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Dynamics of Social Norms in the City

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Abstract

We study how in a city either opposite social norms remain or a particular code of behavior spreads and ultimately prevails. We develop a multicommunity model with overlapping generations. When young, an individual chooses the level of educational effort. The crucial feature is that her decision is influenced by peers living in the area who favor either a social norm valuing education or a social norm discrediting education. When an adult, an individual who cares about her offspring's expected income chooses the family's location. Endogenous location leads to different patterns of social norms in the city. We identify two types of urban equilibrium: a *culturally-balanced* city where social norms are distributed evenly among urban areas and the rate of education is the same in each urban area and a *culturally-divided* city where urban areas oppose on their prevailing social norm and exhibit different rates of education. We then study the dynamics of social norms. We show that there are multiple long-run patterns of social norms. A particular steady state is achieved depending on the initial distribution of social norms support in the population. Finally, we show that the public policies promoting social integration can lead in the long run to a population unanimously discrediting education and getting less education than letting the culturally-divided city arise.

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1 Introduction

The striking fact about urban inequality is that social problems such as crime activities, unemployment, school drop out, teenage childbearing, concentrate in urban areas, for instance the inner-city in the US and the suburbs in Europe. Ethnographic and sociological studies have documented that concentration of social problems in depressed communities may entertain a culture of poverty which may oppose the mainstream culture and may trap their inhabitants into poverty (see among others Wilson, 1987, Anderson 1999 and the survey of Lamont and Small, 2008). Concentration of social problems in some depressed communities is thus a threat for social cohesion and raises the issue of the design of public policies aiming to fight against urban inequality.

The purpose of this paper is to understand why, in some urban areas, subcultures favoring standards of behavior which prove to be detrimental for their inhabitants emerge and perpetuate over time. We focus on particular peer-group effects, that is social norms followed by peers. Youth decisions are driven by the concern to follow some social norms as the obedience of the code of behavior prescribed by a particular social norm may generate reputation benefits while disobedience may incur stigmatization costs.

There is now a widespread consensus on the influence of social interactions on behavioral and economic outcomes. In particular, the youth while taking decisions appear to be strongly influenced by their local environment. For instance, Gaviria and Raphael (2001) find strong evidence of peer-group effects at the school level for drug use, alcohol drinking, cigarette smoking, church going and the likelihood of dropping out of high school. Further, there is evidence that these peer-group effects may be the result of peer pressure. At the school level, it has been widely documented that pupils engage in harassment and other peer pressures in order to enforce norms of behavior (see Bishop, 2003, for a broad review of ethnographic and psychological studies on this issue and also for the study he conducted from the Educational Excellence Alliance's Survey of Student Culture)¹. Further, one explanation of the racial achievement gap in education lies in "acting white" peer externality which refers to stigmatization exerted by peers in case one invests in behaviors characteristic of whites (see Fryer, 2010). Finally, there is empirical evidence that some neighborhood effects on social problems, especially dropping out and teenage childbearing, are characterized by some epidemics, that is once the neighborhood quality has dropped below some threshold then the incidence of social problem sharply increases (see for instance Crane, 2001). In his review of the US literature, Galster (2002) concludes that neighborhood poverty has no effect

¹Studying peer effects in the workplace, Mas and Moretti (2009) find evidence of the influence of productivity coworkers on a worker effort. This influence is stronger for coworkers with whom interactions are frequent corroborating that social pressure is a way to internalize free-riding externalities.

on crime, schooling drop-out when the poverty rate does not exceed 20% whereupon the externality grows rapidly until the poverty rate reaches approximately 40 percent, subsequent increases in the poverty are innocuous.

In order to grasp both the local nature and the dynamic aspect of a social norm influence, we develop a multicommunity model with overlapping generations. Individuals live two periods, childhood and adulthood. When child, each individual decides the level of educational effort to exert. In accordance with the empirical results mentioned above on the strong evidence of peer influence at school, we assume that education decision depends on economic returns to education, cost of effort and also reputation benefits and stigmatization costs generated by the adherence to, respectively the deviation from, the social norm. We consider that a child faces two opposite social norms: one valuing education and prescribing high effort (named the "education" social norm) and the other one depreciating schooling effort and prescribing low effort at school (named the "no education" social norm). We assume that children are heterogenous with respect to their preferences. Individuals are called education believers, respectively non education believers, when they take care about the costs and benefits of adhering or deviating from the "education", respectively "no education", social norm. We capture the local nature of social norm assuming that stigmatization costs and reputation benefits of a social norm depend on the fraction of inhabitants in the urban area who believe in this social norm. When an adult, any individual chooses the place of residence of the family comprised by her offspring and himself. Hence, the intensity of a social norm is endogenous as it will be given by the emerging urban equilibrium. The second key feature of the model is that the population of believers in a social norm evolves over time. We follow in this respect Akerlof (1980)'s argument that a social norm may spread if the number of individuals adhering the social norm is greater than the number of individuals believing in this social norm. Our model thus allows us to study the interplay between the dynamics of opposite social norms and the dynamics of the organization of the city. At date t, a particular urban equilibrium may emerge depending on the population characteristics, that is the number of believers and non-believers in the population. This equilibrium implies particular incentives to educate in each neighborhood. This will drive a new number of believers which gives rise to a new urban configuration at date t + 1. On the whole, the social norms dynamics are driven by the urban configuration that arises.

Our results are threefold. First, we identify multiple urban equilibria that arise at each date t and are characterized by the spatial distribution of believers and non-believers. In particular, a symmetric equilibrium, called "culturally-balanced", may emerge where believers and non-believers are uniformly located in the city leading urban areas to be identical with respect to the social norms mix. It turns out that incentives faced by any child to exert educational effort are independent of

her location. Education rates are identical across urban areas. On the contrary, some asymmetric equilibria, called "culturally-divided", may also arise. They are such that urban areas differ with respect to the prevailing social norm and the implied education rate. Depending on the fraction of believers in the whole population, the "culturally-divided" city can exhibit two types of cultural clash: either a urban area only inhabited by believers unanimously promotes education and contrasts with other locations where both social norms are present, or a urban area only inhabited by non-believers deterring from any education effort opposes other urban areas with both social norms. Second, we study the social norms dynamics which arise under cultural division. We show that social norms dynamics exhibit a contagion process so that once the population of believers in the urban area reaches a threshold the underlying social norm spreads out. In particular, if the number of believers in the "education" social norm is too low then the urban area can be trapped in a low-education equilibrium while high education is promoted in the rest of the city. Hence, initial conditions on the composition of the whole population in believers are key for the long-run equilibrium that may be reached. This model thus highlights how two societies with slight differences in their believers' populations may exhibit very different social norms dynamics and experience varying performances in terms of education. Third, we show that if the culturally-balanced equilibrium is imposed by the government it may reach a low-level equilibrium at the steady state. On the contrary, this same economy would obtain the culturally-divided equilibrium under laissez-faire and would exhibit dynamics reaching a high-level equilibrium. This result corroborates Cutler, Glaeser and Vigdor (2008) findings on the positive impact of isolation on better-educated groups.

Our paper belongs to two strands of literature. First, it is related to the literature on human capital accumulation with neighborhood effects which has been impulsed by Loury (1977) and Bénabou (1993, 1996a,b). We depart from their work as we focus on particular neighborhood effects, that is social norms, that involve reputation or stigmatization effects which shape incentives to educate and follow the norm. We are thus able to study the dynamics of neighborhood effects and characterize the conditions under which a social norm may spread or on the contrary may disappear over time. Second, our paper belongs to the economic literature on social norms which has explored how the influence of social norms helps to explain unemployment (see for instance Akerlof, 1980), trade union membership (see Naylor 1989, Corneo, 1995), decisions about work and benefits in the welfare state (see Lindbeck, Nyberg and Weibull, 1999) or more generally the emergence of cooperation (see, for instance, Tabellini, 2008). We depart from this literature as costs and benefits to follow a social norm depend on the neighborhood population characterizing a urban equilibrium. Third, our paper also belongs to the literature on the formation of oppositional identities (see Akerlof and Kranton, 2000, Battu Mwale and Zénou, 2007, Bisin, Patacchini, Verdier and Zénou, 2011). Compared to this literature, our contribution is that the persistence or the disappearance of a particular social norm relies on the endogenous degree of segregation between believers and non-believers.

The plan of the paper is as follows. In the following section we set up the model. In section 3, we characterize the urban equilibrium which may emerge at each date t. Then in section 4, we study the dynamics of social norms arising when cultural division perpetuates. In section 5, we examine the influence of cultural balance on the dynamics of social norms and long-run economic performances. Section 6 concludes.

2 The Setup

We consider an overlapping generations model of neighborhood formation. The essential features of the model are such that (i) during their life, individuals make educational decisions and location choices, (ii) and that individuals may derive utility from obeying a particular social norm.

2.1 The City

The city is comprised of two residential areas indexed by j = 1, 2. We consider that landowners are absent and without loss of generality we normalize the opportunity cost of building a house to 0. Houses are identical across the city. The inelastic supply of houses within a residential area is of mass L. This land-market is a closed-city model where the population of the city is a continuum of families of mass N. Each family, comprised of a parent and a child, lives in one and only one house. The city can accommodate the entire population and we assume for simplicity that L = N/2. Agents live two periods. When child, an individual faces an educational discrete choice. When adult, an individual has to decide in which neighborhood her family resides.

2.2 Children's Educational Choice and Social Norms

Preferences. Children have to decide whether they exert a high effort denoted by $e = \overline{e}$ or a low effort $\underline{e} < \overline{e}$. The crucial feature of the model is that there exist two opposite social norms regarding schooling behavior. One social norm values the behavior "exerting an effort at school" (exerting \overline{e} in the model) and the other one prescribes the code of behavior "not exerting an effort at school" (exerting \underline{e}). Children differ with respect to their beliefs underlying the code of behavior, *i.e.* their preferences regarding the social norm. There are B_t , respectively $N - B_t$, individuals who believe in the values underlying the norm "exerting an effort at school", respectively "not exerting an effort at school". For sake of simplicity, we will call believers (implicitly "education" believers)

the individuals who believe in the norm "exerting an effort at school" and non-believers (implicitly "non-education" believers) those who believe in "not exerting an effort at school".

One could justify the existence of two opposite social norms by the fact that pupils differ with respect to their identity which dictates particular educational behavior (see Akerlof and Kranton, 2000, Bishop, 2003, 2004).

We specify preferences following the works of Akerlof (1980), Naylor (1989) or Corneo (1993) which study how social norms can overcome the free rider problem arising under voluntary union trade membership or collective strike action. Precisely, preferences for a child living in area j = 1, 2 at date t may be defined as follows:

$$\begin{aligned} U_t^j &= p(e)U_{t+1}(w_r) + (1-p(e))U_{t+1}(w_p) - \varepsilon c(e) + \left(sb_t^j \widehat{n} - (1-s)b_t^j \widehat{h}\right) & \text{for a believer,} \\ U_t^j &= p(e)U_{t+1}(w_r) + (1-p(e))U_{t+1}(w_r) - \varepsilon c(e) - s(1-b_t^j)\widetilde{h} + (1-s)(1-b_t^j)\widetilde{n} & \text{for a non-believen} \end{aligned}$$

with $U_{t+1}(w)$ the utility when parent with income w. She earns income w_r , respectively $w_p < w_r$, with p(e), respectively 1 - p(e), while exerting effort e. We assume for simplicity that $p(\overline{e}) = 1$ and $p(\underline{e}) = 0$. $U_{t+1}(w)$ will be specified in the following section while describing parents preferences. $\varepsilon c(e)$ is the cost of effort with $c(\overline{e}) = \overline{c}$ and $c(\underline{e}) = \underline{c} < \overline{c}$ and ε his innate ability. We assume that children differ with respect to their innate ability, ε . The lower her ε the brighter is the individual. Let us assume that it is uniformly distributed on [0, 1].

The social norm component is $sb_t^j \hat{n} - (1-s)b_t^j \hat{h}$ for a believer and $-s(1-b_t^j)\tilde{h} + (1-s)(1-b_t^j)\tilde{n}$ for a non-believer. s denotes a dummy variable, s = 1 if $e = \bar{e}$ and s = 0 if e = e. Parameters \hat{n} and \hat{h} , respectively \tilde{n} and \tilde{h} , in the believer utility, respectively non-believer utility, are positive. We do not make any particular assumption about the magnitude of \tilde{n} with respect to \hat{n} respectively \tilde{h} with respect to \hat{h} . We denote by B_t^j the number of believers in area j and define the fraction of believers in area j by $b_t^j \equiv B_t^j/L$. Hence, our specification of the social norm component implies that the obedience of the code of behavior the child believes in may provide a reputation benefit. It is equal to $b_t^j \hat{n}$ for a believer and $(1 - b_t^j)\tilde{n}$ for a non-believer. In contrast, the disobedience of the code of behavior may incur a stigmatization cost equal to $b_t^j \hat{h}$ for a believer and $(1 - b_t^j)\tilde{h}$ for a non-believer. We depart from the literature on social norms by considering the existence of neighborhood effects. Incentives to follow the social norm depend on the fraction of people in the neighborhood believing in this norm. Precisely, for any individual, both reputation and stigmatization effects increase with the fraction of the neighborhood population with the same beliefs. We do not consider that cross-population effects such that neighboring with believers, respectively non-believers, generate some reputation and stigmatization effects for non-believers, respectively believers. Introducing such effects would not change the main results of the model. However, it will become clear later that there are still externalities between believers and non-believers generated by the location choice which will determine the socioeconomic composition of urban areas and thus the pattern of neighborhood effects.

We assume that individual's beliefs in a particular norm are public information and are inherited from her parent. We do not consider here any transmission mechanism of preferences \dot{a} la Bisin-Verdier (2001). However, we will see later that parents play a key role on the dynamics of believers via their location choice.

Educational Choice. A believer residing in area j decides to educate if and only if

$$U_{t+1}(w_r) - \varepsilon \overline{c} + b_t^j \widehat{n} > U_{t+1}(w_p) - \varepsilon \underline{c} - b_t^j \widehat{h}$$

which amounts to

$$\varepsilon < \widehat{\varepsilon}(b_t^j) \equiv \frac{(U_{t+1}(w_r) - U_{t+1}(w_p)) + b_t^j(\widehat{n} + h)}{\overline{c} - \underline{c}}.$$
(1)

We see that neighborhood effects are crucial for the education decision. The larger the fraction of believers in the neighborhood the more likely the individual exerts effort \overline{e} , that is $\widehat{e}(b_t^j)$ is increasing with b_t^j and $\widehat{e}(0) = (U_{t+1}(w_r) - U_{t+1}(w_p)) / \Delta c < \widehat{e}(1) = ((U_{t+1}(w_r) - U_{t+1}(w_p)) + \widehat{n} + \widehat{h}) / \Delta c$ with $\Delta c \equiv \overline{c} - \underline{c}$.

A non-believer residing in area j decides to exert high effort if and only if

$$U_{t+1}(w_r) - \varepsilon \overline{c} - (1 - b_t^j) \widetilde{h} > U_{t+1}(w_p) - \varepsilon \underline{c} + (1 - b_t^j) \widetilde{n}$$

leading to

$$\varepsilon < \widetilde{\varepsilon}(b_t^j) \equiv \frac{(U_{t+1}(w_r) - U_{t+1}(w_p)) - (1 - b_t^j)(\widetilde{n} + \widetilde{h})}{\overline{c} - \underline{c}}.$$
(2)

The presence of non-believers in the area brings down incentives to exert high effort. The larger the fraction of non-believers, the lower the probability for the individual to exert effort \overline{e} , *i.e.* $\widetilde{e}(b_t^j)$ is decreasing with $1 - b_t^j$. We have $\widetilde{e}(0) = (U_{t+1}(w_r) - U_{t+1}(w_p) - \widetilde{n} - \widetilde{h})/\Delta c$ and $\widetilde{e}(1) = (U_{t+1}(w_r) - U_{t+1}(w_p))/\Delta c$. Obviously, for any b_t^j , we have $\widetilde{e}(b_t^j) < \widehat{e}(b_t^j)$ leading the rate of education to be higher for believers than for non-believers.

2.3 Parents' location choice

Parents differ with respect to both income and beliefs. We assume that a parent and her child have the same beliefs. We also make a limited altruism assumption, that is parents take care about the expected income of their offspring. Hence, at date t, given that ε is uniformly distributed over [0, 1], any parent with income w_z , z = r, p who lives in area j has the following preferences:

$$U_t(w_z) = u(w_z - \rho_t^j) + a \left(\int_{0}^{\widehat{\varepsilon}(b_t^j)} w_r d\varepsilon + \int_{\widehat{\varepsilon}(b_t^j)}^{1} w_p d\varepsilon \right) \text{ if he is a believer}$$
$$U_t(w_z) = u(w_z - \rho_t^j) + a \left(\int_{0}^{\widehat{\varepsilon}(b_t^j)} w_r d\varepsilon + \int_{\widehat{\varepsilon}(b_t^j)}^{1} w_p d\varepsilon \right) \text{ otherwise.}$$

with u(.) the instantaneous utility function, ρ_t^j the rent paid to live in area j at date t, a an altruism parameter. u(.) is bounded, continuously differentiable, strictly increasing and strictly concave over \mathbb{R}_+ . Letting $\Delta w \equiv (w_r - w_p)$, at date t, any parent w_z must solve the following program:

$$\max_{j} u(w_z - \rho_t^j) + a \left[\varepsilon \Delta w + w_p \right] \text{ with } \varepsilon = \widehat{\varepsilon}(b_t^j) \text{ if he is a believer and } \varepsilon = \widetilde{\varepsilon}(b_t^j) \text{ otherwise.}$$

We look at the following equilibrium:

Definition 1 At date t, the urban configuration $[\rho_t^*, b_t^{1*}]$ is an equilibrium if no one wants to move.

We impose that area 1 is inhabited by a higher fraction of believers, *i.e.* $b_t^1 \ge b_t^2 \equiv (B_t/L) - b_t^1$. In order to study the equilibrium, it will be convenient to examine the bid-rent function which measures the willingness to pay for a parent with income w_z , z = r, p to live in area 1. It is denoted by $\rho^z(b_t^1)$. Assuming without loss of generality that the rent in area 2 equals the opportunity cost of building a house which is normalized to 0, the bid-rent of a parent w_z can be expressed as follows:

$$u(w_z) - u(w - \hat{\rho}^z(b_t^1)) = a\left(\frac{2b_t^1 - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w \text{ if he is a believer}$$
(3)

$$u(w_z) - u(w - \tilde{\rho}^z(b_t^1)) = a\left(\frac{2b_t^1 - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w \text{ otherwise.}$$
(4)

We immediately see that whatever their beliefs, parents are willing to pay to neighbor individuals who believe in the norm "exerting an effort at school". The higher $2b_t^1 - (B_t/L)$ the more attractive is area 1. The reason is that given that parents value the expected income of their offspring they want to live in the neighborhood that most promotes education whatever their beliefs.²

We will denote by \widehat{N}_t^z ($\widehat{N}_t^{z,j}$), respectively \widetilde{N}_t^z ($\widetilde{N}_t^{z,j}$), the number of parents w_z who are believers, respectively non-believers, in the whole population (in area j).

²Given this formalization, parents who are non-believers are willing to pay to locate in the neighborhood of believers as it maximizes the probability that their child has a high expected income. To alleviate such apparent schizophrenia, we could consider that parents suffer from a cost Θ when their offspring does not obey the social norm they believe

2.4 Social Norms Dynamics

We consider that the dynamics of a social norm are assimilated to the dynamics of the number of believers in this social norm. We restrict our attention to the dynamics of believers, B_t , as the dynamics of non-believers $N - B_t$ being immediately deduced from the evolution of B_t . Following Akerlof (1980) "[...] if disobedience is more common, in all likelihood the values responsible for the observance of a social custom are less likely to be passed on from one generation to the next the greater is the disobedience." (p. 749), we specify the social norm dynamics as follows

$$b_{t+1}^{j} - b_{t}^{j} = H(\lambda(b_{t}^{j}) - b_{t}^{j}) \text{ for any } j = 1,2$$
 (5)

with $H: [-1,1] \longrightarrow \mathbb{R}$, H'(.) > 0 and H(0) = 0 and $\lambda(b_t^j)$ denoting the rate of education at date t in area j = 1, 2. The function H(.) is assumed to be positive when $\lambda(b_t^j) > b_t^j$, so that the "education" social norm spreads out in the area j when the rate of education is higher than the percentage of believers.³

According to these dynamics, the population born at date t with B_t believers becomes adult at date t + 1 with B_{t+1} believers.

In order to obtain the rate of education, we make the following assumptions

Assumption 1 Children are myopic in the sense that they do not anticipate they will pay a rent while they will become parents.

Assumption 1 puts aside difficulties that would arise if the children would have to anticipate the future urban equilibrium in order to make their choice of educational effort. It amounts to say that they are not aware of the fact that neighborhood social composition matters and has in. The utility function would then be

$$U_t(w_z) = u(w_z - \rho_t^j) + a \left(\int_{0}^{\widehat{\varepsilon}(b_t^j)} w_r d\varepsilon + \int_{\widehat{\varepsilon}(b_t^j)}^{1} w_p d\varepsilon - \Theta \int_{\widehat{\varepsilon}(b_t^j)}^{1} d\varepsilon \right) \text{ for a believer}$$
$$U_t(w_z) = u(w_z - \rho_t^j) + a \left(\int_{0}^{\widehat{\varepsilon}(b_t^j)} w_r d\varepsilon + \int_{\widehat{\varepsilon}(b_t^j)}^{1} w_p d\varepsilon - \Theta \int_{0}^{\widehat{\varepsilon}(b_t^j)} d\varepsilon \right) \text{ for a non-believer}$$

with Θ the cost of departing from the social norm. The utility function can be rewritten as follows

$$U_t^{z,j} = u(w_z - \rho^j) + x \left[\widehat{\varepsilon}(b_t^j)(a\Delta w + \Theta) + a(\underline{p}w_r + (1 - \underline{p})w_p) - \Theta \right] + (1 - x) \left[\widetilde{\varepsilon}(b_t^j)(a\Delta w - \Theta) + a(\underline{p}w_r + (1 - \underline{p})w_p) \right].$$

The main results would remain while introducing this cost.

³Let us mention that at each date parents choose their place of residence. Hence, the number of believers b_{t+1}^{j} in (5) is obtained given the urban equilibrium that arises at date t. The number of believers b_{t+1}^{j} in (5) can differ from the number of believers that will reside in area j at the urban equilibrium at date t + 1.

some intrinsic value. One could justify myopia by saying that children only interact with the neighborhood population and do not realize that the population may be unevenly distributed in the city. Hence they consider that the belonging to a particular neighborhood has no effect on their educational effort. We thus assume that individuals expect to pay the cost of opportunity of land which is normalized to 0. Given parents preferences, Assumption 1 thus implies that both thresholds $\hat{\varepsilon}(b_t^j)$ and $\tilde{\varepsilon}(b_t^j)$ defined in (1) and (2) can be written as follows

$$\widehat{\varepsilon}(b_t^j) = \frac{(u(w_r) - u(w_p)) + b_t^j(\widehat{n} + \widehat{h})}{\Delta c}$$

and $\widetilde{\varepsilon}(b_t^j) = \frac{(u(w_r) - u(w_p)) - (1 - b_t^j)(\widetilde{n} + \widetilde{h})}{\Delta c}$

Later on, we let $\Delta u \equiv u(w_r) - u(w_p)$.

Assumption 2 Parameters \overline{c} , \underline{c} , w_r , w_p , \widetilde{n} , \widetilde{h} , \widehat{n} , \widehat{h} and the function u(.) are such that (i) $\Delta u/\Delta c < 1$, (ii) $(\Delta c - \Delta u)/(\widehat{n} + \widehat{h}) < 1$, (iii) $\Delta u/(\widetilde{n} + \widetilde{h}) < 1$.

Assumption 2 provides information about how a change of the composition of the population in terms of the fraction of believers in the neighborhood impacts the educational rate of either the believers or the non-believers. Precisely, given both items (i) and (ii), we have $\hat{\varepsilon}(0) < 1 < \hat{\varepsilon}(1)$ ensuring that the fraction of believers exerting high effort lies between a strictly positive number less than 1 and 1 for any $b_t^j \in [0, 1]$. Hence, below some threshold, any increase of the number of believers in the neighborhood strictly increases the number of believers who educate. Any fraction of believers in the neighborhood above this threshold leads all believers to get education. Further, given both items (i) and (iii), we have $\tilde{\varepsilon}(0) < 0 < \tilde{\varepsilon}(1) < 1$. It turns out that the the fraction of non-believers exerting high effort lies between 0 and a strictly positive number lower than 1. Hence, a minimal fraction of believers is needed to incite non-believers to exert educational effort. Once b_t^j has reached this threshold, any further rise of believers in the neighborhood strictly increases the number on non-believers who decide to educate.

Given that ε is uniformly distributed on the interval [0, 1], we can thus derive the fraction of children who educate at date t in area j = 1, 2, denoted by $\lambda(b_t^j)$,

$$\lambda(b_t^j) = b_t^j \min[\widehat{\varepsilon}(b_t^j), 1] + (1 - b_t^j) \max[0, \widetilde{\varepsilon}(b_t^j)]$$

Given (1) and (2), we have for j = 1, 2

$$\lambda(b_t^j) = \min\left[\frac{\left(\Delta u\right)b_t^j + \left(b_t^j\right)^2(\widehat{n} + \widehat{h})}{\Delta c}, b_t^j\right] + \max\left[0, \frac{\Delta u(1 - b_t^j) - (1 - b_t^j)^2(\widetilde{n} + \widetilde{h})}{\Delta c}\right].$$

Letting $\Delta(n+h) \equiv \hat{n} + \hat{h} - (\tilde{n} + \tilde{h})$, we assume the following

Assumption 3 Parameters \overline{c} , \underline{c} , w_r , w_p , \widetilde{n} , \widetilde{h} , \widehat{n} , \widehat{h} and the function u(.) are such that

$$\frac{(\Delta c - \Delta u)}{(\widehat{n} + \widehat{h})} > 1 - \frac{\Delta u}{(\widetilde{n} + \widetilde{h})}$$

Assumption 3 avoids the case where we have for some b_t^j corner situations for both believers and non-believers, *i.e.* $\min[\widehat{\varepsilon}(b_t^j), 1] = 1$ and $\max[0, \widetilde{\varepsilon}(b_t^j)] = 0$ for some b_t^j . Assumption 3 thus guarantees that a variation of b_t^j always leads to a change of the fraction of children exerting high effort. Given Assumption 3, it is always worth living in area 1 where $b_t^1 \ge b_t^2$.

Given Assumptions 1, 2 and 3, letting $A \equiv 1 - (\Delta u/(\tilde{n} + \tilde{h}))$ and $C \equiv (\Delta c - \Delta u)/(\hat{n} + \hat{h}) > A$, the fraction of children who educate then equals:

$$\lambda(b_t^j) = \frac{(\Delta u)b_t^j + (b_t^j)^2 (\widehat{n} + \widehat{h})}{\Delta c} \text{ when } b_t^j \in [0, A]$$
(6)

$$\lambda(b_t^j) = \left(b_t^j\right)^2 \frac{\Delta(n+h)}{\Delta c} + b_t^j \frac{2(\tilde{n}+\tilde{h})}{\Delta c} - \frac{(\tilde{n}+\tilde{h})}{\Delta c} + \frac{\Delta u}{\Delta c} \text{ when } b_t^j \in [A, C]$$

$$\tag{7}$$

$$\lambda(b_t^j) = -\left(b_t^j\right)^2 \frac{(\widetilde{n}+\widetilde{h})}{\Delta c} + b_t^j \left(1 - \frac{\Delta u}{\Delta c} + \frac{2(\widetilde{n}+\widetilde{h})}{\Delta c}\right) + \frac{\Delta u - (\widetilde{n}+\widetilde{h})}{\Delta c} \text{ when } b_t^j \in [C,1].$$
(8)

Let us stress that if item *(ii)* of Assumption 2, respectively item *(iii)* of Assumption 2 were not satisfied then (8), respectively (6), would not be considered and the dynamics would be described by (7). In such a case, as it will become clearer in Section 4, the model would not admit a multiplicity of steady states.

3 Short-Run Equilibrium: Cultural Balance versus Cultural Divide

The type of equilibrium that emerges crucially depends on the identity of individuals who are most willing to pay to live in a better neighborhood. Formally, the slope of the bid-rent is expressed as follows:

$$R(w_z, n+h, \rho_t) \equiv \left. \frac{d\rho^{w_z}(b_t^1)}{db_t^1} \right|_{U_t(w_z) = \overline{U}} = 2 \frac{a\Delta w}{\Delta c} \frac{(n+h)}{u'(w_z - \rho_t)} \text{ for } z = r, p \text{ and } n+h = \widehat{n} + \widehat{h}, \widetilde{n} + \widetilde{h}.$$
(9)

The following lemma provides some information about the ranking of bid-rent slopes:

Lemma 1 At any date t, for any $0 < \rho_t < w_p$, we have (i) $R(w_r, \hat{n} + \hat{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$, (ii) $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ and (iii) $R(w_z, \hat{n} + \hat{h}, \rho_t) \gtrless R(w_z, \tilde{n} + \tilde{h}, \rho_t)$ for z = r, p if and only if $\hat{n} + \hat{h} \gtrless \tilde{n} + \tilde{h}$. (iv) At any date t, we may have for some ρ_t , $R(w_r, \tilde{n} + \tilde{h}, \rho_t) - R(w_p, \hat{n} + \hat{h}, \rho_t) < 0$, respectively $R(w_r, \hat{n} + \hat{h}, \rho_t) - R(w_p, \tilde{n} + \tilde{h}, \rho_t) < 0$, only if $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$, respectively $\hat{n} + \hat{h} < \tilde{n} + \tilde{h}$.

Proof. Obvious from the concavity of u(.).

Parents differ with respect to two dimensions, beliefs and income, which impact their willingness to pay to live in area 1. According to items (i) and (ii), given beliefs of individuals, the bid-rent is steeper for a rich parent. Thus he outbids the poorer agent. If instead we take two individuals with the same income but different beliefs, the bid-rent differential depends on the magnitude of $\hat{n} + \hat{h}$ with respect to $\tilde{n} + \tilde{h}$ (see item (iii)). Item (iv) tells us that the income effect and the beliefs effect may have an opposite impact on the bid-rent function making possible the case where a poor individual outbids a rich one.

In the following we introduce a stability condition which will prove useful in order to select an equilibrium.

Definition 2 Without loss of generality, let us assume that $\frac{B_t}{2L} \leq 1$. Consider an equilibrium $[\rho_t^*, b_t^{1*}]$ with $\frac{B_t}{2L} \leq b_t^{1*} \leq 1$. Take a small $\varepsilon > 0$. The equilibrium $[\rho_t^*, b_t^{1*}]$ is stable if after a perturbation leading to $b_t^1 = b_t^{1*} + \varepsilon$, respectively $b_t^1 = b_t^{1*} - \varepsilon$, there are non-believers, respectively believers, in area 2 who are able to outbid believers, respectively non-believers, in order to live in area 1.

Hence, an equilibrium is stable if after some perturbation, individuals are able to migrate and correct the initial perturbation.

We can first offer the following

Proposition 1 The symmetric equilibrium $[\rho_t^* = 0, b_t^{1*} = \frac{B_t}{2L}]$, called culturally-balanced, always exists. It is not stable, respectively stable, when $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$, respectively $\hat{n} + \hat{h} < \tilde{n} + \tilde{h}$.

Proof. Existence. It is always possible to split the believers' population such that $b_t^1 = b_t^2 = \frac{B_t}{2L}$. According to (3) and (4), we have $\hat{\rho}^z(\frac{B_t}{2L}) = \tilde{\rho}^z(\frac{B_t}{2L}) = 0$ for z = r, p. Hence, no one has an incentive to move.

Stability. Take a culturally-balanced with $\widehat{N}_t^{z,j} > 0$ and $\widetilde{N}_t^{z,j} > 0$ for z = r, p and j = 1, 2. Consider the perturbation $\varepsilon > 0$ that leads to $b_t'^1 = \frac{B_t + \varepsilon}{2L}$ and $b_t'^2 = \frac{B_t - \varepsilon}{2L}$. When $\widehat{n} + \widehat{h} > \widetilde{n} + \widetilde{h}$, according to Lemma 1 we have $\widehat{\rho}^r(\frac{B_t + \varepsilon}{2L}) > \widetilde{\rho}^r(\frac{B_t + \varepsilon}{2L}) > \widetilde{\rho}^p(\frac{B_t + \varepsilon}{2L})$ and $\widehat{\rho}^r(\frac{B_t + \varepsilon}{2L}) > \widehat{\rho}^p(\frac{B_t + \varepsilon}{2L})$. Hence all rich believers migrate to area 1 implying the culturally-balanced equilibrium to be unstable. When $\hat{n} + \hat{h} < \tilde{n} + \tilde{h}$, according to Lemma 1 we have $\hat{\rho}^z(\frac{B_t + \varepsilon}{2L}) < \tilde{\rho}^z(\frac{B_t + \varepsilon}{2L})$ for z = r, p, and $\hat{\rho}^p(\frac{B_t + \varepsilon}{2L}) < \tilde{\rho}^r(\frac{B_t + \varepsilon}{2L})$, then rich non-believers migrate to area 1, thus restoring the culturally-stable equilibrium.

There is always an equilibrium where the believers' population is distributed evenly among both areas. The mix of social norms is the same in both neighborhoods. It turns out that individuals, whatever their income and beliefs, are indifferent between the two residential areas. Without loss of generality, we will consider culturally-balanced equilibria where both areas are comprised by all types of parents, *i.e.* $\hat{N}_t^{z,j} > 0$ and $\tilde{N}_t^{z,j} > 0$ for z = r, p and j = 1, 2. Hence, while studying stability of such an equilibrium, migration of any type of individuals among areas are thus possible. We show that a culturally-balanced equilibrium is not robust to all possible movements of individuals across areas. For instance, if we assume that $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$ and consider a migration of ε rich believers from area 2 to area 1 and a migration of ε rich non-believers in the reverse direction, then, given Lemma 1, rich believers find it worthwhile to migrate to area 1 preventing the culturally-balanced equilibrium from being restored.

The following Proposition focuses on another type of equilibrium

Proposition 2 Consider that $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$. At any date t, depending on the distribution of beliefs and income in the population, an equilibrium such that $b_t^{1*} > b_t^{2*}$ exists and is stable. It is called culturally-divided.

Proof. See Appendix.

Another type of urban equilibrium, called culturally-divided equilibrium, such that the fraction of believers is larger in area 1 than in area 2 exists. Hence, the "education" social norm is more influential in area 1 rather than in area 2 yielding a higher probability of the child to exert \overline{e} . It is thus more attractive for households to reside in this area. Precisely, existence and stability of the culturally-divided equilibrium rely first on values taken by the slopes of the bid-rent function, second, on the demographic characteristics of the population, *i.e.* the magnitude of each type of population \widehat{N}_t^z , \widetilde{N}_t^z , for z = r, p. There are thus various configurations of culturally-divided equilibria. The Appendix provides a complete characterization of them. All the urban configurations presented here are obtained under the assumption that $\widehat{n} + \widehat{h} > \widetilde{n} + \widetilde{h}$. According to Lemma 1, rich believers are always the highest bidders for the better area. In the Appendix we also study the case where $\widehat{n} + \widehat{h} < \widetilde{n} + \widetilde{h}$.

\widetilde{R} Area 1	\widehat{P} \widetilde{P} Area 2	P P Area 1	P̃ Area 2
Ŕ	Ĩ	R R	ã

Figure 1: Two possible urban configurations when $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ_t . \hat{R} , \tilde{P}, \tilde{P} denote the rich believers, rich non-believers, poor believers, poor non-believers, populations. On the left, $\hat{N}_t^r \geq \frac{B_t}{L}$ and $\hat{N}_t^r + \tilde{N}_t^r \geq L$. On the right, $\tilde{N}_t^p > L$.

Figure 1 displays two configurations of a culturally-divided city that possibly exist when the rich believers population is smaller than L and when the rich non-believers are able to outbid poor believers. In addition of the cultural divide, the city on the left exhibits income segregation where all the poor live in area 2. Hence, area 1 is inhabited by a larger fraction of believers than in area 2 and it is also homogenously rich. Nevertheless, area 2 is populated by a fraction of believers that may help individuals to get education. The city on the right arises when the whole population is comprised by a majority of poor non-believers, *i.e.* $\tilde{N}_t^p > L$. Both the rich and the believers populations are located in area 1. Population in area 2 unanimously rejects education. Further, area 2 concentrates poverty. In both urban configurations, the rich population has a larger opportunity to benefit from better neighborhood effects than the poorer one.

On the contrary, when the poor believers outbid rich non-believers, a different configuration of the culturally-divided city may arise where all believers may segregate in area 1 leading area 2 to promote the opposite social norm. This city is depicted below.



Figure 2: A possible urban configuration when $R(w_r, \tilde{n} + \tilde{h}, \rho_t) < R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ_t . Population is such that $B_t < L$ and $\tilde{N}_t^p < L$.

Let us mention that a small increase of the income inequality may lead to a dramatic change of the urban configuration. The urban configuration in Figure 2 is obtained when $R(w_r, \tilde{n} + \tilde{h}, \rho_t) - R(w_p, \hat{n} + \hat{h}, \rho_t) < 0$. A small increase of income inequality, assimilated to an increase of the income gap between w_r and w_p can reverse the ranking between the slopes of both bid-rent curves, *i.e.* $R(w_r, \tilde{n} + \tilde{h}, \rho_t) - R(w_p, \hat{n} + \hat{h}, \rho_t) > 0$. It thus turns out that the city on the left of Figure 1 can emerge. Hence, an increase in income inequality can lead to a more balanced-distribution of believers in the whole city.

Finally, when bid-rents cross more than once, a urban configuration where poor believers and rich non-believers are both indifferent between living in area 1 or area 2 exists. All rich believers who are the highest bidders live in area 1 while the poor non-believers who are the lowest bidders inhabit area 2. It is depicted below.⁴



Figure 3: A urban configuration when bid-rents curves cross more. Both poor believers and rich non-believers are indifferent between the two areas.

The following numerical example provides an illustration of multiple crossing.

A numerical example with multiple equilibria. Let us assume that the instantaneous utility function is $u(x) = -x^2 + 2x + 1$. It is increasing for 0 < x < 1 and strictly concave. Given (3) and (4), we have multiple equilibria if and only if the following equations are simultaneously satisfied for different values of b_t^1

$$\frac{u(w_r) - u(w_r - \rho)}{(\tilde{n} + \tilde{h})} = a\left(\frac{2b_t^1 - \frac{B_t}{L}}{\Delta c}\right)\Delta w \tag{10}$$

$$\frac{u(w_r) - u(w_r - \rho)}{(\widetilde{n} + \widetilde{h})} = \frac{u(w_p) - u(w_p - \rho)}{(\widehat{n} + \widehat{h})}$$
(11)

Parameters are supposed to take the following values $w_r = 0.78$, $w_p = 0.39$, $\Delta w = 0.39$, $\Delta c = 0.75$, $\tilde{n} + \tilde{h} = 0.37$, $\hat{n} + \hat{h} = 0.9$ and a = 0.8. Consider that $\frac{B_t}{L} = 0.495$. We have $\Delta u \equiv u(w_r) - u(w_p) = 0.3237.^5$

⁴The interesting issue of selection of a particular equilibrium among a set of stable equilibria is beyond the scope of our paper.

⁵Note that these values satisfy both assumptions 2 and 3. Assumption 2 (i) $\Delta u/\Delta c = \frac{0.3237}{0.75} < 1$, (ii) $(\Delta c - \Delta u)/(\hat{n} + \hat{h}) = 0.47367 < 1$ (iii) $\Delta u/(\tilde{n} + \tilde{h}) = \frac{0.3237}{0.37} < 1$. Assumption 3 $(\Delta c - \Delta u)/(\hat{n} + \hat{h}) = 0.47367 > 1 - \frac{\Delta u}{(\tilde{n} + \tilde{h})} = 0.12514$.

There exist three urban equilibria:

	ρ	b_t^1	Stability
Culturally-Balanced City	0	$\frac{B_t}{2L} = 0.2475$	no
Culturally-Divided City	0.10453	0.4324	no
Culturally-Divided City	$\hat{\rho}^p(0.495) = 0.13661 > \tilde{\rho}^r(0.495) = 0.13297$	0.495	yes

We can compute that both duples ($\rho = 0, b_t^1 = \frac{B_t}{2L} = 0.2475$) and ($\rho = 0.10453, b_t^1 = 0.4324$) are solutions of (10)-(11). Moreover, according to (9) and Definition 2, the culturally-divided city ($\rho = 0.10453, b_t^1 = 0.4324$) is unstable as $R(w_r, \tilde{n} + \tilde{h}, 0.10453) = 0.21343 < R(w_p, \hat{n} + \tilde{h}, 0.10453) = 0.23579$. The third equilibrium is a culturally-divided one ($\rho = \hat{\rho}^p(0.495) = 0.13661, b_t^1 = 0.495$) and it is stable as $\hat{\rho}^p(0.495) = 0.13661 > \tilde{\rho}^r(0.495) = 0.13297$. Let us mention that the equilibrium with $b_t^{1*} = 0.4324$ would require that $0.4324 > \hat{N}^r > 0, \ \hat{N}^p > 0, \ \tilde{N}^r > L(1 - 0.4324)$ and $\tilde{N}^p > 0$.

4 Social Norms Dynamics in the Culturally-Divided City

We analyze the dynamics of social norms when at each date t a new urban configuration of the city emerges. We restrict the analysis to the case where $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$ and $R(w_p, \hat{n} + \hat{h}, \rho_t) > R(w_r, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ_t . Hence, at each date t a culturally-divided equilibrium with $b_t^{1*} = 1$ if $B_t \ge L$ or $b_t^{1*} = \frac{B_t}{L}$ if $B_t < L$ exists and is stable. Let us remark that no culturally-balanced equilibrium is stable when $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$. This restriction allows us to explore the dynamics of B_t without needing to study dynamics of the four state variables $\hat{N}_t^r, \tilde{N}_t^r, \tilde{N}_t^p$ and \hat{N}_t^p . Otherwise it would be required to make assumptions on how are the believers distributed among poor and rich adults. Such assumptions would necessarily be *ad hoc* in our setup⁶. We just impose that along the transitional path, $\hat{N}_t^z > 0$ and $\tilde{N}_t^z > 0$ for z = r, p which is obtained given Assumption 2, given that $\bar{p} = 1$ and $\underline{p} = 0$ and if the probability to become a believer is never 0 nor 1 for any income z individual.

From (5), we have:

$$B_{t+1} = B_t + L \sum_{j=1}^2 H(\lambda(b_t^j) - b_t^j)$$
(12)

Given (6)-(8) and (12), the "education" social norm dynamics when the culturally-divided

⁶We abstract from any intergenerational transmission mechanism of preferences for social norms that could be studied in a Bisin and Verdier (2001) framework for instance.

equilibrium arises at each date t can be expressed as follows:

$$B_{t+1} = B_t + L \left[H \left(\lambda_t \left(\frac{B_t}{L} \right) - \frac{B_t}{L} \right) + H(0) \right] \text{ when } \frac{B_t}{L} < 1$$

$$B_{t+1} = B_t + L \left[H(0) + H \left(\lambda_t \left(\frac{B_t}{L} - 1 \right) - \left(\frac{B_t}{L} - 1 \right) \right) \right] \text{ when } 1 \le \frac{B_t}{L} \le 2.$$

Assuming that H(x) = x allows us to conduct a tractable analysis of the dynamics as we obtain:

$$B_{t+1} = L\lambda_t \left(\frac{B_t}{L}\right) \text{ when } \frac{B_t}{L} < 1$$
$$B_{t+1} = L\lambda_t \left(\frac{B_t}{L} - 1\right) + L \text{ when } 1 \le \frac{B_t}{L} \le 2$$

where $\lambda_t \left(\frac{B_t}{L}\right)$ and $\lambda_t \left(\frac{B_t}{L}-1\right)$ are given by the dynamical system (6)-(8). In this dynamic setup with multiple urban equilibria, we have thus two notions of stability, the dynamic one and the spatial one. When we will speak about the stability of a steady-state equilibrium we refer to the dynamic one. While the stability of a urban equilibrium is defined in Definition 2.

We are able to offer the following:

Proposition 3 When a culturally-divided equilibrium emerges at each date t, there are three locally stable long-run equilibria: $B_1^{CB} = 0$, $B_2^{CB} = L$, $B_3^{CB} = 2L$.

Proof. See Appendix.

Proposition 3 allows us to depict the following social norms dynamics implied by a culturallydivided city. The complete characterization of the dynamical system is provided in the Appendix.



Figure 4: Social Norms Dynamics in the Culturally-Divided City.

When the culturally-divided equilibrium emerges at each date t, the implied dynamics exhibit three history-dependent stable steady states. Also depicted in the graph are two unstable steady states $B_1^{\star\star}$ and $B_2^{\star\star}$ with $0 < B_1^{\star\star} < L$ and $L < B_2^{\star\star} < 2L$. Consider the case where initially all believers inhabit area 1, *i.e.* $0 \le B_0 \le L$. When $0 \le B_0 \le B_1^{\star\star}$, the initial number of believers is not sufficient to generate incentives that lead to a spread of the "education" social norm. Hence, in the long run, the whole population becomes non-believer and depreciates education. When $B_1^{\star\star} \leq B_0 \leq L$, the city reaches a steady state such that the area 1 is populated exclusively by believers. A cultural clash arises as area 2 promotes exclusively the "no education" social norm. If now the number of believers is larger than L then, according to Proposition 2, believers are located in both urban areas. When $L \leq B_0 \leq B_2^{\star\star}$, believers in area 2 are not numerous enough to promote the "education" social norm and thus urban area 2 ends up with no believers in the long run. On the whole, when $B_1^{\star\star} \leq B_0 \leq B_2^{\star\star}$, in the long run, inhabitants in urban area 2 unanimously depreciate education and fall in a poverty trap. When $B_2^{\star\star} \leq B_0 \leq 2L$, the culturally-divided equilibrium is such that believers in area 2 are numerous enough to generate incentives to invest into education. The number of believers expands in the whole population and in the long run, the city is inhabited by believers and the level of education is maximized and equal to $B_3^{CD} = 2L^7$

⁷In the graph, $B_1^{\star\star} \in [LA, LC]$ and $B_2^{\star\star} \in [LA + L, LC + L]$. This may not be always the case. However, we are

Hence, following Crane (1991) terminology, the dynamics exhibit *epidemics* of social norm. The "education" social norm spreads in one urban area or in the entire city if the believers' population reaches a threshold value. On the contrary, if the believers' population stays below a threshold the population tends to a low-level equilibrium with the "no education" social norm prevailing in the city. Given these tipping dynamics, small differences in initial conditions may lead to dramatic differences in education rates and inequality dynamics.

For each steady state, the long-run distribution of income is the following:

at
$$B_1^{CD}$$
, $N^r = 0$ and $N^p = 2L$
at B_2^{CD} , $N^r = L$ and $N^p = L$
at B_3^{CD} , $N^r = 2L$ and $N^p = 0$.

Obviously, the steady state where the "no education" social norm is the rule of behavior will lead to the highest poverty rate while the steady state promoting education will lead to a homogenously rich population. For these two steady states, the education rate is the same in both urban areas and the rent paid in area 1 becomes nil. Moreover, the city reaching B_2^{CD} is characterized by an intermediate number of poor people and exhibits an uneven distribution of education as both urban areas are opposed with respect to the prevailing social norm.

5 Does a Culturally-Balanced City Lead to Higher Education?

We study the impact on the education rate of a public policy implemented by a central authority that would lead the culturally-balanced city to emerge. Precisely, under the assumption $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$, we design a fiscal policy that allows to restore spatial stability of the culturally-balanced equilibrium. The government implements a property tax paid only by rich people and redistributes the tax proceeds to poor individuals. Taxes and subsidies are defined such that poor non-believers are more willing to pay than rich believers to live in the most attractive urban area after a small perturbation of the culturally-balanced city, *i.e.* $\hat{\rho}^r(\frac{B_t+\varepsilon}{2L}) < \hat{\rho}^p(\frac{B_t+\varepsilon}{2L})$ at each date *t*. Precisely, let us consider a property tax τ_t which proceeds are equally redistributed among the poors. Given this taxation schedule, for each household, the willingness to pay would be such that

$$\frac{u(w_r) - u(w_r - \hat{\rho}^r(b_t^1)(1 + \tau_t))}{(\hat{n} + \hat{h})} = a\left(\frac{2b_t^1 - \frac{B_t}{L}}{\Delta c}\right)\Delta w$$

sure that $B_1^{\star\star} \in [LA, L]$ and $B_2^{\star\star} \in [L + LA, 2L]$ and when $B_1^{\star\star} \in [LA, LC]$, respectively $B_1^{\star\star} \in [LC, L]$, we have $B_2^{\star\star} \in [LA + L, LC + L]$, respectively $B_2^{\star\star} \in [LC + L, 2L]$.

$$\frac{u(w_p) - u\left(w_p - \tilde{\rho}^p(b_t^1) + \tau_t\left(\frac{\hat{\rho}^r(b_t^1)\hat{N}_t^r + \tilde{\rho}^r(b_t^1)\hat{N}_t^r}{\hat{N}_t^p + \tilde{N}_t^p}\right)\right)}{(\tilde{n} + \tilde{h})} = a\left(\frac{2b_t^1 - \frac{B_t}{L}}{\Delta c}\right)\Delta w$$

Given that $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$, $w_r > w_p$, and the concavity of u(.), we have

$$\widehat{\rho}^r(b_t^1)(1+\tau_t) > \widetilde{\rho}^p(b_t^1) - \tau_t \left(\frac{\widehat{\rho}^r(b_t^1)\widehat{N}_t^r + \widetilde{\rho}^r(b_t^1)\widetilde{N}_t^r}{\widehat{N}_t^p + \widetilde{N}_t^p}\right) \text{ for any } b_t^1 > 0$$

However, it is possible to choose τ_t such that $\hat{\rho}^r(\frac{B_t+\varepsilon}{2L}) < \tilde{\rho}^p(\frac{B_t+\varepsilon}{2L})$. Hence, from both Proposition (1) and Lemma (1), the equilibrium $[\rho_t^* = 0, b_t^{1*} = \frac{B_t}{2L}]$ exists and is stable at each date t.

If we consider that at each date t, $b_t^1 = b_t^2 = \frac{B_t}{2L}$ then given (12), the social norm dynamics writes down⁸:

$$B_{t+1} = B_t + 2L\left(\lambda_t\left(\frac{B_t}{2L}\right) - \frac{B_t}{2L}\right)$$

where $\lambda_t \left(\frac{B_t}{2L}\right)$ is given by equations (6)-(8).

Proposition 4 When a culturally-balanced equilibrium emerges at each date t, there are two locally stable long-run equilibria $B_1^{CB} = 0$, $B_2^{CB} = 2L$.

The dynamics exhibit two history-dependent stable steady states. If the city starts with a population of believers above the threshold B^* depicted in Figure 5 then incentives to exert high effort are sufficiently high in both urban areas leading the "education" social norm to spread in the population. Hence, in the long-run, all individuals believe in this social norm, $B_2^{BI} = 2L$. On the contrary, when the initial population of believers is below B^* then the "education" social norm vanishes. In the long-run, the whole city population adheres the "no education" social norm.

⁸Let us remind that a culturally-divided equilibrium could also arise. In order to study the dynamics, we make the crucial assumption that once the culturally-balanced equilibrium arises at date 0 it also arises subsequently, that is we prevent the city from alternating between the culturally-divided equilibrium and the culturally-balanced equilibrium along the transitional path. Nevertheless, a study of such a change of the urban configuration along the transitional path would be interesting but is beyond the scope of the paper.



Figure 5: Social Norms Dynamics in the Culturally-Balanced City.

Each steady state is characterized by a particular long-run distribution:

at
$$B_1^{CB} = 0$$
, $N^r = 0$ and $N^p = N$
at $B_1^{CB} = 0$, $N^r = N$ and $N^p = 0$.

We now turn to the issue whether the culturally-balanced city always perform better than a culturally-divided one in the long run regarding to the rate of education. We consider an economy where the stability of the culturally-balanced equilibrium is guaranteed by the fiscal policy considered above and where the culturally-divided equilibrium exists. Hence, we can show the following:

 $\begin{aligned} & \textbf{Proposition 5} \quad (i) \text{ When } B_1^{\star\star} \in [LA, LC] \text{ and } B_2^{\star\star} \in [LA + L, LC + L], \text{ we have } B^{\star} \in [2LA, 2LC] \\ & and B_1^{\star\star} \leq B^{\star} \leq B_2^{\star\star}. \\ & (ii) \text{ When } B_1^{\star\star} \in [LC, L] \text{ and } B_2^{\star\star} \in [LC + L, 2L], \text{ we have } B^{\star} \in [2LC, 2L], \\ & B_1^{\star\star} \quad \lessapprox \quad B^{\star} \text{ if and only if } L\left(-3\left(\frac{(\widetilde{n} + \widetilde{h})}{\Delta c} - \frac{\Delta u}{\Delta c}\right) + \left(\sqrt{2(1 - \frac{\Delta u}{\widetilde{n} + \widetilde{h}})}\right)\left(\left(\frac{2(\widetilde{n} + \widetilde{h})}{\Delta c} - \frac{\Delta u}{\Delta c}\right)\right)\right) \gtrless 0 \\ & and B^{\star} \quad \leq \quad B_2^{\star\star}. \end{aligned}$

This proposition stresses the fact that depending on the initial conditions (assuming that once a particular urban equilibrium arises initially it arises afterwards), the culturally-balanced city and the culturally-divided city may reach very different steady states. For instance, consider the case where $B_1^{\star\star} \leq B^{\star} \leq B_2^{\star\star}$ and let us start with B_0 such that $B_1^{\star\star} < B_0 < B^{\star}$. Hence, the culturally-divided city, respectively culturally-balanced city, reaches the steady-state $B_2^{CD} =$ L, respectively $B_1^{CB} = 0$. The culturally-divided city performs better in the long run than the culturally-balanced city because in case of concentration of the "education" social norm in area 1 the initial number of believers is sufficient for the "education" social norm to spread in urban area 1 while in case of cultural balancedness the same number of believers is too disseminated in the city for the "education" social norm to spread. However, if the economy starts with B_0 such that $B^{\star} < B_0 < B_2^{\star\star}$, the culturally-balanced city would perform better than the culturally-divided. The culturally-balanced city would end-up with a maximized rate of education while the culturallydivided city would exhibit long-run inequality between both urban areas. Let us mention that the numerical example presented below considers parameters such that we are in case (i) of Proposition 5, *i.e.* $B_1^{\star\star}/L = 0.30515 \leq B^{\star}/L = 0.6103 \leq B_2^{\star\star}/L = 1.3051.$

6 Conclusion

In a multicommunity model, we study how social norms on educational behavior spread in the city. When young, any individual who believes in a social norm has to decide his educational effort depending on some peer-pressure produced by the fraction of people in the neighborhood believing in the same social norm. When an adult, any individual chooses the place of residence for the whole family taking into account the educational prospect of her offspring. At each date, it turns out that multiple urban equilibria arise. First, there exists the culturally-balanced equilibrium where all urban areas are similar with respect to the produced neighborhood effects, that is to the fraction of people obeying a particular social norm. Second, there may also exist culturally-divided equilibria such that the composition of social norms followed by the population differs among urban areas. In particular, some culturally-divided equilibrium with a dramatic cultural contrast may arise: either all believers, or all non-believers, live in the same urban area. We then study the social norms dynamics implied by the culturally-divided city. We show that the dynamics exhibit epidemics so that if the fraction of people believing in a particular social norm is above some threshold then this social norm spreads in the urban area and possibly in the whole city. We also show that for some initial conditions the culturally-divided city experiences higher education in the long run than the culturally-balanced city.

Finally, this model relies on the specific dynamic mechanism that the social norm expands if the number of people following it exceeds the number of believers. Relaxing this assumption would allow us to extend the model in various ways. First, we could introduce cultural transmission à *la* Bisin-Verdier (2001) and study the interactions between endogenous formation of neighborhood and the incentives to transmit tastes for a particular social norm. Second, Acemoglu and Jackson (2012) studies another evolution process of social norms relying on agents' interpretations about the past. It would then be worth investigating a model where endogenous urban segregation shapes the information set of individuals and thus influences how history determines the evolution of social norms.

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7 Appendix

7.1 Proof of Proposition 2

A formal equivalent of Proposition 2 is

Proposition 6 Consider that $\hat{n} + \hat{h} > \tilde{n} + \tilde{h}$. At any date t, we have the following results

- 1. If $\hat{N}_t^r \ge L$ then the equilibrium $[\rho_t^* = \hat{\rho}^r(1), b_t^{1*} = 1]$ exists and it is stable.
- 2. If $\widehat{N}_{t}^{r} < L$, then we have the following results 2.a) If $R(w_{r}, \widetilde{n} + \widetilde{h}, \rho_{t}) > R(w_{p}, \widehat{n} + \widehat{h}, \rho_{t})$ for all ρ then (i) the equilibrium $[\rho_{t}^{*} = \widetilde{\rho}^{r}(\frac{\widehat{N}_{t}^{r}}{L}), b_{t}^{1*} = \frac{\widehat{N}_{t}^{r}}{L}]$ exists if $\widehat{N}_{t}^{r} \geq \frac{B_{t}}{2}$ and $\widetilde{N}_{t}^{r} + \widehat{N}_{t}^{r} > L$. It is stable. (ii) the equilibrium $[\rho_{t}^{*} = \widehat{\rho}^{p}(\frac{L-\widetilde{N}_{t}^{r}}{L}), b_{t}^{1*} = \frac{L-\widetilde{N}_{t}^{r}}{L}]$ exists if $L - \widetilde{N}_{t}^{r} > \frac{B_{t}}{2}$ and $\widetilde{N}_{t}^{r} + \widehat{N}_{t}^{r} < L$. It is stable. (iii) the equilibrium $[\rho_{t}^{*} = \widehat{\rho}^{p}(1), b_{t}^{1*} = 1]$ exists if $\widetilde{N}_{t}^{p} > L$. 2.b) If $R(w_{p}, \widehat{n} + \widehat{h}, \rho_{t}) > R(w_{r}, \widetilde{n} + \widetilde{h}, \rho_{t})$ for all ρ then, (i) the equilibrium $[\rho_{t}^{*} = \widehat{\rho}^{p}(1), b_{t}^{1*} = 1]$ exists if $B_{t} \geq L$. It is stable, (ii) the equilibrium $[\rho_{t}^{*} = \widehat{\rho}^{p}(\frac{B_{t}}{L}), b_{t}^{1*} = \frac{B_{t}}{L}]$ exists if $B_{t} < L$ and $\widetilde{N}_{t}^{p} < L$. It is stable. (iii) the equilibrium $[\rho_{t}^{*} = \widetilde{\rho}^{p}(\frac{B_{t}}{L}), b_{t}^{1*} = \frac{B_{t}}{L}]$ exists if $B_{t} < L$ and $\widetilde{N}_{t}^{p} > L$. It is stable. (iii) the equilibrium $[\rho_{t}^{*} = \widetilde{\rho}^{p}(\frac{B_{t}}{L}), b_{t}^{1*} = \frac{B_{t}}{L}]$ exists if $B_{t} < L$ and $\widetilde{N}_{t}^{p} > L$. It is stable. (iii) the equilibrium $[\rho_{t}^{*} = \widetilde{\rho}^{p}(\frac{B_{t}}{L}), b_{t}^{1*} = \frac{B_{t}}{L}]$ exists if $B_{t} < L$ and $\widetilde{N}_{t}^{p} > L$. It is stable. 2.c) If there exists some $b_{t} \in [\frac{B_{t}}{L}, 1]$ such that $\widetilde{\rho}^{r}(b) = \widehat{\rho}^{p}(b), L = \widehat{N}_{t}^{r} + \widehat{N}_{t}^{p,1} + \widetilde{N}_{t}^{r,1} = \widehat{N}_{t}^{p,2} + \widetilde{N}_{t}^{r,2} + \widetilde{N}_{t}^{p}$ with $\widehat{N}_{t}^{w_{r}} + \widehat{N}_{t}^{p,1} > \widehat{N}_{t}^{p,2}$ then the equilibrium $[\rho^{*} = \widetilde{\rho}^{r}(b_{t}), b_{t}^{1*} = b_{t}]$ exists. It is stable if and only if $R(w_{r}, \widetilde{n} + \widetilde{h}, \rho_{t}^{*}) - R(w_{p}, \widehat{n} + \widehat{h}, \rho_{t}^{*}) > 0$.

Proof. Case 1. When $\widehat{N}_t^r \ge L$, we consider the urban configuration such that urban area 1 is inhabited by only rich believers. It turns out that $b_t^1 = 1 > b_t^2$. Given Lemma 1, with $\rho = \widehat{\rho}^r(1)$,

we have:

$$\begin{split} u(w_r) - u(w_r - \widehat{\rho}^r(1)) &= a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w \\ u(w_r) - u(w_r - \widehat{\rho}^r(1)) &> a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\widetilde{n} + \widetilde{h})\Delta w \\ u(w_p) - u(w_p - \widehat{\rho}^r(1)) &> a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w \\ u(w_p) - u(w_p - \widehat{\rho}^r(1)) &> a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widetilde{h})\Delta w. \end{split}$$

The duple $[\hat{\rho}^r(1), b_t^1 = 1]$ is an equilibrium as no body has an incentive to move. Further, whatever the small perturbation of the equilibrium (immigration of individuals \tilde{N}^r or \hat{N}^p or also \tilde{N}^p in area 1), individuals \tilde{N}^r , \hat{N}^p and \tilde{N}^p still strictly prefer to live in urban area 2. This equilibrium is stable.

Case 2. When $\widehat{N}_t^r < L$, we have to consider three cases.

2.a) $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ . We will construct a urban configuration such that rich individuals whatever their beliefs live in an area which is at least better as the one inhabited by the poor individuals. (i) If $\hat{N}_t^r \geq \frac{B_t}{2}$, and given that $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ then it is possible that urban area is inhabited only by rich individuals such that $b_t^1 = \frac{\hat{N}_t^r}{L} > b_t^2$. Given Lemma 1 and that $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ , with $\rho = \tilde{\rho}^r(\frac{\hat{N}_t^r}{L})$ we have:

$$\begin{split} u(w_r) - u(w_r - \widetilde{\rho}^r(\frac{\widehat{N}_t^r}{L})) &< a\left(\frac{2\frac{\widehat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w \\ u(w_r) - u(w_r - \widetilde{\rho}^r(\frac{\widehat{N}_t^r}{L})) &= a\left(\frac{2\frac{\widehat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widetilde{n} + \widetilde{h})\Delta w \\ u(w_p) - u(w_p - \widehat{\rho}^r(\frac{\widehat{N}_t^r}{L})) &> a\left(\frac{2\frac{\widehat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w \\ u(w_p) - u(w_p - \widehat{\rho}^r(\frac{\widehat{N}_t^r}{L})) &> a\left(\frac{2\frac{\widehat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widetilde{h})\Delta w. \end{split}$$

Hence, no body has an incentive to move. The duple $[\tilde{\rho}^r(\frac{\hat{N}_t^r}{L}), b_t^1 = \frac{\hat{N}_t^r}{L}]$ is thus an equilibrium. Whatever the small perturbation of the area 1 population, individuals \hat{N}_t^r , respectively \hat{N}^p and \tilde{N}^p , still strictly prefer to live in urban area 1, respectively 2. This equilibrium is stable. If $\hat{N}_t^r < \frac{B_t}{2}$, it would not be possible that urban area 1 inhabited only by rich individuals would be such that $b_t^1 > b_t^2$. (ii) If $L - \tilde{N}_t^r > \frac{B_t}{2}$ and $\tilde{N}_t^r + \hat{N}_t^r < L$, and given that $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ then it is possible that urban area is inhabited by the whole rich population and a fraction of the \hat{N}^p population such that $b_t^1 = \frac{L - \tilde{N}_t^r}{L} > b_t^2$. Given Lemma 1 and that $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ , we have:

$$\begin{aligned} u(w_r) - u(w_r - \hat{\rho}^p(\frac{L - \tilde{N}_t^r}{L})) &< a\left(\frac{2\left(\frac{L - \tilde{N}_t^r}{L}\right) - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w \\ u(w_r) - u(w_r - \hat{\rho}^p(\frac{L - \tilde{N}_t^r}{L})) &< a\left(\frac{2\left(\frac{L - \tilde{N}_t^r}{L}\right) - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w \\ u(w_p) - u(w_p - \hat{\rho}^p(\frac{L - \tilde{N}_t^r}{L})) &= a\left(\frac{2\left(\frac{L - \tilde{N}_t^r}{L}\right) - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w \\ u(w_p) - u(w_p - \hat{\rho}^p(\frac{L - \tilde{N}_t^r}{L})) &> a\left(\frac{2\left(\frac{L - \tilde{N}_t^r}{L}\right) - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w. \end{aligned}$$

No one has an incentive to move. The duple $[\hat{\rho}^p(\frac{L-\tilde{N}_t^r}{L}), b_t^1 = \frac{L-\tilde{N}_t^r}{L}]$ is thus an equilibrium. Whatever the small perturbation of the area 1 population, individuals $\hat{N}_t^{w_r}$ and \tilde{N}^r , respectively \tilde{N}^p still strictly prefer to live in urban area 1, respectively 2. This equilibrium is stable. (iii) If $\tilde{N}_t^p > L$, given Lemma 1 and that $R(w_r, \tilde{n} + \tilde{h}, \rho_t) > R(w_p, \hat{n} + \hat{h}, \rho_t)$ for all ρ then, if urban area 1 is inhabited by the whole believer population, that is $b_t^1 = 1$, we have

$$u(w_r) - u(w_r - \tilde{\rho}^p(1)) < a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_r) - u(w_r - \tilde{\rho}^p(1)) < a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

$$u(w_p) - u(w_p - \tilde{\rho}^p(1)) < a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_p) - u(w_p - \tilde{\rho}^p(1)) = a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

No one has an incentive to move. The duple $[\tilde{\rho}^p(1), b_t^1 = 1]$ is thus an equilibrium. After a small reduction of the number of believers in area 1, individuals \hat{N}_t^r , \tilde{N}^r and \hat{N}^p still strictly prefer to live in urban area 1. This equilibrium is stable.

Items (i), (ii) and (iii) cover the whole possible cases.

2.b) $R(w_p, \hat{n} + \hat{h}, \rho_t) > R(w_r, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ . Poor believers are thus able to outbid rich non-believers to live in the better neighborhood. We thus have to construct urban configuration where all believers live in a area at least as good as the area inhabited by non-believers. (i) If

 $B_t \ge L$, given Lemma 1 and that $R(w_p, \hat{n} + \hat{h}, \rho_t) > R(w_r, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , then if urban area 1 is inhabited by the whole believer population, that is $b_t^1 = 1$, we have:

$$u(w_r) - u(w_r - \hat{\rho}^p(1)) < a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_r) - u(w_r - \hat{\rho}^p(1)) > a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

$$u(w_p) - u(w_p - \hat{\rho}^p(1)) = a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_p) - u(w_p - \tilde{\rho}^p(1)) > a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w.$$

No one has an incentive to move. The duple $[\hat{\rho}^p(1), b_t^1 = 1]$ is thus an equilibrium. After a small reduction of the number of believers in area 1, individuals \hat{N}_t^r , respectively \tilde{N}^r and \tilde{N}^p , still strictly prefer to live in urban area 1, respectively 2. This equilibrium is stable. (ii) If $B_t < L$ and $\tilde{N}_t^p < L$, given Lemma 1 and that $R(w_p, \hat{n} + \hat{h}, \rho_t) > R(w_r, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , then if urban area 1 is inhabited by the whole believer population, that is $b_t^1 = \frac{B_t}{L}$, with $\rho = \tilde{\rho}^r(\frac{B_t}{L})$ we have:

$$u(w_r) - u(w_r - \tilde{\rho}^r(\frac{B_t}{L})) < a\left(\frac{B_t}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_r) - u(w_r - \tilde{\rho}^r(\frac{B_t}{L})) = a\left(\frac{B_t}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

$$u(w_p) - u(w_p - \tilde{\rho}^r(\frac{B_t}{L})) < a\left(\frac{B_t}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_p) - u(w_p - \tilde{\rho}^r(\frac{B_t}{L})) > a\left(\frac{B_t}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w.$$

No one has an incentive to move. The duple $[\tilde{\rho}^r(\frac{B_t}{L}), b_t^1 = \frac{B_t}{L}]$ is thus an equilibrium. After a small reduction of the number of believers in area 1, individuals \hat{N}_t^r and \hat{N}^p , respectively \tilde{N}^p , still strictly prefer to live in urban area 1, respectively 2. This equilibrium is stable. (iii) If $B_t < L$ and $\tilde{N}_t^p > L$, given Lemma 1 and that $R(w_p, \hat{n} + \hat{h}, \rho_t) > R(w_r, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , then if urban area 1

is inhabited by the whole believer population, that is $b_t^1 = \frac{B_t}{L}$, with $\rho = \tilde{\rho}^p(\frac{B_t}{L})$ we have:

$$\begin{split} u(w_r) &- u(w_r - \widetilde{\rho}^p(\frac{B_t}{L})) < a\left(\frac{B_t}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w \\ u(w_r) &- u(w_r - \widetilde{\rho}^p(\frac{B_t}{L})) < a\left(\frac{B_t}{\Delta c}\right)(\widetilde{n} + \widetilde{h})\Delta w \\ u(w_p) &- u(w_p - \widetilde{\rho}^p(\frac{B_t}{L})) < a\left(\frac{B_t}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w \\ u(w_p) &- u(w_p - \widetilde{\rho}^p(\frac{B_t}{L})) = a\left(\frac{B_t}{\Delta c}\right)(\widetilde{n} + \widetilde{h})\Delta w. \end{split}$$

No one has an incentive to move. The duple $[\tilde{\rho}^{p}(\frac{B_{t}}{L}), b_{t}^{1} = \frac{B_{t}}{L}]$ is thus an equilibrium. After a small reduction of the number of believers in area 1, individuals \hat{N}_{t}^{r} , \hat{N}^{p} and \tilde{N}^{r} still strictly prefer to live in urban area 1. This equilibrium is stable.

2.c) Let us now consider that indifference curves may cross more than once. If for some $b_t \in [\frac{B_t}{2L}, 1]$, we have ρ such that

$$u(w_r) - u(w_r - \rho) < a\left(\frac{2b_t - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_r) - u(w_r - \rho) = a\left(\frac{2b_t - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

$$u(w_p) - u(w_p - \rho) = a\left(\frac{2b_t - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_p) - u(w_p - \rho) > a\left(\frac{2 - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

then $[\rho = \tilde{\rho}^r(b_t) = \hat{\rho}^p(b_t), b_t^1 = b_t]$ is an equilibrium as no one has an incentive to move. Let us consider a move of ε individuals \hat{N}^p from area 2 in area 1 and a reverse move of individuals \tilde{N}^r . If and only if $R(w_r, \tilde{n} + \tilde{h}, \rho) - R(w_p, \hat{n} + \tilde{h}, \rho) > 0$, individuals \tilde{N}^r who have been jarred out from area 1 are able to outbid individuals \hat{N}^p to live in area 1.

7.2 Existence and stability of a culturally-divided equilibrium when $\hat{n} + \hat{h} < \tilde{n} + \tilde{h}$

Proposition 7 Consider that $\hat{n} + \hat{h} < \tilde{n} + \tilde{h}$. At any date t,

- 1. If $R(w_r, \hat{n} + \hat{h}, \rho_t) < R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , no culturally-divided equilibrium exists.
- 2. If $R(w_r, \hat{n} + \hat{h}, \rho_t) > R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ then (i) the equilibrium $[\rho_t = \hat{\rho}_t^r(\frac{\hat{N}_t^{r,1}}{L}), b_t^1 = \frac{\hat{N}_t^{r,1}}{L}]$ exists if $\hat{N}_t^r > \frac{B_t}{2}$ and $L - \tilde{N}_t^r = \hat{N}_t^{r,1} > \frac{B_t}{2}$. It is stable.

(ii) the equilibrium
$$[\rho_t = \widetilde{\rho}_t^p(\frac{\widehat{N}_t^r}{L}), b_t^1 = \frac{\widehat{N}_t^r}{L}]$$
 exists if $\widehat{N}_t^r > \frac{B_t}{2}, \widetilde{N}_t^r + \widehat{N}_t^r < L$ and $\widetilde{N}_t^r + \widehat{N}_t^r + \widetilde{N}_t^p > L$. It is stable.

Proof. Case 1. If $R(w_r, \hat{n} + \hat{h}, \rho_t) < R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , non-believers individuals are able to outbid believers to live in a better neighborhood. Given that $R(w_r, \hat{n} + \hat{h}, \rho_t) < R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ and Lemma 1, it is thus impossible to have an equilibrium with $b_t^1 > b_t^2$ as all nonbelievers living in area 2 would have an incentive to move in area 1 and can outbid its non-believers inhabitants.

Case 2. If $R(w_r, \hat{n} + \hat{h}, \rho_t) > R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , rich believers are able to outbid poor non-believers. Given that $R(w_r, \hat{n} + \hat{h}, \rho_t) > R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ and Lemma 1, we can build an equilibrium with $b_t^1 > b_t^2$. Two cases must be considered. (i) When $\hat{N}_t^r > \frac{B_t}{2}$ and $L - \tilde{N}_t^r = \hat{N}_t^{r,1} > \frac{B_t}{2}$, given Lemma 1 and that $R(w_r, \hat{n} + \hat{h}, \rho_t) > R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , then if urban area 1 is inhabited by the whole rich non-believers population and a number $\hat{N}_t^{r,1}$ of rich believers, that is $b_t^1 = \frac{\hat{N}_t^{r,1}}{L}$, with $\rho = \hat{\rho}^r(\frac{\hat{N}_t^{r,1}}{L})$ we have:

$$u(w_r) - u(w_r - \hat{\rho}^r(\frac{\widehat{N}_t^{r,1}}{L})) < a\left(\frac{2\frac{\widehat{N}_t^{r,1}}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widetilde{n} + \widetilde{h})\Delta w$$

$$u(w_r) - u(w_r - \hat{\rho}^r(\frac{\widehat{N}_t^{r,1}}{L})) = a\left(\frac{2\frac{\widehat{N}_t^{r,1}}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w$$

$$u(w_p) - u(w_p - \hat{\rho}^r(\frac{\widehat{N}_t^{r,1}}{L})) > a\left(\frac{2\frac{\widehat{N}_t^{r,1}}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widetilde{n} + \widetilde{h})\Delta w$$

$$u(w_p) - u(w_p - \hat{\rho}^r(\frac{\widehat{N}_t^{r,1}}{L})) > a\left(\frac{2\frac{\widehat{N}_t^{r,1}}{L} - \frac{B_t}{L}}{\Delta c}\right)(\widehat{n} + \widehat{h})\Delta w$$

The duple $[\hat{\rho}^r(\frac{\widehat{N}_t^{r,1}}{L}), b_t^1 = \frac{\widehat{N}_t^{r,1}}{L}]$ is an equilibrium as no body has an incentive to move. Further, whatever the small perturbation of the equilibrium, individuals \widetilde{N}^r , respectively \widehat{N}^p and \widetilde{N}^p , still strictly prefer to live in urban area 1, respectively 2. This equilibrium is stable.

(ii) When $\widehat{N}_t^r > \frac{B_t}{2}$, $\widetilde{N}_t^r + \widehat{N}_t^r < L$ and $\widetilde{N}_t^r + \widehat{N}_t^r + \widetilde{N}_t^p > L$, given Lemma 1 and that $R(w_r, \widehat{n} + \widehat{h}, \rho_t) > R(w_p, \widetilde{n} + \widetilde{h}, \rho_t)$ for all ρ , then if urban area 1 is inhabited by the whole rich population,

 $b_t^1 = \frac{\widehat{N}_t}{L}$ and $\rho = \widetilde{\rho}^p(\frac{\widehat{N}_t^r}{L})$ we have:

$$u(w_r) - u(w_r - \tilde{\rho}^p(\frac{\hat{N}_t^r}{L})) < a\left(\frac{2\frac{\hat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

$$u(w_r) - u(w_r - \tilde{\rho}^p(\frac{\hat{N}_t^r}{L})) < a\left(\frac{2\frac{\hat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\hat{n} + \hat{h})\Delta w$$

$$u(w_p) - u(w_p - \tilde{\rho}^p(\frac{\hat{N}_t^r}{L})) = a\left(\frac{2\frac{\hat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w$$

$$u(w_p) - u(w_p - \tilde{\rho}^p(\frac{\hat{N}_t^r}{L})) > a\left(\frac{2\frac{\hat{N}_t^r}{L} - \frac{B_t}{L}}{\Delta c}\right)(\tilde{n} + \tilde{h})\Delta w.$$

The duple $[\tilde{\rho}^{p}(\frac{\hat{N}_{t}^{r}}{L}), b_{t}^{1} = \frac{\hat{N}_{t}^{r}}{L}]$ is an equilibrium as no body has an incentive to move. Further, whatever the small perturbation of the equilibrium, individuals \tilde{N}^{r} , \hat{N}^{r} , respectively \tilde{N}^{p} , still strictly prefer to live in urban area 1, respectively 2. This equilibrium is stable.

When $\hat{N}_t^r > \frac{B_t}{2}$, $\tilde{N}_t^r + \hat{N}_t^r < L$ and $\tilde{N}_t^r + \hat{N}_t^r + \tilde{N}_t^p < L$, given Lemma 1 and that $R(w_r, \hat{n} + \hat{h}, \rho_t) > R(w_p, \tilde{n} + \tilde{h}, \rho_t)$ for all ρ , it is impossible to have an equilibrium such that $b_t^1 > b_t^2$ as area 2 would be inhabited by poor believers.

When the rich believers are able to outbid poor non-believers, a culturally-divided equilibrium with income segregation may emerge if the size of the rich believers population living in area 1 is large enough to make this location more attractive: either all the rich individuals live in area 1 (item (i)) either the whole poor population resides in area 2 (item (ii)).

Let us remark that under the assumption $\hat{n} + \hat{h} < \tilde{n} + \tilde{h}$ the constraint that area 1 must contain more believers than in area 2 is more stringent. Consider case 2 where $R(w_r, \hat{n} + \hat{h}, \rho) > R(w_p, \tilde{n} + \tilde{h}, \rho)$. The urban configuration where area 2 is inhabited by only poor believers cannot be an equilibrium as area 2 becomes more attractive than area 1.

7.3 Proof of Proposition 3

Proof. From (6)-(8) and (12), the dynamics of a culturally-divided equilibrium are characterized as follows:

$$B_{t+1} = \begin{cases} \frac{(\Delta u)B_t + \frac{B_t^2}{L}(\hat{n}+\hat{h})}{\Delta c} \text{ when } B_t \in [0, LA] \\ \frac{B_t^2}{L} \frac{\Delta(n+h)}{\Delta c} + B_t \frac{2(\tilde{n}+\tilde{h})}{\Delta c} + L\left(\frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c}\right) \text{ when } B_t \in [LA, LC] \\ -\frac{B_t^2}{L} \frac{(\tilde{n}+\tilde{h})}{\Delta c} + B_t(1 - \frac{\Delta u}{\Delta c} + \frac{2(\tilde{n}+\tilde{h})}{\Delta c}) + L\left(\frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c}\right) \text{ when } B_t \in [LC, L] \\ \frac{B_t^2}{L} \frac{(\hat{n}+\hat{h})}{\Delta c} + B_t\left(\frac{\Delta u - 2(\hat{n}+\hat{h})}{\Delta c}\right) + L\frac{(\hat{n}+\hat{h}) - \Delta u}{\Delta c} + L \text{ when } B_t \in [L, L + LA] \\ \frac{B_t^2}{L} \frac{\Delta(n+h)}{\Delta c} + 2B_t\left(\frac{(\tilde{n}+\tilde{h}) - \Delta(n+h)}{\Delta c}\right) + L\left(\frac{\Delta(n+h)}{\Delta c} - \frac{3(\tilde{n}+\tilde{h})}{\Delta c} + \frac{\Delta u}{\Delta c} + 1\right) \text{ when } B_t \in [L + LA, L + LC] \\ -\frac{B_t^2}{L} \frac{(\tilde{n}+\tilde{h})}{\Delta c} + B_t(1 - \frac{\Delta u}{\Delta c} + \frac{4(\tilde{n}+\tilde{h})}{\Delta c}) + L\left(\frac{2\Delta u - 4(\tilde{n}+\tilde{h})}{\Delta c}\right) \text{ when } B_t \in [L + LC, 2L]. \end{cases}$$

 $\begin{array}{l} B_{t+1}(B_t) \text{ is a continuous function of } B_t. \text{ It is easy to check that } B_{t+1}(B_t) \text{ is increasing over}\\ [0,2L]. Further, \ B_{t+1}(0) = 0. \text{ From Assumption 1, we know that } B_{t+1}'(0) = \frac{(\Delta u)}{\Delta c} < 1. \text{ Hence,}\\ \lim_{B_t \to 0^+} (B_{t+1}(B_t) - B_t) < 0. \text{ We have } B_{t+1}(L) = L. \text{ From Assumption 1, we have } 0 < \lim_{B_t \to L^-} B_{t+1}'(B_t) = \\ 1 - \frac{\Delta u}{\Delta c} < 1 \text{ and } 0 < \lim_{B_t \to L^+} B_{t+1}'(B_t) = \frac{\Delta u}{\Delta c} < 1. \text{ Hence, } B_{t+1}(B_t) \text{ must cross the 45^{\circ} line at } B_t = L\\ \text{from above, formally we must have } \lim_{B_t \to L^-} (B_{t+1}(B_t) - B_t) > 0 \text{ and } \lim_{B_t \to L^+} (B_{t+1}(B_t) - B_t) < 0.\\ \text{We have } B_{t+1}(2L) = 2L. \text{ From Assumption 1, we know that } 0 < B_{t+1}'(2L) = 1 - \frac{(\Delta u)}{\Delta c} < 1. \text{ Hence,}\\ B_{t+1}(B_t) \text{ must cross the 45^{\circ} line at } B_t = 2L \text{ from above, formally } \lim_{B_t \to 2L} (B_{t+1}(B_t) - B_t) > 0. \text{ As }\\ B_{t+1}(0) = 0, \lim_{B_t \to 0^+} (B_{t+1}(B_t) - B_t) < 0, \text{ and given that } B_{t+1} \text{ crosses twice the the 45^{\circ} line from above at } B_t = L \text{ and } B_t = 2L, \text{ it must also cross twice the 45^{\circ} line from below. We deduce that there exist } B_1^{\star\star} \text{ and } B_2^{\star\star} \text{ with } 0 < B_1^{\star\star} < L < B_2^{\star\star} < 2L \text{ such that } B_{t+1}(B_1^{\star\star}) = B_1^{\star\star} \text{ and } \\\\ B_{t+1}(B_2^{\star\star}) = B_2^{\star\star}. \end{aligned}$

Further, we can provide some information about $B_1^{\star\star}$ and $B_2^{\star\star}$:

(i) We have $B_1^{\star\star} > LA$ and $B_2^{\star\star} > L + LA$. It amounts to show that $B_{t+1}(LA) < LA$ which is equivalent to $B_{t+1}(L+LA) < L+LA$. Hence, $B_{t+1}(LA) < LA$ can be written as follows

$$\frac{\Delta u}{\Delta c} + \frac{A(\widehat{n} + \widehat{h})}{\Delta c} < 1$$

Given that $A \equiv 1 - \frac{\Delta u}{(\tilde{n} + \tilde{h})}$ we have

$$\frac{\Delta u}{\Delta c} + \left(\frac{\widehat{n} + \widehat{h}}{\Delta c} - \frac{\Delta u}{(\widetilde{n} + \widetilde{h})} * \frac{(\widehat{n} + \widehat{h})}{\Delta c}\right) < 1$$

which is equivalent to

$$\frac{\Delta u}{\Delta c} \left(1 - \frac{(\widehat{n} + \widehat{h})}{(\widetilde{n} + \widetilde{h})} \right) + \left(\frac{\widehat{n} + \widehat{h}}{\Delta c} \right) < 1$$

which can be written as

$$\frac{1}{\frac{\Delta u}{\Delta c}\left(-\frac{\Delta(n+h)}{(\tilde{n}+\tilde{h})}\right) + \left(\frac{(\hat{n}+\hat{h})}{\Delta c}\right)} > 1.$$

Multiplying both the numerator and denominator of the LHS of this inequality by $\Delta c/(\hat{n}+\hat{h})$ leads to

$$\frac{\frac{\Delta c}{(\widehat{n}+\widehat{h})}}{\frac{\Delta u}{(\widehat{n}+\widehat{h})}\left(-\frac{\Delta(n+h)}{(\widetilde{n}+\widetilde{h})}\right)+1} > 1$$

which is equivalent to

$$\frac{\Delta c}{(\widehat{n}+\widehat{h})}-\frac{\Delta u}{(\widehat{n}+\widehat{h})}>1-\frac{\Delta u}{(\widetilde{n}+\widetilde{h})}$$

which is Assumption 2. Hence, the result.

 $(ii) \text{ If } B_{t+1}(LC) \ge LC \Leftrightarrow B_{t+1}(L+LC) \ge L+LC \Leftrightarrow (C)^2 L \frac{\Delta(n+h)}{\Delta c} + CL \frac{2(\tilde{n}+\tilde{h})}{\Delta c} + L\left(\frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c}\right) \ge LC \text{ then } LA \le B_1^{\star\star} \le LC \text{ and that } LA + L \le B_2^{\star\star} \le LC + L.$

 $\begin{array}{l} (iii) \text{ If } B_{t+1}(LC) < LC \Leftrightarrow B_{t+1}(L+LC) \geq L+LC \Leftrightarrow (C)^2 L \frac{\Delta(n+h)}{\Delta c} + CL \frac{2(\tilde{n}+\tilde{h})}{\Delta c} + L \left(\frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c} \right) < LC \text{ then } LC \leq B_1^{\star\star} \leq L \text{ and } LC + L \leq B_2^{\star\star} \leq 2L. \quad \blacksquare \end{array}$

7.4 Proof of Proposition 4

Proof. From (6)-(8), the rate of education in a culturally-balanced equilibrium equals at each date t

$$\lambda_t \left(\frac{B_t}{2L}\right) = \begin{cases} \frac{(\Delta u)\frac{B_t}{2L} + \left(\frac{B_t}{2L}\right)^2(\hat{n}+\hat{h})}{\Delta c} & \text{when } 0 \le B_t \le 2LA \\ \left(\frac{B_t}{2L}\right)^2 \frac{\Delta(n+h)}{\Delta c} + \frac{B_t}{2L} \frac{2(\tilde{n}+\tilde{h})}{\Delta c} - \frac{(\tilde{n}+\tilde{h})}{\Delta c} + \frac{\Delta u}{\Delta c} & \text{when } 2LA \le B_t \le 2LC \\ - \left(\frac{B_t}{2L}\right)^2 \frac{(\tilde{n}+\tilde{h})}{\Delta c} + \frac{B_t}{2L} (1 - \frac{\Delta u}{\Delta c} + \frac{2(\tilde{n}+\tilde{h})}{\Delta c}) + \frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c} & \text{when } 2LC \le B_t \le 2L. \end{cases}$$

Hence, from (12), we have

$$B_{t+1} = \begin{cases} B_t^2 \frac{1}{2L} \frac{(\hat{n} + \hat{h})}{\Delta c} + B_t \frac{(\Delta u)}{\Delta c} \text{ when } 0 \le B_t \le 2LA \\ B_t^2 \frac{1}{2L} \frac{\Delta(n+h)}{\Delta c} + B_t \frac{2(\tilde{n} + \tilde{h})}{\Delta c} + 2L \left(\frac{\Delta u - (\tilde{n} + \tilde{h})}{\Delta c}\right) \text{ when } 2LA \le B_t \le 2LC \\ -B_t^2 \frac{1}{2L} \frac{(\tilde{n} + \tilde{h})}{\Delta c} + B_t \left(1 - \frac{\Delta u}{\Delta c} + \frac{2(\tilde{n} + \tilde{h})}{\Delta c}\right) + 2L \left(\frac{\Delta u - (\tilde{n} + \tilde{h})}{\Delta c}\right) \text{ when } 2LC \le B_t \le 2L. \end{cases}$$

 $B_{t+1}(B_t)$ is a continuous function of B_t . It is easy to check that $B_{t+1}(B_t)$ is increasing over [0, 2L]. Further, $B_{t+1}(0) = 0$. From Assumption 1, we know that $B'_{t+1}(0) = \frac{(\Delta u)}{\Delta c} < 1$. Hence, $\lim_{B_t \to 0^+} (B_{t+1}(B_t) - B_t) < 0$. We have $B_{t+1}(2L) = 2L$. From Assumption 1, we know that $0 < B'_{t+1}(2L) = 1 - \frac{(\Delta u)}{\Delta c} < 1$. Hence, $B_{t+1}(B_t)$ must cross the 45° line from above at $B_t = 2L$, formally $\lim_{B_t \to 2L^-} (B_{t+1}(B_t) - B_t) > 0$. As $B_{t+1}(0) = 0$ and $\lim_{B_t \to 0^+} (B_{t+1}(B_t) - B_t) < 0$ and $B_{t+1}(B_t)$ must cross the 45° line from above at $B_t = 2L$, it is easy to deduce that $B_{t+1}(B_t)$ must cross the 45° line from below once. Hence, there exists $0 < B^* < 2L$ such that $B_{t+1}(B^*) = B^*$.

Let us provide information on B^* :

(i) We can first show that $B^* > 2LA$. This amounts to show that $B_{t+1}(B_t) < B_t$ for $B_t \in [0, 2LA]$. Hence, this inequality is equivalent to

$$\frac{(\Delta u) + B_t(\hat{n} + \hat{h})}{\Delta c} < 1$$

For $B_t = 2LA$, we have

$$\frac{\Delta u}{\Delta c} + \frac{A(\hat{n} + \hat{h})}{\Delta c} < 1$$

Given that $A \equiv 1 - \frac{\Delta u}{(\tilde{n} + \tilde{h})}$ we have

$$\frac{\Delta u}{\Delta c} + \left(\frac{\widehat{n} + \widehat{h}}{\Delta c} - \frac{\Delta u}{(\widetilde{n} + \widetilde{h})} * \frac{(\widehat{n} + \widehat{h})}{\Delta c}\right) < 1$$

which is equivalent to

$$\frac{\Delta u}{\Delta c} \left(1 - \frac{(\widehat{n} + \widehat{h})}{(\widetilde{n} + \widetilde{h})} \right) + \left(\frac{\widehat{n} + \widehat{h}}{\Delta c} \right) < 1$$

which can be written

$$\frac{1}{\frac{\Delta u}{\Delta c} \left(-\frac{\Delta(n+h)}{(\tilde{n}+\tilde{h})}\right) + \left(\frac{(\hat{n}+\hat{h})}{\Delta c}\right)} > 1.$$

Multiplying both the numerator and denominator of the LHS of this inequality by $\Delta c/(\hat{n}+\hat{h})$ leads to

$$\frac{\frac{\Delta c}{(\hat{n}+\hat{h})}}{\frac{\Delta u}{(\hat{n}+\hat{h})}\left(-\frac{\Delta(n+h)}{(\tilde{n}+\tilde{h})}\right)+1} > 1$$

which is equivalent to

$$\frac{\Delta c}{(\widehat{n}+\widehat{h})} - \frac{\Delta u}{(\widehat{n}+\widehat{h})} > 1 - \frac{\Delta u}{(\widetilde{n}+\widetilde{h})}$$

which is Assumption 2. Hence, the result.

(ii) If $B_{t+1}(2LA) \leq 2LA \Leftrightarrow \frac{(\Delta u) + A(\hat{n} + \hat{h})}{\Delta c} \leq 1$ and $B_{t+1}(2LC) > 2LC \Leftrightarrow (C)^2 L \frac{\Delta(n+h)}{\Delta c} + CL \frac{2(\tilde{n} + \tilde{h})}{\Delta c} + L\left(\frac{\Delta u - (\tilde{n} + \tilde{h})}{\Delta c}\right) > LC$ then $2LA \leq B^* \leq 2LC$. Considering item (ii) of Proposition 3, we can see that when $LA \leq B_1^{**} \leq LC$ and that $LA + L \leq B_2^{**} \leq LC + L$ we also have $2LA \leq B^* \leq 2LC$.

(iii) If $B_{t+1}(2LC) \leq 2LC \Leftrightarrow (C)^2 L \frac{\Delta(n+h)}{\Delta c} + CL \frac{2(\tilde{n}+\tilde{h})}{\Delta c} + L\left(\frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c}\right) \leq LC$ then $2LC \leq B^* \leq 2L$. Considering item (iii) of Proposition 3, we can see that when $LC \leq B_1^{\star\star} \leq L$ and $LC + L \leq B_2^{\star\star} \leq 2L$ we also have $2LC \leq B^* \leq 2L$.

7.5 Proof of Proposition 5

Proof. We already know that when $B_1^{\star\star} \in [LA, LC]$ and $B_2^{\star\star} \in [LA + L, LC + L]$, respectively $B_1^{\star\star} \in [LC, L]$ and $B_2^{\star\star} \in [LC + L, 2L]$, we have $B^{\star} \in [2LA, 2LC]$, respectively $B^{\star} \in [2LC, 2L]$. Let us write the dynamical system for the culturally-balanced equilibrium

$$B_{t+1}^{*} = \begin{cases} B_{t}^{2} \frac{1}{2L} \frac{(\hat{n} + \hat{h})}{\Delta c} + B_{t} \frac{(\Delta u)}{\Delta c} \text{ when } 0 \leq B_{t} \leq 2LA \\ B_{t}^{2} \frac{1}{2L} \frac{\Delta(n+h)}{\Delta c} + B_{t} \frac{2(\tilde{n} + \tilde{h})}{\Delta c} + 2L \left(\frac{\Delta u - (\tilde{n} + \tilde{h})}{\Delta c}\right) \text{ when } 2LA \leq B_{t} \leq 2LC \\ -B_{t}^{2} \frac{1}{2L} \frac{(\tilde{n} + \tilde{h})}{\Delta c} + B_{t} (1 - \frac{\Delta u}{\Delta c} + \frac{2(\tilde{n} + \tilde{h})}{\Delta c}) + 2L \left(\frac{\Delta u - (\tilde{n} + \tilde{h})}{\Delta c}\right) \text{ when } 2LC \leq B_{t} \leq 2L. \end{cases}$$

and for the culturally-divided equilibrium

$$B_{t+1}^{\star\star} = \begin{cases} \frac{(\Delta u)B_t + \frac{B_t^2}{L}(\hat{n}+\hat{h})}{\Delta c} \text{ when } 0 \leq B_t \leq LA \\ \frac{B_t^2}{L} \frac{\Delta(n+h)}{\Delta c} + B_t \frac{2(\tilde{n}+\tilde{h})}{\Delta c} + L\left(\frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c}\right) \text{ when } LA \leq B_t \leq LC \\ -\frac{B_t^2}{L} \frac{(\tilde{n}+\tilde{h})}{\Delta c} + B_t (1 - \frac{\Delta u}{\Delta c} + \frac{2(\tilde{n}+\tilde{h})}{\Delta c}) + L\left(\frac{\Delta u - (\tilde{n}+\tilde{h})}{\Delta c}\right) \text{ when } LC \leq B_t \leq L \\ \frac{B_t^2}{L} \frac{(\hat{n}+\hat{h})}{\Delta c} + B_t\left(\frac{\Delta u - 2(\hat{n}+\hat{h})}{\Delta c}\right) + L\frac{(\hat{n}+\hat{h}) - \Delta u}{\Delta c} + L \text{ when } L \leq B_t \leq L + LA \\ \frac{B_t^2}{L} \frac{\Delta(n+h)}{\Delta c} + 2B_t\left(\frac{(\tilde{n}+\tilde{h}) - \Delta(n+h)}{\Delta c}\right) + L\left(\frac{\Delta(n+h)}{\Delta c} - \frac{3(\tilde{n}+\tilde{h})}{\Delta c} + \frac{\Delta u}{\Delta c} + 1\right) \text{ when } L + LA \leq B_t \leq L + LC \\ -\frac{B_t^2}{L} \frac{(\tilde{n}+\tilde{h})}{\Delta c} + B_t(1 - \frac{\Delta u}{\Delta c} + \frac{4(\tilde{n}+\tilde{h})}{\Delta c}) + L\left(\frac{2\Delta u - 4(\tilde{n}+\tilde{h})}{\Delta c}\right) \text{ when } L + LC \leq B_t \leq 2L. \end{cases}$$

(i) Let us show that $B^* > B_1^{**}$ when $B_1^{**} \in [LA, LC]$ and $B^* \in [2LA, 2LC]$. If 2LA > LC, it is obvious that $B^* > B_1^{**}$. If 2LA < LC, we are going to show that

$$B_{t+1}^{\star\star}(B_t) > B_{t+1}^{\star}(B_t)$$
 for any $B_t \in [LA, LC] \cap [2LA, 2LC]$.

We have

$$B_{t+1}^{\star\star}(B_t) - B_{t+1}^{\star}(B_t) = \frac{B_t^2}{2L} \frac{\Delta(n+h)}{\Delta c} - L\left(\frac{\Delta u - (\widetilde{n} + \widetilde{h})}{\Delta c}\right)$$

According to Assumption 1, we have $\Delta u < (\tilde{n} + \tilde{h})$, we thus deduce that $B_{t+1}^{\star\star}(B_t) - B_{t+1}^{\star}(B_t) > 0$. This implies that $B_{t+1}^{\star\star}(.)$ intersects the 45° line before $B_{t+1}^{\star}(.)$.

Let us consider the case $B_1^{\star\star} \in [LC, L]$ and $B^{\star} \in [2LC, 2L]$. If 2C > 1, it is obvious that $B^{\star} > B_1^{\star\star}$. If 2C < 1, we are going to study for any $B_t \in [LC, L] \cap [2LC, 2L]$

$$B_{t+1}^{\star\star}(B_t) - B_{t+1}^{\star}(B_t) = -\frac{B_t^2}{2L} \frac{(\widetilde{n} + \widetilde{h})}{\Delta c} - L\left(\frac{\Delta u - (\widetilde{n} + \widetilde{h})}{\Delta c}\right)$$

As $\Delta u - (\tilde{n} + \tilde{h}) < 0$ given Assumption 1, we have

$$B_{t+1}^{\star\star}(B_t) \stackrel{\leq}{\leq} B_{t+1}^{\star}(B_t)$$
 if and only if $L((\widetilde{n} + \widetilde{h}) - \Delta u) \stackrel{\leq}{\leq} \frac{B_t^2}{2L}(\widetilde{n} + \widetilde{h})$

which is equivalent to

$$B_{t+1}^{\star\star}(B_t) \stackrel{\leq}{\equiv} B_{t+1}^{\star}(B_t)$$
 if and only if $L\sqrt{2(1-\frac{\Delta u}{\widetilde{n}+\widetilde{h}})} \stackrel{\leq}{\equiv} B_t$.

We will thus have

$$B_1^{\star\star} \stackrel{\leq}{=} B^{\star} \text{ if and only if } B_{t+1}^{\star\star} \left(L \sqrt{2(1 - \frac{\Delta u}{\widetilde{n} + \widetilde{h}})} \right) \stackrel{\geq}{=} L \sqrt{2(1 - \frac{\Delta u}{\widetilde{n} + \widetilde{h}})}$$

which is equivalent to

$$B_1^{\star\star} \stackrel{\leq}{\equiv} B^{\star} \text{ if and only if } L\left(-3\left(\frac{(\widetilde{n}+\widetilde{h})}{\Delta c} - \frac{\Delta u}{\Delta c}\right) + \left(\sqrt{2(1-\frac{\Delta u}{\widetilde{n}+\widetilde{h}})}\right)\left(\left(\frac{2(\widetilde{n}+\widetilde{h})}{\Delta c} - \frac{\Delta u}{\Delta c}\right)\right)\right) \stackrel{\geq}{\equiv} 0$$

(iii) Let us consider the case $B_2^{\star\star} \in [LA + L, LC + L]$ and $B^{\star} \in [2LA, 2LC]$ and let us show that $B^{\star} < B_2^{\star\star}$. If 2LC < LA + L, it is obvious that $B^{\star} < B_2^{\star\star}$. If 2LC > LA + L, we are going to show that

$$B_{t+1}^{\star}(B_t) > B_{t+1}^{\star\star}(B_t)$$
 for any $B_t \in [LA + L, LC + L] \cap [2LA, 2LC]$

We have

$$B_{t+1}^{\star}(B_t) - B_{t+1}^{\star\star}(B_t) = -B_t^2 \frac{1}{2L} \frac{\Delta(n+h)}{\Delta c} + 2B_t \frac{\Delta(n+h)}{\Delta c} + L\left(\frac{\Delta u + (\tilde{n}+\tilde{h}) - \Delta(n+h)}{\Delta c} - 1\right) > 0$$

The above function has its maximum in $B_t = L$ and we have

$$B_{t+1}^{\star}(2LC) - B_{t+1}^{\star\star}(2LC) = 2L \frac{\Delta(n+h)}{\Delta c} + L \left(\frac{\Delta u + (\tilde{n}+\tilde{h}) - \Delta(n+h)}{\Delta c} - 1 \right)$$
$$= L \left(\frac{(\hat{n}+\tilde{h}) - (\Delta c - \Delta u)}{\Delta c} \right).$$

According to Assumption 2 $\frac{\Delta c - \Delta u}{(\hat{n} + \hat{h})} < 1$, hence $B_{t+1}^{\star}(2LC) - B_{t+1}^{\star\star}(2LC) > 0$. Hence, $B_{t+1}^{\star}(B_t) > B_{t+1}^{\star\star}(B_t)$ for any $B_t \in [LA + L, LC + L] \cap [2LA, 2LC]$ implying that $B_{t+1}^{\star}(.)$ intersects the 45° line before $B_{t+1}^{\star\star}(.)$ and that $B^{\star} < B_2^{\star\star}$. Let us now consider the case where $B_2^{\star\star} \in [LC + L, 2L]$ and $B^{\star} \in [2LC, 2L]$ and let us show that $B^{\star} < B_2^{\star\star}$. We are going to show that

$$B_{t+1}^{\star}(B_t) > B_{t+1}^{\star\star}(B_t)$$
 for any $B_t \in [LC + L, 2L] \cap [2LC, 2L]$.

We have

$$B_{t+1}^{\star}(B_t) - B_{t+1}^{\star\star}(B_t) = \frac{B_t^2}{2L} \frac{(\tilde{n} + \tilde{h})}{\Delta c} - B_t \frac{2(\tilde{n} + \tilde{h})}{\Delta c} + 2L\left(\frac{\tilde{n} + \tilde{h}}{\Delta c}\right)$$
$$= \frac{(\tilde{n} + \tilde{h})}{\Delta c} \left(\frac{B_t}{\sqrt{2L}} - \sqrt{2L}\right)^2 > 0.$$

Hence, $B_{t+1}^{\star}(.)$ intersects the 45° line before $B_{t+1}^{\star\star}(.)$ and $B_2^{\star\star} > B^{\star}$.