar, except that the two branches would simply say 50% rather than “Red” or “Black”. Figure A1 displays the typical decision that subject had to make. Subjects were faced with a choice between two acts denoted as options A and B. They could not state indifference. By choosing between the two acts, the subject narrowed down the interval in which her indifference value should fall.

After narrowing down the interval thrice, we presented subjects with a scrollbar (Figure A2). The scrollbar allowed subjects to specify their indifference value up to $€1$ precision. The starting point of the scrollbar was in the middle of the interval determined by their previous choices. The range of the scrollbar was wider than this interval, so that subjects could correct any mistakes they might have made. The data on the use of the scrollbar also give an indication of the quality of the data. If many subjects would provide answers that did not align with their previous choices, possibly even violating stochastic dominance, this might signal poor understanding of the task. After specifying a value with the scrollbar, subjects were asked to confirm their choice (Figure A3). If they cancelled their choice, the process started over. If subjects confirmed their choice, they moved on to the next elicitation.

We included a number of repetitions to test for consistency. First, in each of the six standard sequences (the short and the long gain sequences and the loss sequence for both risk and uncertainty), we repeated the second-to-last iteration in the elicitation of $x_2^i, i = +,-$. Repeating the second-to-last iteration is a strong test of consistency, as subjects were probably close to indifference at the end of the iteration process. Furthermore, at the end of eliciting the long gain sequence, we elicited x_4^+ again, for both risk and uncertainty. Together, these repetitions and the way in which subjects used the scrollbar allowed us to gain insight into the quality of the data.

4.3 Analyses

4.3.1 Analyses of utility curvature

Two different methods were used to investigate utility curvature.[3] For the first, nonparametric, method, we calculated the area under the utility function. The domain of U was normalized to $[0,1]$, by transforming every gain x_j^+ to the value x_j^+/x_6^+ and every loss x_j^- to x_j^-/x_6^-.[4] If utility is linear, the area under this normalized curve equals ½. For gains, we consider utility to be convex [concave] if the area under the curve is smaller [larger] than ½. For losses, utility is considered to be convex [concave] if the area under the curve is larger [smaller] than ½.

We also analyzed the utility function by parametric estimation. We employed the power family, x^α, as it is the most commonly employed parametric family. For gains [losses] $\alpha > 1$ corresponds to convex [concave] utility, $\alpha = 1$ corresponds to linear utility, and $\alpha < 1$ corresponds to concave [convex] utility. Estimation was done using nonlinear least squares. To test for robustness, we also performed a mixed-effects estimation in which each individual parameter was estimated as the sum of a fixed effect, common to all subjects, and an individual-specific random effect. The results were similar. A potential problem in estimating a model like binary PT using nonlinear least squares is collinearity between utility and the event weights, which implies that the obtained estimates may not be uniquely identified. The trade-off method avoids this problem by keeping event weighting fixed, while eliciting utility and, hence, the obtained estimates are uniquely identified.

4.3.2 Loss aversion

In the literature, loss aversion has been defined in a multitude of ways. Abdellaoui et al. (2007) concluded that the definitions proposed by Kahneman and Tversky (1979) and Köbberling and Wakker (2005) were empirically most useful, and we will use these. Other definitions (Wakker and Tversky 1993, Bowman et al. 1999, Neilson 2002) turned out to be too strict for empirical purposes, leaving many subjects unclassified.

Kahneman and Tversky (1979) defined loss aversion as $-U(-x) > U(x)$ for all $x > 0$. To measure loss aversion coefficients, we computed $-U(-x_j^+)/U(x_j^+)$ and $-U(-x_j^-)/U(x_j^-)$ for $j = 1, \ldots, 6$,

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[3] We also used a third, nonparametric, method based on changes in the slope of utility. This method led to similar conclusions.

[4] One subject violated monotonicity so that x_6^- was not the largest loss. For this subject we transformed losses x_j^- to $x_j^-/\{\min_{i=1,\ldots,6} x_i^-\}$.

whenever possible.[5] Usually $U(-x_j^+)$ and $U(-x_j^-)$ could not be observed directly and had to be determined through linear interpolation. Some subjects occasionally violated stochastic dominance. As a result, their utility was not unique and one amount could have multiple utilities. For these amounts, utility was undefined. A subject was classified as loss averse if $-U(-x)/U(x) > 1$ for all observations, as loss neutral if $-U(-x)/U(x) = 1$ for all observations, and as gain seeking if $-U(-x)/U(x) < 1$ for all observations. To account for response error, we also used more a lenient approach, classifying subjects as loss averse, loss neutral, or gain seeking if the above inequalities held for more than half of the observations.

Köbberling and Wakker (2005) defined loss aversion as the kink of utility at the reference point (Benartzi and Thaler 1995 suggested a similar definition). Formally, they defined loss aversion as $U'_\uparrow(0)/U'_\downarrow(0)$, where $U'_\uparrow(0)$ represents the left derivative and $U'_\downarrow(0)$ the right derivative of U at the reference point. To operationalize this empirically, we computed each subject's coefficient of loss aversion as the ratio of $U(x_1^-)/x_1^-$ over $U(x_1^+)/x_1^+$, because x_1^- and x_1^+ are the loss and gain closest to the reference point. Given that $U(x_1^-) = -U(x_1^+)$, this ratio is equal to $x_1^+/-x_1^-$. Hence, our method immediately gives an approximation of Köbberling and Wakker's (2005) loss aversion coefficient without the need to further measure utility. A subject was classified as loss averse if this ratio exceeded 1, as loss neutral of it was equal to 1, and as gain seeking if it was smaller than 1.

5. Results

Three subjects violated stochastic dominance in the first stage of the measurement procedure. This undermines their subsequent answers and they were removed from the analyses. For the remaining 72 subjects, we could determine the entire utility function, for both gains and losses and under both risk and uncertainty. Of these 72 subjects, 14 violated stochastic dominance at least once. Violations of stochastic dominance potentially signal a lower degree of understanding or a lower degree of effort put in the task. We, therefore, also analyzed the data including only the 58 subjects who never violated stochastic dominance, but this led to similar conclusions.

[5] These computations required that $-x_j^+$ was contained in $[x_6^-, 0)$ and $-x_j^-$ in $(0, x_6^+]$.

5.1 Consistency checks

Overall, consistency was satisfactory. Subjects made the same choice in 63.7% of the repeated choices. Reversal rates round ⅓ are common in the literature (Stott 2006). Moreover, our consistency test was strict, as subjects were close to indifference in the repeated choice and, hence, reversals were more likely. There were no differences in consistency between risk and uncertainty.

The correlation between the original measurement and the repeated measurement of x_4^+ was almost perfect. For risk, Kendall's τ was 0.924, for uncertainty it was 0.938.

As a final indication of consistency, we compared whether the final answer provided by using the slider fell within the interval as set up by the bisection procedure. Subjects provided answers that aligned with their original choices. Furthermore, when a subject's final answer was outside the bisection interval, it typically only violated the final choice, probably indicating that they were close to indifference at this point.

5.2 A test of binary PT

As explained in Section 4, we elicited two sequences of gains, a longer one based on $\ell = -€300$, which we use in the main analysis, and a shorter one based on $\ell_{alt} = €0$. If our subjects behaved according to binary PT, then the values of x_2^+ and x_3^+ in the short sequence should be equal to those obtained in the long sequence.

We found support for binary PT, both for risk and for uncertainty. The correlation between the obtained values was substantial. Under risk, Kendall's τ was 0.564 for x_2^+ and 0.518 for x_3^+. Under uncertainty, these values were 0.694 for x_2^+ and 0.625 for x_3^+. All correlation coefficients were different from 0 (P < 0.001). Moreover, for uncertainty, we could not reject the hypotheses that the values of x_2^+ and x_3^+ obtained in the short sequence were equal to those obtained in the long sequence (Wilcoxon test, both P > 0.684). For risk, the values of x_2^+ differed marginally (P = 0.055), but the values of x_3^+ did not differ (P = 0.138). Hence, even though x_3^+ was chained to x_2^+, the marginal difference for x_2^+ did not carry over to x_3^+.

5.3 Ambiguity aversion

The measurement of L and x_1^+ in stage 1 of our method provide insight into subjects' ambiguity attitudes. Let L_r and L_u denote the elicited values of L for risk and uncertainty, respectively. Then,

$2000_{.5}L_r \sim 0$ and $2000_E L_u \sim 0$. A subject is ambiguity averse if $2000_{.5}L_r \succ 2000_E L_r$. By transitivity, $2000_E L_u \succ 2000_E L_r$ and, thus, $L_u > L_r$. This was true for 63.9% of our subjects (Binomial test, $p = 0.024$) and the median elicited value of L_u ($-€612.50$) indeed exceeded the median value of L_r ($-€750$) (Wilcoxon test, *P* = 0.012). Hence, we found evidence of ambiguity aversion in the measurement of L.

Ambiguity aversion also predicts that $x_{1,r}^+$, the value of x_1^+ measured under risk will exceed $x_{1,u}^+$, the value of x_1^+ measured under uncertainty. This follows by transitivity from $x_{1,r}^+ \sim 2000_{.5}0 \succ 2000_E 0 \sim x_{1,u}^+$. However, this was only true for 44.4% of our subjects and we could not reject the hypothesis that x_1^+ was the same for risk and for uncertainty (Wilcoxon test, *P* = 0.807).

5.4 The utility for gains and losses

Figure 1, Panel A displays the utility for gains and losses under risk, based on the median data. Figure 1, Panel B shows the same graph for uncertainty. Taken at face value, the utility functions seem similar. They are consistent with the typical finding of convex utility for losses and concave utility for gains. Furthermore, the utility function appears considerably steeper for losses than for gains, indicating loss aversion.

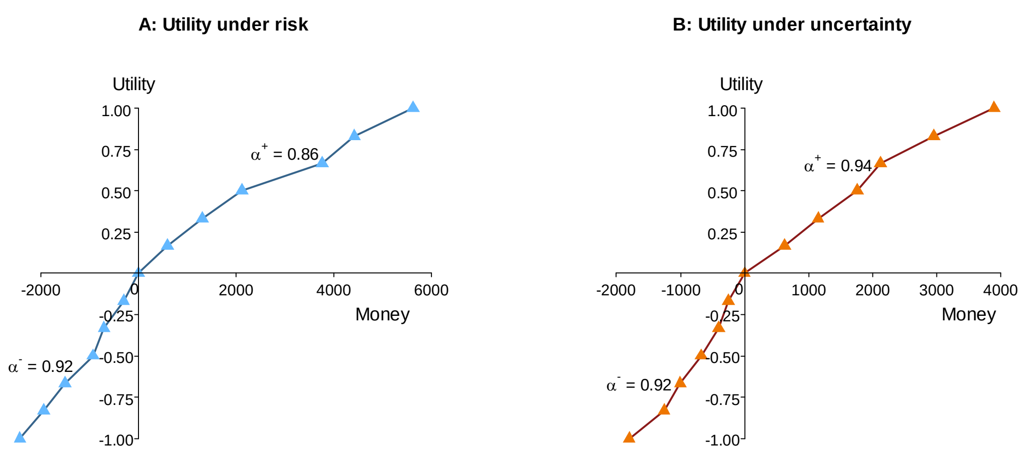


Figure 1: The utility for gains and losses under prospect theory based on the median data. The figure displays the utility for gains and losses under prospect theory based on the median responses of our subjects. $\alpha^+[\alpha^-]$ indicates the estimated power coefficient for gains [losses]. Panel A displays utility under risk. Panel B displays utility under uncertainty.

To investigate these patterns more thoroughly, we move to the individual level analysis. Table 2 shows that the classification of subjects according to the shape of their utility function was very similar for risk and uncertainty and there were no differences in the overall distribution of classifications between conditions (Fisher's exact test, P = 0.943). Utility under risk and uncertainty were related (Kendall's τ = 0.389 for gains and 0.455 for losses, P < 0.001 in both cases) and the common pattern was that of an S-shaped utility function: concave for gains and convex for losses. Less than 20% of the subjects behaved according to the traditional assumption in decision theory that utility is concave throughout.

Table 2: Classification of subjects according to the shape of their utility function

The table classifies the subjects according to the shape of their utility function based on the area under the normalized utility function. Panel A displays the results under risk. Panel B displays the results under uncertainty.

Panel A: Risk

	Losses			
Gains	Concave	Convex	Linear	Total
Concave	13	31	1	45
Convex	15	8	1	24
Linear	2	0	1	3
Total	30	39	3	72

Panel B: Uncertainty

	Losses			
Gains	Concave	Convex	Linear	Total
Concave	13	30	0	43
Convex	18	10	2	30
Linear	1	0	0	1
Total	32	40	2	72

The parametric results confirmed the above conclusions. Table 3 shows the estimated power functions at the individual level. Utility was mostly concave for gains and convex for losses. Under risk, 32 subjects had S-shaped utility. Under uncertainty, this was the case for 30 subjects.

Table 3: Summary of individual parametric fittings of utility for gains

The table depicts the results of fitting power functions on each subject's choices individually. Shown are the median and interquartile range (IQR) for the resulting estimates.

	Risk		Uncertainty	
	Gains	Losses	Gains	Losses
Median	0.857	0.924	0.937	0.898
IQR	[0.616-1.062]	[0.649-1.154]	[0.716-1.188]	[0.675-1.356]

Wilcoxon signed rank tests on these power function estimates indicated that there was no difference in curvature for losses between risk and uncertainty (P = 0.866). There was some indication that utility for gains was more concave under risk (P = 0.027).[6] The power coefficients of utility under risk and under uncertainty were moderately correlated: Kendall's τ was 0.373 for gains and 0.423 for losses.

Figure 2 shows the relationship between individual estimates for the power coefficients under risk and uncertainty. The dashed lines correspond to the case where subjects have exactly the same coefficients in the two domains. Most estimates were relatively close to the dashed lines and there was no strong indication that subjects had different curvature under risk than under uncertainty.

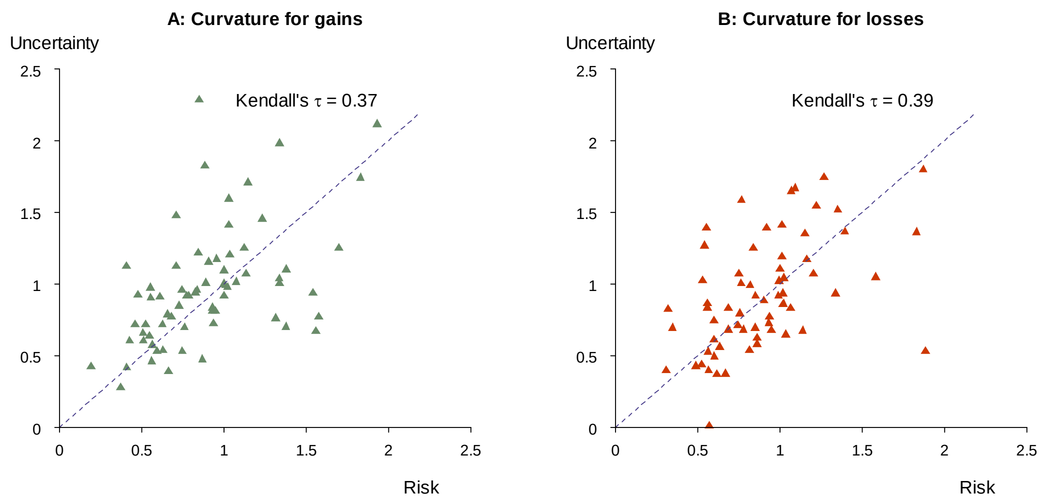


Figure 2: The relationship between individual power coefficients under risk and uncertainty. Panel A displays the power coefficients for gains. Panel B displays the power coefficients for losses. Subjects who had a power coefficient in excess of 2.5 are not shown in the graphs (4 for gains, 7 for losses). The dashed lines correspond to the case where subjects had exactly the same coefficients under risk and uncertainty.

5.5. Loss Aversion

Figure 3 displays the relationships between the medians of x_j^+ and $-x_j^-$ under risk and under uncertainty. An advantage of our method is that it immediately reveals that there is loss aversion in the sense of Kahneman and Tversky (1979) when $x_j^+ > -x_j^-$.[7] Hence, there is no need to measure the entire utility function to obtain insight into the presence or absence of loss aversion. As Figure 3

[6] The difference was no longer significant if we restrict attention to the 58 subjects who never violated stochastic dominance.

[7] For a given j, x_j^+ and $-x_j^-$ have the same utility by construction, $U(x_j^+) = -U(x_j^-)$, and, thus, $x_j^+ > -x_j^-$ implies that $U(x_j^+) < -U(-x_j^+)$, consistent with Kahneman and Tversky's definition of loss aversion ($U(x) < -U(-x)$ for all x > 0).

clearly shows, $-x_j^-$ was below x_j^+ for all j, both under risk and under uncertainty. An estimate of the degree of loss aversion is obtained by regressing the x_j^+ on $\left(-x_j^-\right)$. The $\beta's$ in Figure 3 display the coefficients from this regression. Both $\beta's$ (for risk and uncertainty) were different from unity (P < 0.001) and the values that we obtained were close to those observed previously for risk. We could not reject the hypothesis that the values of β were the same for risk and uncertainty (P = 0.431), which can be taken as an indication that loss aversion was similar under risk and uncertainty.

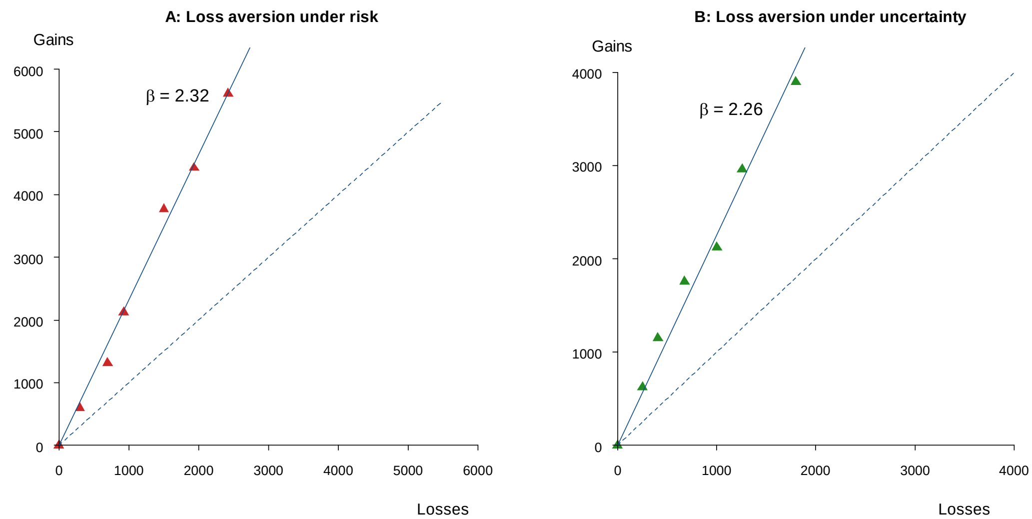


Figure 3: The relationship between median gains and median losses with the same absolute utility. Panel A displays the relationship between median gains and losses under risk. Panel B displays the same relationship under uncertainty. The dashed line corresponds to the case where gains and losses of the same absolute utility would be of equal size. The straight line with slope β corresponds to the best fitting linear equation through all points.

Moving to the individual level, we found that $x_j^+ > -x_j^-$ for all j (Wilcoxon tests, all P < 0.001). Furthermore, $x_j^+/-x_j^-$ did not differ between risk and uncertainty for any j (Wilcoxon tests, all P > 0.254).

Table 4 shows the results of the individual analyses of loss aversion based on Kahneman and Tversky's (1979) and Köbberling and Wakker's (2005) measures. The table clearly shows evidence of loss aversion, irrespective of the definition used and regardless of whether we took response errors into account. According to both definitions, the median loss aversion coefficients for risk and uncertainty did not differ (Wilcoxon test, P > 0.257 in both tests) and were moderately correlated (Kendall's τ > 0.368, P < 0.001 in both tests).

Table 4: Results under the various definitions of loss aversion

The table depicts the results under the two definitions of loss aversion for both risk and uncertainty. The table displays how the coefficient is defined and the number of loss averse, gain seeking, and loss neutral subjects in both conditions. The numbers in parentheses for Kahneman and Tversky's definition correspond with the case where response errors are not taken into account. Furthermore, the table depicts the median and interquartile range (IQR) for each measure of loss aversion under both definitions.

Definition	Coefficient	Condition	Median [IQR]	Loss averse	Gain seeking	Loss neutral
Kahneman and Tversky (1979)	$\frac{-U(-x)}{U(x)}$	Risk	2.19 [1.06, 5.59]	58(46)	10(6)	1(1)
		Uncertainty	2.48 [1.10, 7.16]	54(50)	16(10)	0(0)
Köbberling and Wakker (2005)	$\frac{x_1^+}{-x_1^-}$	Risk	1.86 [1.06, 4.47]	56	13	3
		Uncertainty	2.00 [1.21, 6.50]	57	14	1

Finally, the two measures of loss aversion were substantially correlated. For risk, Kendall's τ was 0.740 and for uncertainty it was 0.799 (all $P < 0.001$ in both cases). It is comforting to observe that these two distinct measures, one of a local nature and relying on a single kink in the slope of the utility function, and the other global and relying on different absolute utilities associated with the same absolute money amounts in the positive and negative domain, showed a high degree of consistency in classifying subjects.

5.6 Reflection

The aggregate findings reported earlier suggest that the power coefficients were similar in the gain and loss domains. This implies that the utility for losses is the mirror image of the utility for gains and is referred to as *reflection*.[8] It is of interest to test whether reflection also held at the individual level. Practically, this would allow us to infer utility for both gains and losses by only measuring it in one of these domains. Theoretically, it would provide support for the idea that utility in both domains is caused by the same psychophysical response to changes relative to the reference point. Reflection is a central result in Tversky and Kahneman (1992) and is widely adopted in theoretical and empirical analyses based on prospect theory (e.g. Barberis et al. 2001).

[8]Reflection is also defined as risk [ambiguity] attitudes for losses being the mirror image of risk [ambiguity] attitudes for gains. As risk [ambiguity] attitudes are jointly determined by utility and event weighting under binary PT, it is clear that this definition differs from the one we use here.

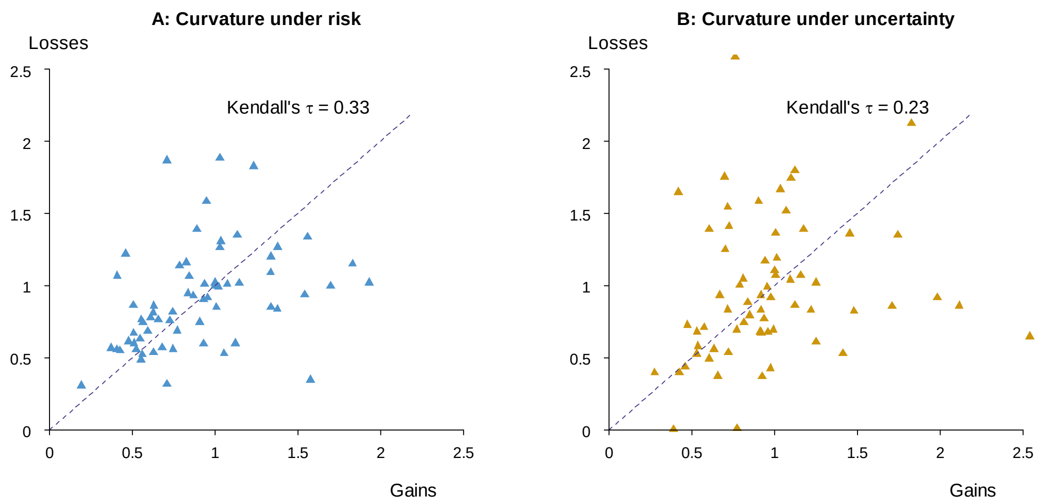


Figure 4: The relationship between individual power coefficients for gains and losses. Panel A displays the power coefficients under risk. Panel B displays the power coefficients under uncertainty. Subjects who had a power coefficient in excess of 2.5 are not shown in the graphs (6 for risk, 9 for uncertainty). The dashed lines correspond to the case where subjects had exactly the same coefficients for gains and losses.

We found little indication that reflection should be rejected. Based on the area measure, there was some, albeit marginal, difference in curvature between gains and losses (Wilcoxon test: P = 0.067). For uncertainty, there was no difference (Wilcoxon test, P = 0.724). Reflection also implies that the power coefficients for gains and losses should be identical. We could not reject this hypothesis, neither for risk (Wilcoxon test: P = 0.128) nor for uncertainty (P = 0.814).

On the other hand, both the area measure and the power coefficients, were only slightly correlated under uncertainty, and moderately correlated under risk. For the area measure, Kendall's τ was 0.317 under risk ($P < 0.001$), and 0.191 under uncertainty (P = 0.018). For the power coefficients, Kendall's τ was 0.325 under risk ($P < 0.001$), and 0.231 under uncertainty (P = 0.004). Figure 5 displays the relation between the power coefficients for both risk and uncertainty. The straight line corresponds to reflection. Both for risk and for uncertainty, reflection approximately held for most subjects, but for some it was a poor working hypothesis, particularly under uncertainty.

6. Discussion

Ambiguity models differ in whether they allow different utility functions for risk and uncertainty. Under binary prospect theory, which includes the multiple priors models and prospect theory as special cases, utility is independent of the source of uncertainty and, hence, the same for risk and uncertainty. Ambiguity aversion is modeled through a difference in event weighting. We tested empirically whether the assumption of identical utility functions is justified and obtained support for it. We could not reject the hypothesis that utility and loss aversion were the same under risk and under uncertainty. We also obtained convincing evidence for reference-dependence: utility was concave for gains, but convex for losses and there was substantial loss aversion. Finally, the elicited standard sequences were similar for different stimuli supporting the central condition underlying binary prospect theory (Köbberling and Wakker 2003), which had not been tested before.

Our findings pose a descriptive challenge for models that explain ambiguity aversion through a difference in utility curvature between risk and uncertainty alone, like the popular smooth ambiguity model. We observed that standard sequences were similar for risk and uncertainty. In Appendix C we show that this implies under the smooth model that the utility function under uncertainty cannot be a concave or convex transformation of the utility function under risk, even on small preference intervals. Hence, the transformation function has an irregular shape, which complicates its use in applications.

It is interesting that loss aversion under risk and under uncertainty were similar. If loss aversion reflects the psychological intuition that losses loom larger than gains then one would expect that measurements of loss aversion are related across domains. Previous evidence of this correlation gave mixed results. Gaechter et al. (2007) found a positive correlation between loss aversion in a risky and in a riskless task, but Abdellaoui et al. (forthcoming) found that loss aversion under risk and loss aversion in intertemporal choice were uncorrelated. Several studies have found that loss aversion is fickle and subject to framing (e.g. Novemsky and Kahneman 2005, Ert and Erev 2008, Abdellaoui et al. forthcoming). We found that loss aversion was stable under risk and uncertainty if the elicitation method is held constant.

In many decisions probabilities are unknown. People are often not neutral towards ambiguity and it is often important to take ambiguity attitudes into account. Our study contributes to the application of ambiguity models in empirical studies and decision analysis by providing a new parameter-free method to measure utility and loss aversion under uncertainty that is robust to event weighting and that can easily be implemented. Our method extends the trade-off method by allowing for standard sequences that contain both gains and losses and that go through the reference point. It provides a straightforward way of exploring whether decision makers are loss averse without the need to elicit the entire utility function. As stage 1 of our method shows, three elicitations suffice to measure loss aversion in the sense of Köbberling and Wakker (2005) and with a few more measurements loss aversion in the sense of Kahneman and Tversky (1979) can be verified.

Our main conclusions, that both utility and loss aversion were the same for risk and for uncertainty, were not caused by the fact that subjects faced the same stimuli for risk and uncertainty. A simple heuristic that subjects might have used was to simplify the uncertain decision task by assuming that the probability of their preferred color in the ambiguous urn was ½. Then, the decisions under risk and uncertainty would be the same and our conclusions would naturally follow. Our data did not corroborate this hypothesis. The value of the loss L stated in the first stage of our method was significantly lower under ambiguity (Wilcoxon test, $P < 0.001$), consistent with ambiguity aversion. Consequently, the subsequent choices that subjects faced were markedly different for risk and uncertainty. Even though the choices were different, the obtained utilities were similar for risk and for uncertainty.

An easy response strategy in the trade-off method is to let the outcomes of the standard sequence increase by the difference between the gauge outcomes ($\mathcal{L}$ and ℓ in the sequence of gains $\mathcal{G}$ and $\mathcal{g}$ in the sequence of losses). This would bias the results in the direction of linear utility. We checked for this heuristic but found little evidence to support it, even allowing for response error.

The trade-off method is chained in the sense that previous responses are used in the elicitation of subsequent choices. Chaining may lead to error propagation, where errors made in one particular choice affect later choices. We checked for the impact of error propagation using the simulation methods developed by Bleichrodt and Pinto (2000) and Abdellaoui et al. (2005). In both simulations, we confirmed the conclusions from those studies that the impact of error propagation was negligible.[9] We also repeated the parametric analysis of utility accounting for serial correlation in the error terms.[10] The estimates were identical to the ones reported in Section 5. Hence, we conclude that the chained nature of our measurements had no noticeable impact on the results either.

Let us finally discuss incentives. We used hypothetical outcomes because we wanted to detect utility curvature. For small money amounts little utility curvature is usually observed and the equality of utility for risk and for uncertainty would then automatically follow. A second reason for not using real incentives is that we wanted to include losses. Ambiguity attitudes differ between gains and losses and loss aversion is important in explaining risk and ambiguity attitudes. Because we used substantial losses, we could not implement real incentives: it is impossible to find subjects willing to participate in an experiment in which they can lose substantial amounts of money. Given that all but

[9] Bleichrodt et al. (2010) also concluded that error propagation was negligible in their measurements using the trade-off method.

[10] We assumed that the error terms followed an AR(1) process $\epsilon_t = \rho\epsilon_{t-1} + u_t$ with u_t normally distributed with expectation 0 and variance σ^2 and estimated this using generalized least squares.

one of the questions involved losses, we could not play out one of the gain questions for real either. Subjects would know immediately which question would be played out for real. The literature on the importance of real incentives is mixed. Most studies found that for small to modest stakes there was little or no effect of using real instead of hypothetical choices for the kind of tasks that we asked our subjects to perform (Bardsley et al. 2010). Therefore, we concluded that the limited potential advantage of using real incentives did not outweigh the advantages of being able to use larger outcomes and losses.

7. Conclusion

We performed a critical test of ambiguity models, such as multiple priors and prospect theory, that assume that utility is source-independent and the same for risk and for uncertainty. We verified this assumption and found support for it, suggesting that utility primarily reflects attitudes towards outcomes. Our findings pose a descriptive challenge for models that capture ambiguity attitudes through a difference in utility between risk and uncertainty. Moreover, we found that reference-dependence of utility was important both in modeling attitudes towards risk and in modeling attitudes towards ambiguity. Utility was S-shaped, concave for gains and convex for losses and we observed clear evidence for loss aversion with most subjects being loss averse and the median loss aversion coefficients varying between 1.86 and 2.48.

To apply ambiguity models in practical decision analysis requires methods to measure their parameters. It is often believed that this is complex. We present an easily applicable method to measure utility and loss aversion under uncertainty. Our method makes the trade-off method robust to sign-dependence and allows the elicitation of standard sequences that include gains, losses, and the reference point. It requires no simplifying assumptions about utility and event weighting and takes account of heterogeneity in individual preferences. We hope that our method will foster the use of ambiguity models in empirical research and practical applications.

Appendix

A: Display of the experimental questions.

Figure A1. Choice screen under uncertainty.

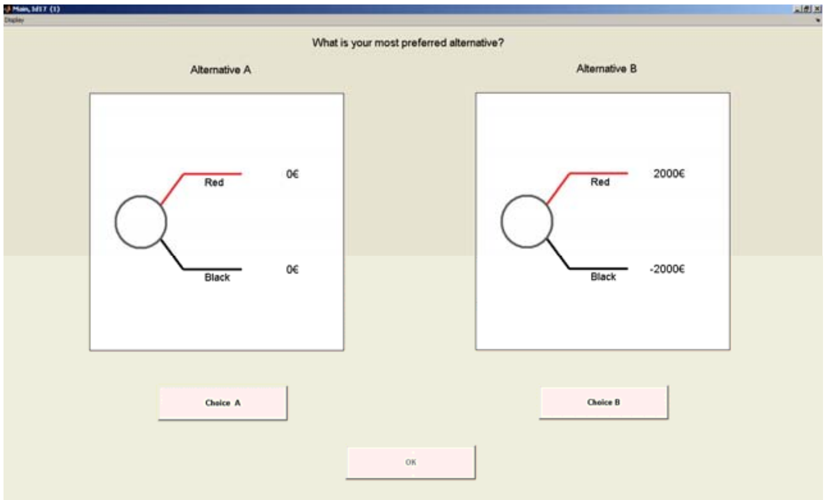


Figure A2. Scrollbar screen under uncertainty.

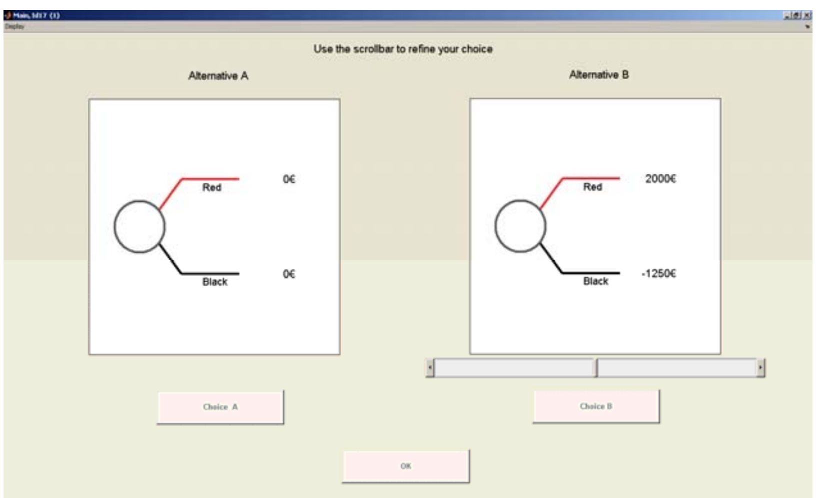
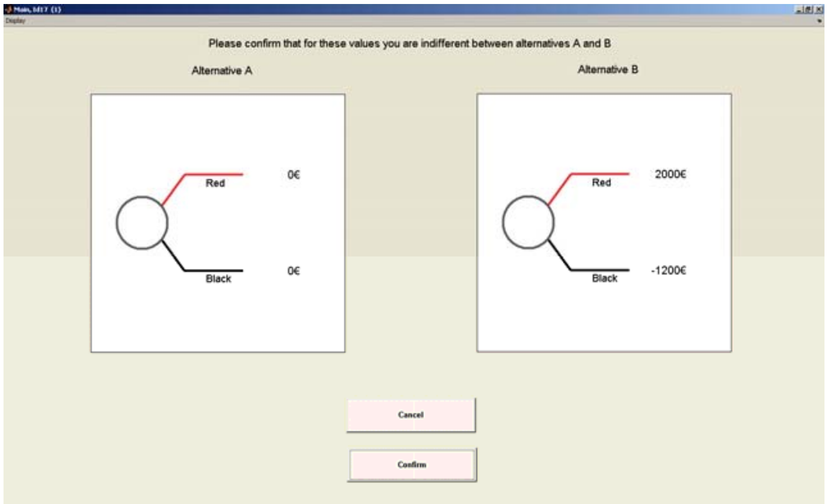


Figure A3. Confirmation screen under uncertainty.



B: Three illustrations of the bisection method under risk.

	Offered choices in elicitation L	Offered choices in elicitation x_1^+	Offered choices in elicitation x_2^-
1	**0** vs. (2000, 0.5; -2000)	(2000,0.5;0) vs. **1000**	**(300,0.5;-200)** vs. (800,0.5;-700)
2	0 vs. **(2000, 0.5; -1000)**	**(2000,0.5;0)** vs. 500	**(300,0.5;-200)** vs. (800,0.5;-450)
3	**0** vs. (2000, 0.5; -1500)	(2000,0.5;0) vs. **750**	(300,0.5;-200) vs. **(800,0.5;-325)**
Slider	Start value: -1250 Interval: [-2000,-500]	Start value: 625 Interval: [250,1000]	Start value: -388 Interval: [-576,-200]

C: Proof that equal utility midpoints for risk and uncertainty imply ambiguity neutrality or volatile ambiguity attitudes under the smooth model.

In our experiment we ask indifferences $x_j^+{}_E\mathcal{L} \sim x_{j-1}^+{}_E\ell$. Under the smooth model this implies:

$$\sum_{i=1}^m \pi_i\varphi(p_iU(x_j^+)+(1-p_i)U(\mathcal{L})) - \sum_{i=1}^m \pi_i\varphi(p_iU(x_{j-1}^+)+(1-p_i)U(\ell)) = \sum_{i=1}^m \pi_i\varphi(p_iU(x_1^+)+(1-p_i)U(\mathcal{L})) - \sum_{i=1}^m \pi_i\varphi(p_iU(x_0)+(1-p_i)U(\ell)) \qquad \text{(A1)}$$

Or

$$\sum_{i=1}^m \pi_i(\varphi(p_iU(x_j^+)+(1-p_i)U(\mathcal{L})) - \varphi\left(p_iU(x_{j-1}^+) + (1-p_i)U(\ell)\right)) = \sum_{i=1}^m \pi_i(\varphi(p_iU(x_1^+)+(1-p_i)U(\mathcal{L})) - \varphi\left(p_iU(x_0)+(1-p_i)U(\ell)\right)) \qquad \text{(A2)}$$

Suppose utility midpoints are the same for risk and uncertainty. Because the π_i sum to one, we also have

$$\sum_{i=1}^{m} \pi_i \left(p_i U\left(x_j^+\right) + \left(1-p_i\right)U(\mathcal{L})\right) - \left(p_i U\left(x_{j-1}^+\right) + \left(1-p_i\right)U(\ell)\right) =$$

$$\sum_{i=1}^{m} \pi_i \left(p_i U(x_1^+) + \left(1-p_i\right)U(\mathcal{L})\right) - \left(p_i U(x_0^+) + \left(1-p_i\right)U(\ell)\right) \tag{A3}$$

If φ is strictly concave or strictly convex (A2) and (A3) can never be jointly true. Hence, either φ is linear or it has both convex and concave parts on any interval $[x_{j-1}^+, x_j^+]$, $j = 1, \dots, k_G$.

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