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## Optimal production channel for private labels: Too much or too little innovation?

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# Optimal production channel for private labels: Too much or too little innovation?\*

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## Abstract

We analyze the impact of the private label production channel on innovation. A retailer may either choose a competitive fringe or rely on a brand manufacturer to produce its private label. The trade-off between the two channels is a choice between too much or too little innovation, i.e. quality investment, on the private label. On the one hand, when choosing the competitive fringe, the retailer over-invests to increase its buyer power. On the other hand, when the brand manufacturer is selected, a hold-up effect leads to under-investment. In addition, selecting the brand manufacturer may create economies of scale that spur innovation.

*JEL-Classification: L14, L15, L42*

*Keywords: Private label, vertical relations, buyer power, innovation.*

# 1 Introduction

The sale of private label goods in supermarkets has been increasing since the seventies and has now reached approximately 25% of global supermarket sales, compared with 15% in 2003. In Europe, the market share of such products is particularly high in Switzerland (46%); the UK (42.5%); and Germany, Spain, Belgium and France, in which private-label goods command market shares of 28-30%. The two leading supermarket chains in the world, Wal-Mart and Carrefour, respectively sold 38% and 35% of private labels in volume in 2008.<sup>1</sup>

If private labels were initially positioned as low-quality "me-too" products, their quality has significantly improved. In 2010, a survey conducted in the U.S. found that "44% of grocery shoppers believe store-brand products are of better quality today than they were five years ago".<sup>2</sup> Accordingly, the average price level of many private-label categories has increased (see, e.g., Connor and Peterson, 1992). Private labels are also increasingly innovative. In the UK, the proportion of new product development in the food and non-alcoholic drink categories has traditionally been higher for brands than for private labels. However, although brands held a 55% share of total new product development in 2010, the balance switched in 2011 in favor of store brands, which accounted for 54% of new product development.<sup>3</sup>

In most countries, three sources of private-label production generally co-exist. First, the retailer may buy the private label from small- and medium-sized firms or directly hold the production facilities. Second, the national brand producers themselves often supply the private-label goods to retailers. Finally, the retailer can entrust the production of its private label to powerful manufacturers that, which have specialized in the production of private labels only and may work for several retailers at a time.<sup>4</sup> According to the Private Label Manufacturer Association, about 40% of private labels are still produced by small firms, another 40% are produced by the national brand producers, and approximately 20% are made by large specialized private label manufacturers.<sup>5</sup> According to Quelch and Harding (1996), more than 50% of U.S. manufacturers of branded consumer packaged goods make private label goods as well. Some national brand manufacturers are leaders in the private label goods production. For instance, Heinz is a major supplier of private label baby food.

Finally, large firms that specialize in the production of private labels and that have reached a critical mass through successive mergers represent a growing part of total private label production.<sup>6</sup>

Our purpose in this paper is to understand the main drivers of a retailer's choice of its private label production channel and to define the consequences of this choice for product innovation on both the national brand and the private label and on consumer surplus and welfare. We consider a demand framework where *ex ante* (before quality investments) some consumers have a preference for the national brand while others are indifferent between the brand and the private label. We study a game in which, a monopolist retailer can either entrust a competitive fringe or rely on the national brand manufacturer for the production of its private label. If the retailer selects a competitive fringe, it acts as if it were vertically integrated and therefore innovates itself. If instead the retailer chooses the national brand manufacturer to produce the private label, the brand manufacturer produces and innovates on both goods.<sup>7</sup> Qualities are not contractible. After the innovation stage firms bargain over a fixed fee for the supply of the good(s) produced by the national brand manufacturer. In the last stage, the retailer chooses the products to put on its shelves and sets their prices.

The trade-off between the two channels is primarily a choice between too much and too little innovation on the private label. On the one hand, when choosing the competitive fringe, the retailer over-invests to increase its buyer power versus the national brand manufacturer. This effect is all the stronger when its buyer power is initially weak. On the other hand, when the national brand manufacturer is selected, a standard hold-up effect leads to under-investment. This effect is reinforced when the retailer's buyer power is stronger. In addition, the choice of the brand manufacturer may create economies of scale that spur innovation. In equilibrium, whenever the buyer power is not too high and when some consumers have a strong preference for the national brand, the retailer selects the brand manufacturer (resp. the competitive firm) to produce the private label. This choice may be detrimental to welfare because consumers can be hurt by too little innovation on the private label.

The economic literature has mostly analyzed the retailer's rationale for launching private labels. The industrial organization and marketing literature has often presented private

labels as a means for retailers to better discriminate demand and to increase their buyer power. A direct argument is that retailers can exchange one private label supplier for another more easily: the retailer owns the label, not the manufacturer. Another reason outlined by Mills (1995) is that a retailer may use a private label as leverage in its bargaining with brand manufacturers. Finally, private labels can also be used more directly to increase differentiation among stores, i.e to increase store loyalty and thus increase both the retailer's market power and its buyer power (see Bergès et al., 2004, for a survey).

Few papers have analyzed the retailer's choice of production channel for private labels. To our knowledge, only Bergès-Sennou (2006), Tarziján (2007) and Bergès and Bouamra-Mechemache (2011) have directly analyzed this issue. Both Bergès-Sennou (2006) and Tarziján (2007) rule out the issue of quality investments. The former focuses on the effect of consumer loyalty to a store and/or a national brand on the choice of production channel for its private label and finds that the retailer prefers to entrust the national brand producer with the private label when the bargaining power of the producer or the consumer loyalty to the national brand is sufficiently low. The latter analyzes how the national brand producer may have an incentive to also produce the private label. This second analysis accounts for potential synergies a national brand manufacturer may benefit from when also manufacturing a private label and balances them with cannibalization effects. Indeed, when the private label is produced by the national brand producer, its perceived quality may increase. In contrast to these two papers, in which the qualities of products are fixed, our paper focuses on innovation issues.

Bergès and Bouamra-Mechemache (2011) do consider quality investment issues. However, in contrast with our work, they focus on quality investments in the private label only. Moreover, they assume that quality is contractible in the sense that when the brand manufacturer is entrusted to also manufacture the private label, the retailer chooses (via a contract) the quality of its private label. We depart from their analysis by considering only non contractible investments and thus inherent hold-up issues.

Our paper is also related to the literature that addresses the role of buyer power in determining investment decisions within a vertical chain. On the one hand, the presence

of large buyers may induce suppliers to invest more in order to make up for their loss of bargaining power, which eventually may increase consumer surplus and total welfare. Focusing on technology adoption, Inderst and Wey (2003, 2007) show that buyer power may increase suppliers' incentives to choose a technology with lower incremental cost at higher quantities because such technology enables them to be stronger in their bargaining with large buyers. In contrast, Battigalli et al. (2007) show that buyer power may weaken the producer's incentive to engage in quality improvement due to the hold-up problem. The latter authors show under which conditions the retailers also suffer from too low investment by the producer.

This article is organized as follows. Section 2 derives the model assumptions. Section 3 analyzes the two major private label production channels, a competitive fringe and the national brand producer itself. In Section 4, we determine the optimal choice of private label production channel for the retailer according to its buyer power and the initial brand advantage. Section 5 derives the implication for consumer surplus and welfare. In Section 6, we provide an extension in which entrusting the brand producer with the production of the private label does not avoid the duplication of investment costs, and show that the retailer may still want to entrust the brand producer with the production of the private label. Section 8 concludes the paper.

## 2 The model

We consider a framework in which a monopolist retailer,  $R$ , may sell two different goods, a national brand  $B$  supplied by a brand producer  $P$  and a private label  $L$  that is produced either through a competitive fringe of small producers denoted by  $f$  or by  $P$  itself. Firms may innovate by investing in the quality of both  $B$  and  $L$ . The qualities of  $B$  and  $L$  are denoted  $k_B$  and  $k_L$  respectively. These qualities affect the gross surplus of consumers. The quality of the brand is chosen by  $P$ , and that of the private label is chosen either by  $P$  or  $R$  depending on whether the private label is supplied by  $P$  or by the fringe  $f$ .

Consumers are heterogeneous in their tastes. The total mass of consumers is normalized to 1. A share  $\lambda$  of the population are "brand lovers" who, absent any difference in quality

and price, have a strict preference for good B represented by a parameter  $\delta$ . The remaining fraction  $(1 - \lambda)$  of consumers are “standard consumers” who, absent any difference in quality and price, have exactly the same demand for B and L.

We adopt a linear specification in which  $v > 0$  is the maximum willingness of a standard consumer to pay for a product with a null quality. Given the prices  $p_B$  for the national brand and  $p_L$  for the private label, the surplus of a brand lover is

$$S_\delta = (v + \delta + k_B)q_B + (v + k_L)q_L - \frac{(q_B + q_L)^2}{2} - p_B q_B - p_L q_L \quad (1)$$

if it purchases a quantity  $q_i$  of good  $i$  ( $i = B, L$ ), while a standard consumer then earns a surplus:

$$S_0 = (v + k_B)q_B + (v + k_L)q_L - \frac{(q_B + q_L)^2}{2} - p_B q_B - p_L q_L. \quad (2)$$

The total consumer surplus is  $S = \lambda S_\delta + (1 - \lambda)S_0$ . The demands  $D_i$  for  $i = B, L$  derived from these surpluses are as follows:

- If  $p_L < v + k_L$ , then the two goods can be sold and we have:

$$D_B = \begin{cases} v + \lambda\delta + k_B - p_B & \text{if } p_B \in [0, k_B - k_L + p_L], \\ \lambda(v + k_B + \delta - p_B) & \text{if } p_B \in [k_B - k_L + p_L, p_L + \delta + k_B - k_L], \\ 0 & \text{if } p_B > p_L + \delta + k_B - k_L, \end{cases}$$

and demand for the private label is given by:

$$D_L = \begin{cases} 0 & \text{if } p_B \in [0, k_B - k_L + p_L], \\ (1 - \lambda)(v + k_L - p_L) & \text{if } p_B \in [k_B - k_L + p_L, p_L + \delta + k_B - k_L], \\ v + k_L - p_L & \text{if } p_B > p_L + \delta + k_B - k_L, \end{cases}$$

- If, however,  $p_L \geq v + k_L$ , then  $D_L = 0$ , and again we have three cases:

$$D_B = \begin{cases} v + \lambda\delta + k_B - p_B & \text{if } p_B \in [0, v + k_B], \\ \lambda(v + k_B + \delta - p_B) & \text{if } p_B \in [v + k_B, v + \delta + k_B], \\ 0 & \text{if } p_B > v + \delta + k_B, \end{cases}$$



This demand specification initially introduced by Soberman and Parker (2004) is sustained by a survey conducted in the U.S. in 2010, that showed that 19% of consumers believe that it is worth paying more for name-brand products.<sup>8</sup> We assume that  $\delta < v$ ; that is, even if all consumers were “brand lovers”, selling the national brand cannot double the market size.

The costs of quality investments and the choice of these investments depend on the supplier of the private label. We assume that quality investments are not contractible. If the private label is supplied by a competitive fringe, P can make a quality investment  $k_B$  at a cost  $C(k_B)$  and R invests in the quality  $k_L$  of its private label and supports the associated cost  $C(k_L)$ . It is *as if* the retailer were vertically integrated with the fringe, an assumption that is justified by the existence of perfect competition among fringe firms.<sup>9</sup>

By contrast, when R entrusts the production of its private label to the producer, P chooses both qualities  $k_B$  and  $k_L$ ; in that case, the associated cost, which is borne entirely by the producer, is  $C(\text{Max}[k_B, k_L])$ . Indeed, we assume here that for a given level of quality investment there is no additional cost to offer two goods instead of one: the firm can differentiate the private label from the national brand at no cost. The difference is only a matter of packaging and the associated packaging cost is neglected.<sup>10</sup> Moreover, if  $k_B > k_L$  (resp.  $k_L > k_B$ ) the producer has to pay the cost  $C(k_B)$  (resp.  $C(k_L)$ ) and can offer the other product at any downgraded level of quality  $k_L < k_B$  (resp.  $k_B < k_L$ ) without additional cost. Investment costs are quadratic and identical for all firms and products:  $C(k_i) = \frac{k_i^2}{2}$  where  $i = B, L$ . Note that in this model we focus on deterministic quality investments. Indeed, innovation in the consumer-packaged-good industries primarily consists of ensuring constant quality improvements, and radical innovations are rare events (Pauwels and Srinivasan, 2004; Steiner, 2004).<sup>11</sup>

We assume that investment costs are the only costs borne by the producers: the marginal cost of production is assumed to be constant and is normalized to 0.

The timing of the game is as follows:

### 1. Choice of the private label production channel

R and P may sign a contract that associates a transfer  $Y$  (positive or negative) with the delegation of the production of L. This case is named “Channel P” and is denoted

by the superscript  $P$ .

Otherwise, the competitive fringe produces L. This case is named “Channel f” and is denoted by the superscript  $f$ .

## 2. Innovation

- Channel P: P chooses both qualities  $k_L$  and  $k_B$ , and bears the associated cost  $C(\text{Max}[k_L, k_B])$ .
- Channel f: P invests  $k_B$  in good B at a cost  $C(k_B)$  and simultaneously, R invests  $k_L$  in good L at a cost  $C(k_L)$ .

Firms can no longer invest in quality after the end of this innovation stage.

## 3. Bargaining

- Channel P: R and P bargain over a fixed transfer for the delivery of both B and L to the retailer. The producer has no outside option, whereas the retailer can still sell a private label of quality  $k_L = 0$  (no investment can be done).
- Channel f: R and P bargain over a fixed transfer for the delivery of B. The producer has no outside option; the retailer can still sell a private label of quality  $k_L$ .<sup>12</sup>

## 4. Sales

R sells either one or both goods to consumers and sets the retail prices  $p_L$  and  $p_B$ .

We consider subgame perfect equilibrium and proceed by backward induction.

Because the last stage is not affected by the production channel choice, we solve it here. Qualities  $k_B$  and  $k_L$  are fixed, and R chooses prices that maximize the industry profit. Three cases may arise: first, R may sell only L to all consumers; second, R may sell the two goods B and L and thus discriminate between brand lovers and others; finally, R may sell only B to all consumers.

- When only the private label is sold, the industry profit is  $(v + k_L - p_L)p_L$ , and R sets the optimal price  $p_L = \frac{v+k_L}{2}$ .

- When both the private label and the national brand are sold, the industry profit is  $\lambda(v + \delta + k_B - p_B)p_B + (1 - \lambda)(v + k_L - p_L)p_L$ , and R sets the optimal prices  $p_L = \frac{v + k_L}{2}$  and  $p_B = \frac{v + \delta + k_B}{2}$ .
- When only the national brand is sold, the industry profit is  $(v + \lambda\delta + k_B - p_B)p_B$ , and R sets the optimal price  $p_B = \frac{v + \lambda\delta + k_B}{2}$ .

Which option is the most profitable depends on the qualities of the two products; the industry revenue, denoted  $\pi(k_L, k_B)$ , is as follows:

$$\pi(k_L, k_B) = \begin{cases} \frac{1}{4}(v + k_L)^2 & \text{for } k_B \in [0, k_L - \delta], \\ \frac{\lambda}{4}(v + \delta + k_B)^2 + \frac{1-\lambda}{4}(v + k_L)^2 & \text{for } k_B \in (k_L - \delta, \sqrt{(k_L + v)^2 + \lambda\delta^2} - v], \\ \frac{1}{4}(v + \lambda\delta + k_B)^2 & \text{for } k_B > \sqrt{(k_L + v)^2 + \lambda\delta^2} - v. \end{cases} \quad (3)$$

For given quality levels, the industry revenue is independent of whether the private label is produced by a competitive fringe or by the brand manufacturer. Given our assumption  $\delta < v$ , when R sells only the national brand, it strictly prefers to sell B at a lower price to all consumers rather than sell it at a higher price to brand lovers only.

In Stage 3, we adopt a standard Nash bargaining approach to model the negotiation between R and P. The exogenous bargaining power of R relative to P is a parameter  $\alpha \in [0, 1]$ . In equilibrium, R (respectively P) earns its outside option profit plus a share  $\alpha$  (resp.  $1 - \alpha$ ) of its incremental gain from trade with P. Because the national brand producer cannot supply its products to any other firm than the retailer, its outside option in all subgames is 0. On the contrary, in case of a failure in the bargaining with P, the retailer can always turn to the competitive fringe and sell only the private label on the final market. The outside option profit of R is then denoted by:

$$\bar{\pi}(k_L) = \frac{1}{4}(v + k_L)^2. \quad (4)$$

### 3 Private label production channel and innovation

In this section, we solve Stages 2 and 3 of the game to highlight how the choice of the private label production channel affects innovation with respect to both the national brand and

the private label. We first solve the subgame “Channel P” in which P produces the private label and is the only firm that invests in quality improvements. We then solve the subgame “Channel f”, in which a competitive fringe produces L and both P and R invest in quality.

### 3.1 Channel P

In this subsection, R entrusts P with the production of the private label; P thus chooses both qualities  $k_L$  and  $k_B$  and pays the associated investment cost  $C(\max[k_L, k_B])$ . In stage 4, the revenue of the industry is given by  $\pi(k_L, k_B)$  which is defined by (3).

At the bargaining stage, the sharing of  $\pi(k_L, k_B)$  between the producer and the retailer depends on the relative bargaining power of each firm, given by  $\alpha$  and their outside options. P has no outside option profit whereas the outside option profit of R  $\bar{\pi}(0)$  comes from the sale of a private label without any quality upgrading. Nash bargaining leads to the following profits:

$$\Pi_P^P(k_L, k_B) = (1 - \alpha) [\pi(k_L, k_B) - \bar{\pi}(0)] - C(\max[k_L, k_B]) \quad (5)$$

$$\Pi_R^P(k_L, k_B) = \bar{\pi}(0) + \alpha [\pi(k_L, k_B) - \bar{\pi}(0)] \quad (6)$$

**Lemma 1.** *For any quality investment  $k$ , the qualities maximizing  $\pi(k_L, k_B)$  subject to  $k_B, k_L \leq k$  are  $k_L = k_B = k$ .*

*Proof.* See Appendix A.1. □

Regardless the investment made by the producer it is always optimal for P, as well as for the total industry profits, to sell both goods at similar quality; P has no incentive to downgrade the quality of the private label. Indeed, it suffices to differentiate packages to be able to discriminate among consumers: with identical qualities brand lovers do not buy the private label. Then, the total industry profit net of investment cost is

$$\Pi^P(k_L, k_B) = \pi(k_L, k_B) - C(\max[k_B, k_L]). \quad (7)$$

with  $k_B = k_L = k$ . The quality maximizing the industry profit  $\Pi^P(k, k)$  is  $v + \lambda\delta$  and we

denote by  $\Pi^{P*}$  the corresponding maximum total industry profit with Channel P. We will henceforth refer to this quality level as the “optimal” quality, in the sense that it is optimal from the point of view of the vertical structure.

The equilibrium quality investment of the producer, however, is determined by its marginal benefit  $(1 - \alpha) [\partial\pi/\partial k_B + \partial\pi/\partial k_L]$ : the equilibrium quality is thus strictly lower than the optimal quality. Given that the producer has to share the marginal benefit of its investment with the retailer, it always underinvests in quality. This so-called “hold-up effect” increases with the bargaining power of the retailer  $\alpha$ .

**Proposition 1.** *There exists a unique equilibrium of the subgame Channel P.*

*In this equilibrium P chooses the same quality for both B and L,*

$$k_B^P = k_L^P = (v + \lambda\delta) \frac{1 - \alpha}{1 + \alpha},$$

*and both goods are sold to consumers.*

*Proof.* See Appendix A.2. □

Because of the hold-up effect, the producer under-invests relative to the optimal quality:  $k^P < \delta\lambda + v$ . Replacing  $k^P$  in eq. (7), the corresponding total industry profit obtained with channel P is denoted  $\Pi^P$  and the difference  $\Pi^P - \Pi^{P*} \leq 0$  is only due to the hold-up inefficiency. The difference is brought to 0 when  $\alpha = 0$  because then all the power is in the hands of P and there is no more hold-up.

### 3.2 Channel f

In this subsection, R entrusts a firm from a competitive fringe with the production of its private label. P (resp. R) thus chooses quality  $k_B$  (resp.  $k_L$ ) and pays the associated investment cost  $C(k_B)$  (resp.  $C(k_L)$ ). In the bargaining stage, the total revenue from sales  $\pi(k_L, k_B)$  is shared among the two firms. The outside option revenue of the retailer amounts to the revenue it would earn if it sold only its private label to all consumers,  $\bar{\pi}(k_L) = \frac{1}{4}(v + k_L)^2$ . By contrast, P has no outside option revenue. Accordingly the profits

are:

$$\Pi_P^f(k_L, k_B) = (1 - \alpha) [\pi(k_L, k_B) - \bar{\pi}(k_L)] - C(k_B) \quad (8)$$

$$\Pi_R^f(k_L, k_B) = \bar{\pi}(k_L) + \alpha [\pi(k_L, k_B) - \bar{\pi}(k_L)] - C(k_L) \quad (9)$$

An equilibrium of Channel f is completely characterized by a pair of qualities chosen by firms. Indeed, it corresponds to a Nash equilibrium of the game with profits given by (8) and (9). In the last stage, three cases may arise: either both goods are sold or only one or the other is sold. Depending on the values of the parameters  $\alpha$ ,  $\delta$ , and  $\lambda$ , we will show that all three situations may arise along an equilibrium path and that one, two or three equilibria may co-exist. We proceed by first considering three equilibrium candidates defined by their pairs of qualities and then determining under which conditions each candidate indeed represents an equilibrium. Furthermore, in cases involving co-existence of equilibria, there is always an equilibrium that Pareto dominates the others and we select this one. Henceforth we refer to such a Pareto dominating equilibrium as a dominating equilibrium. Consequently, the equilibrium qualities of the subgame Channel f can be denoted  $k_B^f$ ,  $k_L^f$  without ambiguity.

**Lemma 2.** *There are three possible types of equilibrium of the subgame Channel f :*

$$(k_L^f, k_B^f) = \begin{cases} (k_L^{BL}, k_B^{BL}) &= (v \frac{1-\alpha\lambda}{1+\alpha\lambda}, \frac{\lambda(1-\alpha)(v+\delta)}{2-\lambda(1-\alpha)}) & \text{if both } B \text{ and } L \text{ are sold,} \\ (k_L^L, k_B^L) &= (v, 0) & \text{if only } L \text{ is sold,} \\ (k_L^B, k_B^B) &= (v \frac{1-\alpha}{1+\alpha}, \frac{(1-\alpha)(v+\lambda\delta)}{1+\alpha}) & \text{if only } B \text{ is sold.} \end{cases}$$

*Proof.* See appendix A.3. □

Note that in the coexistence equilibrium, both  $k_B^{BL}$  and  $k_L^{BL}$  strictly decreases in the retailer's buyer power. The fact that  $k_B^{BL}$  decreases in  $\alpha$  is not surprising because it arises directly from the hold-up effect. More interestingly, the quality of the private label  $k_L^{BL}$  also decreases in  $\alpha$ . Indeed, R obtains its buyer power from two sources: first from its exogenous buyer power parameter,  $\alpha$ , and second from its outside option revenue  $\bar{\pi}$ . When  $\alpha$  is low, the retailer's buyer power is mainly driven by its outside option revenue, and the retailer thus has a greater incentive to raise it by increasing  $k_L$ .

The domain of existence of each type of equilibrium is now determined by checking the incentives of P and R to shift from one case to the other. Note that whenever there is a multiplicity of equilibria, there is always one equilibrium that Pareto dominates the others. We assume henceforth that, in case of coexistence, the dominant equilibrium is played. Appendix A.4 gives the complete proof.

**Proposition 2.** *In the subgame Channel f, if  $\lambda < \frac{\sqrt{17}-3}{2}$ :*

- *there exists a dominant equilibrium, denoted (BL), where both the national brand and the private label are sold if:*

$$\delta > \delta_1 = v \max \left\{ \frac{2 - (1 - \alpha)\lambda}{\sqrt{1 + \alpha\lambda}} - 1, \frac{\sqrt{2(2 - (1 - \alpha)\lambda)}}{1 + \alpha\lambda} - 1 \right\}$$

- *Otherwise, the unique equilibrium, denoted (L), is such that only the private label is sold.*

When  $\lambda \geq \frac{\sqrt{17}-3}{2}$ :

- *If  $\alpha < 2\lambda - 1$  and  $\delta \in [\delta_3, \delta_4]$ , there exists a dominant equilibrium, denoted (B), such that only the brand is sold.*
- *Otherwise, if  $\delta > \delta_1$  the dominant equilibrium is (BL) and if  $\delta \leq \delta_1$ , the unique equilibrium is (L).*

*Proof.* See Appendix A.4 for the full proof and the expressions of thresholds  $\delta_3$  and  $\delta_4$ .  $\square$

In Proposition 2, the parameter  $\delta$  is used to characterize the frontiers between the three types of equilibria.

Figure 1 depicts the equilibrium frontiers depending on the value of  $\lambda$ :

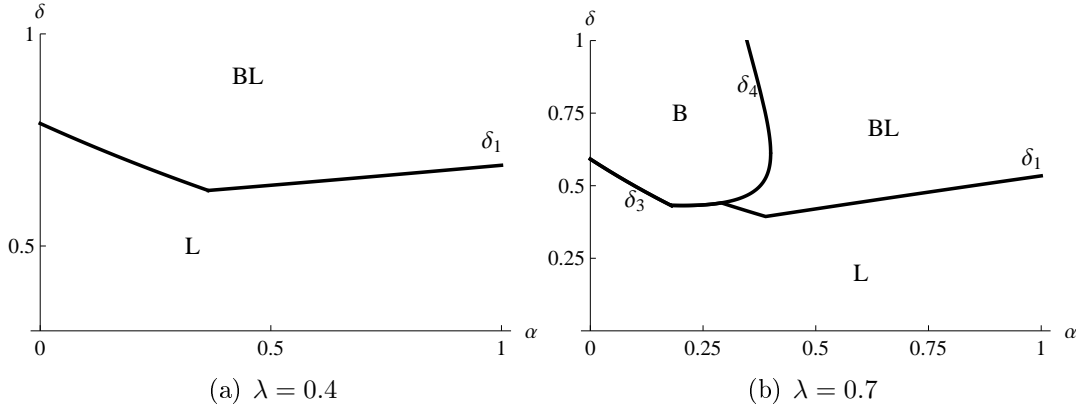


Figure 1: Equilibria (B), (BL) and (L), according to  $\delta$  and  $\alpha$ , for a low and a large  $\lambda$ .

We discuss first the left-hand figure, in which  $\lambda$  is low.

First, when the buyer power  $\alpha$  is low, the retailer's quality investment is high. When the initial advantage of the brand  $\delta$  is sufficiently low ( $\delta < \delta_1$ ), it is too costly for the producer to maintain the coexistence, and the latter prefers to make no investment, hence deviating from a coexistence equilibrium candidate towards a situation where only L is sold. In contrast, when  $\delta$  is above  $\delta_1$ , it becomes profitable for the producer to maintain the coexistence because of its large initial advantage  $\delta$ .

Second, for high values of buyer power, the quality investment of P is low. Whenever the initial advantage of the brand  $\delta$  is sufficiently low, the retailer has no incentive to discriminate by selling the brand and instead prefers to sell a better-quality private label to all consumers. When  $\delta$  is high enough, however, the retailer finds it more profitable to sell both B and L to discriminate consumers rather than to sell only L.

We now discuss the right-hand figure, in which the equilibrium (B) exists. Note that (B) arises when  $\lambda$  is high because more consumers are willing to pay for the brand. In this case, (B) is favored by a high value of  $\delta$  and a low buyer power  $\alpha$ . As mentioned above, the lower the buyer power, the higher the retailer's quality investment. This may still discourage coexistence, but in this case, in which  $\delta$  is sufficiently high, it is more profitable to give up on the private label rather than the brand.

The total industry profit (net of R&D costs) is:

$$\Pi^f(k_L, k_B) = \pi(k_L, k_B) - C(k_B) - C(k_L). \quad (10)$$



We determine the optimal brand and private label qualities, that is the qualities that would maximize the industry profit given by (10) and we denote the corresponding industry profit by  $\Pi^{f*}$ . Note here that the optimal qualities differ from those determined in Channel P because the cost function differs. As in the previous case, firms do not choose these optimal quality investments. There are two types of distortions at stake.

When the producer invests (in (B) and (BL)) there is a hold-up effect, similar to the effect that occurs in the subgame Channel P: P's investment is determined by its own marginal benefit,  $(1 - \alpha)\partial\pi/\partial k_B$ , instead of being determined by the marginal benefit of the industry,  $\partial\pi/\partial k_B$ . In addition, another extreme form of hold-up effect can arise in Channel f because of the retailer's outside option. Even though the marginal benefit of the producer's investment, and thus the value of  $k_B$ , does not depend on  $\bar{\pi}$ , the producer's incentive to invest at all is influenced by the outside option of the retailer. If the retailer's quality  $k_L$  is too high, then  $\bar{\pi}(k_L)$  is too high for the producer to earn a positive profit from selling its good, and P may therefore decide to neither invest nor supply the brand. In this case, only the private label is sold.

The outside option of the retailer plays an opposite role with respect to the retailer itself. Indeed, to increase its outside option and therefore its bargaining power vis-à-vis the producer, the retailer tends to over-invest in quality. This only affects cases where the producer indeed earns a share of the profit, that is, when both goods are sold or only the national brand is sold. In these two cases, the quality of the private label that would maximize the net industry profit would be respectively  $\frac{v(1-\lambda)}{1+\lambda}$  and 0, and the quality choice of R is higher than these optimal quality levels due to this "over-investment effect". The "over-investment effect" decreases with the bargaining power of the retailer  $\alpha$ .

To summarize, the difference between  $\Pi^f$  and  $\Pi^{f*}$  results both from a hold-up effect on the brand quality, which increases in  $\alpha$ , and from an over-investment effect on the private label quality, which decreases in  $\alpha$ .

## 4 Choice of the private label production channel

In stage 1, R chooses the production channel that leads to the highest total industry profit. Indeed, as long as total industry profit is higher in a given channel, in stage 1 P and R can always find a transfer  $Y$  such that both are strictly better off than if R chose the other channel. Therefore, R chooses channel P if total industry profit in the subgame “Channel P” is larger than total industry profit in the subgame “Channel f”. More formally, R chooses Channel P if and only if  $\Delta_{P,f} \stackrel{def}{=} \Pi^P - \Pi^f > 0$ . Henceforth, we will simply refer to “Channel X” as the subgame in which Channel X is chosen by the retailer.

There are several effects at stake that determine whether one option is preferred over the other. Note that if it were possible for the two firms to write complete contracts, the best option would always be to make P supply both B and L because it would prevent a costly replication of investments in quality ( $\Pi^{P*} > \Pi^{f*}$ ). However, contracts are incomplete, and quality investments are not contractible. This induces inefficiencies in both channels that may explain why the retailer may wish to manufacture its private label through a competitive fringe. One way to disentangle these effects is to write the comparison

$$\Delta_{P,f} = \underbrace{[\Pi^{P*} - \Pi^{f*}]}_{\text{cost duplication}} + \underbrace{[\Pi^P - \Pi^{P*}]}_{\text{hold-up in Channel P}} + \underbrace{[\Pi^{f*} - \Pi^f]}_{\substack{\text{hold-up and over-investment} \\ \text{in Channel f}}} \quad (11)$$

The first term casts the positive effect related to the absence of cost duplication in Channel P. It represents the difference between the optimal industry profits without cost duplication and with duplication. The second term is negative and corresponds to the loss resulting from the hold-up effect in Channel P. This term is equal to 0 for  $\alpha = 0$  and increases with the bargaining power of the retailer. Finally, the third term is positive and encompasses the gain in Channel f when correcting both for the hold-up and the over-investment effects described in subsection 7.1. The precise characteristics of this term depend upon the type of equilibrium in Channel f.

The following proposition establishes the conditions under which R chooses Channel P in stage 1.

**Proposition 3.** *There exist two thresholds  $\delta^*$  and  $\delta^{**}$  such that whenever  $\delta^* < \delta < \delta_1$  or  $\delta > \max\{\delta_1, \delta^{**}\}$ ,  $R$  chooses Channel P.*

*Proof.* See Appendix A.5 □

In figure 2, we draw the thresholds mentioned in Proposition 3 together with the frontier between the equilibria (L) and (BL) highlighted in 7.1, for  $\lambda = 0.4$  and  $v = 1$ .

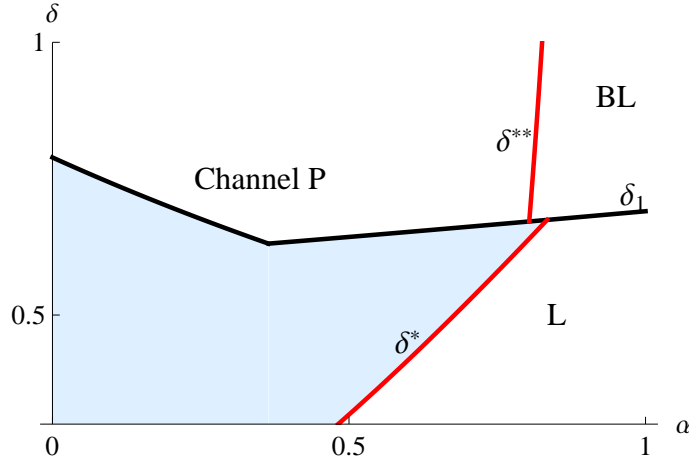


Figure 2: Channel P *vs* Channel f as a function of  $\alpha$  and  $\delta$ .

Consider first the case in which the equilibrium of Channel f is to sell the two goods (BL). As explained above by eq. (11), switching instead to Channel P has three distinct effects on quality decisions. First, it avoids the duplication of investment costs which tends to increase the quality of the two goods. Second, it creates a hold-up effect on the quality investment of the private label. Indeed, the producer now invests in the quality for the private label, and has to share the gains realized from this investment with the retailer. This second effect tends to lower the quality of the private label. Finally, switching to Channel P destroys the incentive of  $R$  to over-invests in the private label's quality. Although this last effect also reduces  $k_L$ , it brings it closer to the industry-profit maximizing quality.

As a result of these three effects, when the equilibrium in Channel f is (BL) and the retailer chooses Channel P, the quality of the national brand always increases while the effect on the quality of the private label is ambiguous. The following lemma summarizes the results.

**Lemma 3.** *If  $\lambda > \frac{1+\alpha-\sqrt{1+\alpha(2+(9-8\alpha)\alpha)}}{2(-1+\alpha)\alpha}$ , there exists a threshold  $\hat{\delta} = \frac{2\alpha v(1-\lambda)}{(1-\alpha)\lambda(1+\alpha\lambda)} > 0$  such that whenever  $\delta > \hat{\delta}$ , we have  $k_L^P > k_L^f$ . Otherwise, we have  $k_L^P < k_L^f$ .*

*Proof.* Straightforward from the comparison between  $k_L^P$  and  $k_L^f$  specified in lemma 2 and in proposition 1.  $\square$

Therefore, a situation in which the retailer chooses Channel f may, despite the duplication of investment costs, increase the joint profit of P and R. The benefit of selecting Channel f increases when  $\alpha$  is large and  $\delta$  is low. Indeed, as the bargaining power of the retailer increases, the over-investment problem is reduced, which mitigates the third effect, whereas the hold-up problem becomes stronger, which increases the second effect. The first effect is all the stronger when  $\delta$  is large because the joint-profit of the industry then increases more with the quality of the national brand.

Consider now the case in which the equilibrium in Channel f is to sell only the private label (L). We obtain the following corollary:

**Corollary 1.** *Whenever  $\delta^* < \delta < \delta_1$ , selecting Channel P is the only way to maintain the brand on the retailer's shelves.*

*Proof.* This result derives from Propositions (3) and (2).  $\square$

This effect arises in the shaded area indicated in Figure 2. In Channel P, because the investment in quality is common to the two goods, the producer never has an incentive to invest 0 on the national brand. The opportunity for P to also produce L enables the industry to maintain the diversity of the products offered to consumers. When  $\delta < \delta_1$ , by investing itself in the private label quality, the retailer would indeed over-invest and thus discourage any investment and sale of the brand. Another reason why supplying the private label together with the national brand may allow the producer to maintain its product on the shelves is that the retailer's outside option when both goods are sold is lower in Channel P. Therefore, for any given value of  $k_B$ , there are more cases in which the producer can earn a profit by selling the national brand. Because of the hold-up effect, however, it is still possible that the retailer will select a fringe firm to produce its private label. Again, the benefit of

entrusting the production of the private label to P is then increasing in  $\delta$  and decreasing in  $\alpha$ .

**Corollary 2.** *If an equilibrium (B) exists in Channel f, selecting Channel P is the only way to maintain the private label on the retailer's shelves.*

*Proof.* This result derives from Propositions (3) and (2). □

The insight of the above corollary is as follows. When the equilibrium (B) exists, the industry profit in Channel f is maximized through the sale of only one good B to all consumers; however, in equilibrium (B), R has to sell B at the same price to all consumers. Now, by choosing Channel P, the same good can be sold at different prices and under different packages to the two types of consumers. Therefore, choosing Channel P enables the retailer to discriminate among consumers and thus raises the joint profit of the industry. Another inefficiency disappears when R chooses Channel P. Indeed, when (B) is an equilibrium, R spends  $C(k_L^B) > 0$  to raise its share of the joint profit, but this investment has no effect on the joint industry profit. When P supplies the private label, R no longer invests and therefore avoids an inefficient duplication of investment costs.

This case also illustrates the cannibalization of sales, a drawback often mentioned in the debate on whether the national brand producer should begin producing private labels (see Quelch and Harding, 1996): The brand loses market share to the benefit of the private label.

Finally, note that in this framework entrusting a powerful specialized private label manufacturer would combine both the hold-up and the cost duplication inefficiencies and would thus never arise in equilibrium.<sup>13</sup>

## 5 Effect on consumer surplus and welfare

We replace equilibrium qualities in the expressions of surplus  $S_\delta$  and  $S_0$  defined by (1) and (2). When the private label is produced by a fringe firm, the consumer surplus is :

$$S^f = \begin{cases} \frac{v^2}{2} & \text{if only L is sold.} \\ \lambda \frac{(v+\delta)^2}{2(2-(1-\alpha)\lambda)^2} + (1-\lambda) \frac{v^2}{2(1+\alpha\lambda)^2} & \text{if B and L are sold.} \\ \frac{(v+\delta\lambda)^2}{2(1+\alpha)^2} & \text{if only B is sold.} \end{cases}$$

When the private label is produced by P, the consumer surplus is given by:

$$S^P = \lambda \frac{[2v + (1-\alpha)\delta\lambda + (1+\alpha)\delta]^2}{8(1+\alpha)^2} + (1-\lambda) \frac{[2v + (1-\alpha)\lambda\delta]^2}{8(1+\alpha)^2}.$$

In equilibrium, Channel P is selected whenever this strategy raises the joint profit of the industry relative to the alternative choice of Channel f. The effect on consumer surplus may, however, be ambiguous.

First, note that in all cases R sets the monopoly prices on the retail market. As a consequence, only two factors affect the consumer surplus: the quality of the products sold and the ability of the retailer to discriminate among brand lovers and standard consumers. By comparing consumer surplus, we obtain the following proposition:

**Proposition 4.** *If Channel P is chosen in equilibrium, it may hurt consumer surplus:*

- *when the resulting decrease in the quality of the private label L is too large relative to the increase in the quality of the national brand B.*
- *when choosing Channel P enables R to discriminate among consumers instead of selling B at the same price to all consumers.*

*Proof.* This result derives from the comparison between  $S^f$  and  $S^P$ . □

The figure below illustrates the areas in which the choice of Channel P is beneficial (areas with "+" sign) and the areas in which it is detrimental (shaded areas with "-" sign) to consumers:

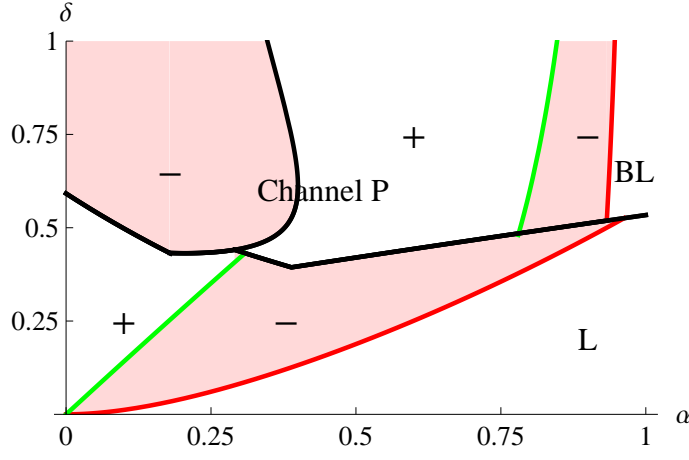


Figure 3: Effect on consumer surplus for  $\lambda > \bar{\lambda}$ .

The shaded area in the North-West represents the negative effect of discrimination on consumer surplus. Provided Channel f is chosen, this effect arises only when (B) is an equilibrium. In particular, when  $\lambda < \bar{\lambda}$ , this negative effect on consumer surplus never appears. In the other distinct shaded area, it is the negative effect of lowering the quality of L that explains the negative effect on the consumer surplus. When  $\alpha$  is sufficiently enough, this effect appears because the hold-up effect remains important when R chooses Channel P; thus, the quality investment on L is greatly lowered, while the increase in the quality investment on B is not so large.

This result calls for some remarks regarding the effect of buyer power on total welfare.

**Corollary 3.** *If Channel P is chosen in equilibrium, it increases welfare except when buyer power becomes too large.*

*Proof.* Straightforward from Propositions (4) and (3). □

When buyer power ( $\alpha$ ) is low, the existence of Channel P as a private label production channel is strictly beneficial to welfare: in that case, welfare is the highest in Channel P, which is precisely the channel chosen by the retailer. In the opposite case, when buyer power is high, the existence of Channel P has no impact on welfare. The only ambiguous case is the case in which the bargaining powers of the producer and the retailer are balanced. Indeed, in that case, Channel P can be chosen while total welfare is the highest in Channel f. As a consequence, at the frontier between the two types of equilibria, Channel f and Channel P,

we observe a positive jump in total welfare. This case seems particularly relevant nowadays, because the retailing sector is presently characterized by increasing buyer power and an increased share of national brand producers in the private label market.

**Proposition 5.** *If Channel f is chosen in equilibrium, it is always beneficial both for industry profit and social welfare.*

*Proof.* This result derives from Propositions (4) and (3). □

Indeed, if Channel f is chosen, it means that the positive effect of avoiding both a duplication of investment costs and the over-investment effect is insufficient to compensate for the lowering in the quality of L due to the hold-up effect. A fortiori in that case, choosing Channel f benefits consumers even more because, they are affected only by the quality investments and not by the form of the costs (with or without duplication). Therefore, consumers and industry interests are always aligned.

## 6 Cost duplication and choice of the private label production channel

In the previous section, despite the benefit of avoiding a duplication of costs, selecting Channel P was not always optimal for the industry because of strategic effects on quality improvements. In this section, we assume that choosing Channel P implies the same duplication of investment costs as choosing Channel f: P has to pay two separate costs to invest in the quality of the brand and in the quality of the private label if it intends to sell both goods. Interestingly, we now show that although one of the main benefits of choosing channel P, i.e., avoidance of the duplication of costs, has been withdrawn, the retailer may still select Channel P only as a result of the balance of opposite strategic effects on quality improvements.

We briefly discuss the effects at play in the following.

**Proposition 6.** *Entrusting the national brand producer with the production of the private label may be profitable for the industry even without scale economies.*



*Proof.* The full resolution of this case is given in Appendix (A.6).  $\square$

Channel f is unchanged; therefore qualities, prices and quantities sold are exactly the same as in the previous case. The equilibrium in this subgame is given by Proposition 2.

Channel P is modified by the duplication of cost. The national brand producer now has to pay an investment cost  $C(k_B)$  to increase the quality of the national brand by  $k_B$ , and an additional investment cost  $C(k_L)$  to simultaneously increase the quality of the private label by  $k_L$ . The total industry profit is therefore denoted  $\tilde{\Pi}^P(k_L, k_B)$  and is identical to the total industry profit in Channel f,  $\Pi^f(k_L, k_B)$ . Therefore, the maximum profit in each case is the same and  $\tilde{\Pi}^{P*} = \Pi^{f*}$ . In Channel P, the producer chooses both qualities and both are still sub-optimal because of the hold-up effect. The resulting total industry profit is denoted  $\tilde{\Pi}^P$  and we obtain the simplified comparison of the two production channels where only strategic effects are at stake:

$$\Delta_{P,f} = \underbrace{[\Pi^{f*} - \Pi^f]}_{\substack{\text{hold-up and over-investment} \\ \text{in Channel f}}} - \underbrace{[\tilde{\Pi}^{P*} - \tilde{\Pi}^P]}_{\text{hold-up in Channel P}} \quad (12)$$

In Channel P either both B and L are sold or only B. To keep the insight simple, we consider a case in which both goods are sold in both Channel f and Channel P.<sup>14</sup> In this case, the quality of the national brand is independent of the choice of production channel :  $k_B^f = k_B^P$ . Indeed, for P, both the marginal benefits and marginal costs associated with  $k_B$  are the same in both channels. In contrast, the quality of the private label depends on the production channel, although it is suboptimal in both cases. With Channel f the quality  $k_L^f$  is too high because of the over-investment effect and with Channel P it is too low because of the hold-up effect. The comparison of the two Channels is a comparison between these two sub-optimal situations, but a simple insight can be given through the comparative statics in  $\alpha$ .

When  $\alpha = 0$ , that is, when the bargaining power is completely in the hands of the producer, it is clear that the hold-up effects in both Channel P and Channel f disappear. The second term in equation (12) is thus equal to 0. By contrast, the first term is strictly

positive because the over-investment distortion remains; therefore, R selects Channel P.

When  $\alpha = 1$ , it is clear that the over-investment distortion disappears. Moreover, the hold-up effect on the brand is the same in both the Channel f and Channel P cases (i.e.  $k_B^f = k_B^P$ ). However, an additional hold-up effect on the private label arises only in Channel P. The second term in eq. (12) is thus strictly higher than the first term, and Channel f is always preferred to Channel P.

By continuity, R chooses Channel P in equilibrium as long as  $\alpha$  is small enough, and chooses Channel f otherwise. Consumers, however, are always better off with Channel f, because of the higher quality of the private label in that case.<sup>15</sup>

## 7 Managerial Perspectives

### 7.1 Individual profitability

Given that transfers are possible in stage 1, P and R can always find a mutually beneficial agreement to share a higher industry profit. Absent such transfers, the private profitability of each alternative now matters. We examine here whether, absent any transfer in stage 1, choosing channel P instead of Channel f can be privately profitable for the producer and the retailer.

**Result 1.** *Regardless of the equilibrium market structure in Channel f, a switch to Channel P is always profitable for the producer.*

The brand producer is always better off manufacturing both goods rather than producing only its national brand. For any quality investment the comparison of the two channels unambiguously favors Channel P: the producer's investment generates more direct revenue and the producer suffers less from the opportunism of the retailer. Although the industry gross revenue may be larger in Channel f than in Channel P, due to the larger private label quality, the gains from trade and hence the profit of the producer are lower in the Channel f.

Focusing now on the retailer, two cases are worth distinguishing, depending on whether the private label is or is not sold in Channel f.

**Result 2.** *When only the national brand is sold in Channel f, switching to Channel P decreases the profit of the retailer.*

When only the national brand is sold in Channel f, switching to Channel P unambiguously increases total revenue. However, the brand is the only good sold in Channel f when brand lovers are over represented (i.e. when  $\lambda$  large) and the bargaining power of the retailer is low. The former limits the gain from discrimination. The latter implies that the ability of the retailer to strategically choose its outside option is all the more valuable. Taken as a whole, the retailer always loses profits when the structure changes from a situation in which only the brand is sold in Channel f to a situation in which both goods are sold in Channel P.

**Result 3.** *When the private label is sold in Channel f (with or without the national brand), switching to Channel P has an ambiguous effect on the retailer's profit. The retailer may have a larger profit in switching to channel P when  $\lambda$  and  $\delta$  are sufficiently high and  $\alpha$  is intermediate.*

When the private label is sold, the comparison between the two options depends on the values of the parameters. Although the retailer loses its ability to strategically manipulate its outside option it may benefit from switching to Channel P because of the induced increase of industry revenue.

The trade-offs at stake could be described by considering the change in the retailer's bargaining power. When  $\alpha$  increases there are two opposite effects on the retailer's comparison of the two options. On the one hand, when its bargaining power increases, the retailer has less incentive to strategically inflate its outside option. This makes Channel f less appealing for large values of  $\alpha$ . On the other hand, when  $\alpha$  increases, the hold-up effect on the producer's investment is exacerbated, and this hold-up effect reduces the relative merit of Channel P. These two opposite effects of  $\alpha$  on the net gain to the retailer of switching from Channel f to Channel P are such that switching is profitable for the retailer only at intermediate values of  $\alpha$ . Note that even in the case where only L is sold in Channel f, the retailer may nonetheless find it profitable in a few cases to switch to Channel P.

## 7.2 Managerial guidelines

In contrast to Quelch and Harding (2012), who dwell upon the risk to national brand manufacturers of also producing a private label for a retailer, our model emphasizes the advantages of such production. As shown in Section 7.1, selling the private label may have several positive effects for the national brand manufacturer: it spurs innovation on the brand, it limits the buyer power of the retailer and finally, it may enable the retailer to discriminate and thus extract more consumer surplus.

Another direct advantage to a national brand producer of producing a private label is that it enables the producer to control the gap in quality between the brand and the private label. By removing innovation from the hands of the retailer, it may prevent excessive innovation on the private label that would be detrimental to the market share of the brand. In the extreme case where only the private label would be sold otherwise, it enables the national brand producer to avoid exclusion (See Corollary 1).

Loss to the national brand of market shares to the benefit of the private label is a drawback often mentioned in the debate on whether the national brand producer should begin making private labels (see Quelch and Harding, 2012). The present work provides a good illustration of cannibalization of sales: entrusting the national brand producer with the manufacturing of the private label may enable the retailer to maintain the private label on its shelves (See Corollary 2). However, this cannibalization effect does not harm the producer; it derives from a better discrimination of consumers, which increases total industry profit, and it is always profitable to the producer.

Our paper also emphasizes that retailers should proceed with caution when choosing to entrust the national brand manufacturer with the manufacturing of their private label. We point out two main risks in such a managerial decision: it reduces the retailer's bargaining power towards the national brand manufacturer and it may deter innovation on the private label.

Moreover, the retailer always loses profits when only the brand would have been sold absent the agreement with the producer to make the private label. Interestingly, as found in Section (7.1), ensuring that the private label is sold on the market is not a good reason

for a contract with the producer; the gains resulting from better discrimination are always offset by the loss in buyer power.

Finally, we acknowledge that the effects we describe are short-term effects and that some of them could be exacerbated or reversed in the long run. For instance, in the long run, the parameters  $\delta$  and  $\lambda$ , both of which represent the consumers' exogenous preference for the national brand with respect to the private label, could be affected. From the producer's point of view, controlling the quality gap between the national brand and the private label may in the long run increase the consumers' preference for the brand. In contrast, if it is publicly revealed to consumers that the national brand manufacturer produces the private label, this may negatively affect the consumers' preference for the brand. From the retailer's point of view, when entrusting the national brand producer with the manufacturing of its private label enables the presence of the private label on the retailer's shelves, an effective cannibalization of sales could arise in the long run if this negatively affects the consumers' preference for the brand.

## 8 Conclusion

This article analyzes the choice by a retailer of the supply channel for its private label. The analysis emphasizes the role played by innovation in both the quality of the private label (the outsourced good) and the brand (an imperfect substitute) on the comparison between supply channels. It shows that a retailer may prefer to entrust a brand producer with the manufacturing of a private label rather than produce a private label on its own. Two main forces are at work here. First, entrusting the brand producer with production of the private label may avoid duplication of R&D costs, which tends to increase the qualities of the two goods. Second, this choice destroys the incentive of the retailer to over-invest in the private label quality so as to gain buyer power towards the brand manufacturer. When the buyer power of the retailer is not too strong and the preference for the national brand is sufficient, these two positive effects prevail over the hold-up effect that pushes the producer to under-invest in quality.

The choice of production channel determines not only the qualities of the two goods

but also which goods appear on the retailers' shelves. In some cases, entrusting the brand producer with the production of the private label implies that two products are sold instead of one. In some circumstances, it ensures that the brand is sold by preventing the retailer from over-investing; in others, it enables the retailer to sell the private label and to discriminate among consumers.

When the national brand producer also manufactures the private label in equilibrium, consumers may be hurt because of too little innovation on the private label. Considering the managerial incentives of firms, we have primarily highlighted the advantages associated with a national brand producer's also manufacturing the private label product, whereas the retailer instead faces drawbacks. The balance of power between the retailer and the national brand producer clearly switches in favor of the latter when the national brand producer also produces the private label. The insight is clear because the producer thereby has more control of strategic decisions.

Finally, it would be interesting for further research to incorporate retail competition in the analysis. In particular, retail competition could explain the noticeable emergence of large size specialized manufacturers in the production of private labels.

## Notes

<sup>1</sup>Source: Planet Retail, 2008.

<sup>2</sup>Survey by Mintel research, quoted in "Private Label Gets a Quality Reputation, Causing Consumers to Change Their Buying Habits", CHICAGO, Jan. 20, 2011 /PRNewswire/.

<sup>3</sup><http://www.storebrandsdecisions.com/news/2012/05/29/mintel-private-label-product-development-outpaces-cpg-in-the-uk> .

<sup>4</sup>For instance, Richelieu Foods in the U.S., a private label food manufacturing company founded in 1862 that produces frozen pizza, salad dressings, sauces, marinades, condiments and deli salads to be marketed by other companies as their store brands, makes more than \$200 million in yearly sales.

<sup>5</sup><http://idei.fr/doc/conf/inra/2011/bonvallet.pdf>

<sup>6</sup>Krüger, a private label producer of chocolate, chocolate spread and instant beverages, made 1.3 billion euros in sales in 2007; Bakkavör, an Icelandic private label producer of fresh products, made 26.3 million euros in sales after a merger with the British Geest and the French 4G. See "Concentration dans les marques d'hypers", P. Deniel and Y. Dougin in *L'Usine Nouvelle* n° 3057 (2007).

<sup>7</sup>We explain in section 4 why a large specialized manufacturer cannot be chosen as private label producer in our set-up.

<sup>8</sup>Survey by Mintel Research. See footnote 2.

<sup>9</sup>With perfect competition among fringe firms, R captures its joint profit with the fringe, and because quality is non-contractible, a competitive fringe would never invest. Thus, a quality investment can only be realized if the retailer bears the full cost of innovation. We have in mind the following process: the retailer invests in R&D and calls for bids from fringe firms for the production of its private label.

<sup>10</sup>Our results are qualitatively unchanged without scale economies on quality investment (cf. Section 6).

<sup>11</sup>Radical innovations represent approximately 6% of total innovation output (Martos-Partal, 2012).

<sup>12</sup> A strictly similar outcome would be reached if firms were bargaining over a two-part tariff. If firms were bargaining over a linear tariff the outcome would be different due to double-marginalization.

<sup>13</sup>Note, however, that whenever producing both the national brand and the private label reduces the share of brand lovers, the retailer may turn to a specialized private label manufacturer instead of the national brand producer to supply the private label. In the same vein, if the specialized private label manufacturer has a cost advantage over the competitive fringe to invest in quality, which may be justified by its larger size, the retailer may prefer resorting to the specialized manufacturer than to a competitive fringe.

<sup>14</sup>We prove in Appendix A.6 that the situation in which the two goods B and L are sold may indeed arise in equilibrium when Channel P is chosen. Note that due to the duplication of cost, the case in which only B is sold often appears in equilibrium. However, if only one good is sold in Channel f or Channel P, the same kind of arbitrage arises. To entrust P with the production of the private label is a way to prevent the retailer from inflating the quality of the private label so as to increase its share of the pie.

<sup>15</sup>If only B is sold in Channel P, then consumers are better off with Channel P for low value of  $\alpha$ .

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# A Appendix

## A.1 Proof of Lemma 1

Assume for the sake of clarity that P sets qualities for the two goods in a two-stage process: first P sets a level of investment  $k$  that defines the maximum quality that P can then choose for each good; second P chooses for good  $i$  a level of quality  $k_i \leq k$ .

In the second stage of this process, given  $k$ , P chooses the two qualities  $k_B$  and  $k_L$  by maximizing its gross profit (investment costs are sunk)  $(1 - \alpha)(\pi(k_L, k_B) - \bar{\pi}(0))$ , subject to the constraint that  $k_B \leq k$  and  $k_L \leq k$ . The monopoly profit  $\pi(k_L, k_B)$  is increasing with respect to both qualities (see eq. (3)). Therefore, the gross profit of the producer is increasing with respect to both qualities. The profit maximizing qualities therefore are  $k_L = k_B = k$ . Furthermore, for any  $k$ , we have  $k \in \left(k - \delta, \sqrt{(k + v)^2 + \delta^2 \lambda} - v\right]$ , which means that for all values of  $\alpha \in [0, 1]$ ,  $\lambda \in [0, 1]$ ,  $v > 0$  and  $\delta \in [0, v]$ , if the two goods have identical qualities, then they are both sold.

## A.2 Proof of Proposition 1

From Lemma 1, the producer maximizes  $\Pi_P^P(k, k)$ . From the expression of P's profit (5) it chooses

$$k_B^P = k_L^P = \frac{(v + \lambda\delta)(1 - \alpha)}{1 + \alpha}.$$

Profits are then:

$$\begin{aligned} \Pi_P^P &= \frac{(1-\alpha)}{4} \left( \frac{2(\delta\lambda+v)^2}{1+\alpha} + \delta^2(1-\lambda)\lambda - v^2 \right), \\ \Pi_R^P &= (1-\alpha)\frac{v^2}{4} + \alpha \left[ \frac{(\delta\lambda+v)^2}{(1+\alpha)^2} + \frac{\delta^2(1-\lambda)\lambda + (1-\alpha)v^2}{4} \right]. \end{aligned}$$

## A.3 Proof of Lemma 2

In the continuation game in which Channel f is chosen, R maximizes  $\Pi_R^f(k_L, k_B)$  with respect to  $k_L$  and P maximizes  $\Pi_P^f(k_L, k_B)$  with respect to  $k_B$ . The function  $\pi(k_L, k_B)$  is continuous, differentiable by part and concave by part with respect to  $k_B$  and to  $k_L$ . The corresponding

first order conditions are satisfied (there is no corner solution because in each corner one of the two firms' gross profit is zero): both goods are sold or only one of them is sold. Therefore, a subgame equilibrium is necessarily of one of three types:

1. **One candidate, denoted (L), is such that the private label only is sold.**

Equilibrium investment is thus  $k_L^L = v$  and  $R$  earns a profit  $\Pi_R^L = \frac{v^2}{2}$ .  $P$  anticipates that its product will not be sold in equilibrium, and therefore does not invest:  $k_B^L = 0$  and  $\Pi_P^L = 0$ .

2. **Another candidate, denoted (BL), is such that both the private label and the brand are sold.**  $R$  sets  $k_L^{BL} = \frac{v-\alpha v\lambda}{1+\alpha\lambda}$  and  $P$  sets  $k_B^{BL} = \frac{(1-\alpha)(v+\delta)\lambda}{2-(1-\alpha)\lambda}$ . The corresponding equilibrium profits are :

$$\begin{aligned}\Pi_R^{BL} &= \frac{4\alpha v\delta\lambda(1+\alpha\lambda)+2\alpha\delta^2\lambda(1+\alpha\lambda)+v^2(4-\lambda(1-\alpha\lambda)(4-\lambda-\alpha(2-(2-\alpha)\lambda)))}{2(2-(1-\alpha)\lambda)^2(1+\alpha\lambda)}, \\ \Pi_P^{BL} &= \frac{(1-\alpha)\lambda(2v\delta(1+\alpha\lambda)^2+(\delta+\alpha\delta\lambda)^2-v^2(3-\lambda(2+\alpha^2\lambda)))}{2(2-(1-\alpha)\lambda)(1+\alpha\lambda)^2}.\end{aligned}$$

3. **Another candidate, denoted (B), is such that only the brand is sold.**  $R$  sets

$k_L^B = \frac{v(1-\alpha)}{(1+\alpha)}$  and  $P$  sets  $k_B^B = \frac{(1-\alpha)(v+\delta\lambda)}{1+\alpha}$ . The equilibrium profits are:

$$\Pi_R^B = \frac{(1+(2-\alpha)\alpha)v^2+4\alpha v\delta\lambda+2\alpha\delta^2\lambda^2}{2(1+\alpha)^2} \quad \text{and} \quad \Pi_P^B = \frac{(1-\alpha)(2(1+\alpha)v\delta\lambda+(1+\alpha)\delta^2\lambda^2-(1-\alpha)v^2)}{2(1+\alpha)^2}.$$

## A.4 Proof of Proposition 2

We now determine the domains of existence of equilibria (L), (BL) and (B).

**Existence of (L):** Whenever  $\delta \leq \delta_2 = v \min \left\{ \left( \frac{2}{\sqrt{1+\alpha\lambda}} - 1 \right), \left( \sqrt{2(2-\lambda+\alpha\lambda)} - 1 \right), \frac{(\sqrt{2(1+\alpha)}-1)}{\lambda} \right\}$ .

- First this equilibrium candidate may exist if and only if:  $\delta < k_L^L - k_B^L = v$ . This condition is always true by assumption.
- Potential deviation of  $P$  towards (BL) by investing  $k_B^{BL}$  is possible only when  $\delta > k_L^L - k_B^{BL}$ , or  $\delta > v(1 - (1 - \alpha)\lambda)$ . Otherwise,  $B$  would still not be sold; thus,  $P$  would be strictly better off by not investing. By deviating,  $P$  earns  $\Pi_P^{BL}(k_L^L, k_B^{BL})$

which is strictly positive when  $\delta > v(\sqrt{2}\sqrt{2-\lambda+\alpha\lambda}-1)$ . To summarize, if  $\delta \leq v(\sqrt{2}\sqrt{2-\lambda+\alpha\lambda}-1)$  there is no profitable deviation by P towards (BL).

- Potential deviation of R towards (BL) by setting  $k_L^{BL}$  is possible when  $\delta > k_L^{BL}$ . Indeed, if  $\delta < k_L^{BL}$  B would still not be sold. Moreover, this deviation is strictly profitable only if  $\Pi_R^{BL}(k_B^L, k_L^{BL}) > \Pi_R^L(k_B^L, k_L^L)$ , i.e if  $\delta > v\left(\frac{2}{\sqrt{1+\alpha\lambda}}-1\right)$ . To summarize, if  $\delta \leq v\left(\frac{2}{\sqrt{1+\alpha\lambda}}-1\right)$ , there is no profitable deviation by R towards (BL).
- Potential deviation of R towards (B) by setting  $k_L^B$  is impossible. Indeed, it would imply  $v \geq \sqrt{(k_L^B + v)^2 + \lambda\delta^2}$ , which cannot be true because  $k_L^B > 0$  and  $\delta, \lambda \geq 0$ .
- Potential deviation of P towards (B) is possible if:

$$\delta \geq \min \left\{ v, \frac{2v}{(1+\alpha)^2 - (1-\alpha)^2\lambda} \left( 1 - \alpha - (1+\alpha)\sqrt{(1-\alpha)^2 - \frac{\alpha(2+\alpha)}{\lambda}} \right) \right\}.$$

which ensures that only B is sold on the market. Additionally, this deviation is only strictly profitable for P if  $\Pi_P^B(k_B^B, k_L^L) > \Pi_P^L(k_B^L, k_L^L)$ , i.e. if  $\delta > \min \left\{ v, \frac{v(\sqrt{2(1+\alpha)}-1)}{\lambda} \right\}$ .

**Existence of (BL):** (BL) exists whenever  $\delta \geq \delta_1 = v \max \left\{ \left( \frac{2-(1-\alpha)\lambda}{\sqrt{1+\alpha\lambda}} - 1 \right), \left( \frac{\sqrt{2(2-(1-\alpha)\lambda)}}{(1+\alpha\lambda)} - 1 \right) \right\}$ ,

or:

$$\delta < \min \left\{ \frac{v(1-\alpha-(1+\alpha)\sqrt{X})}{1+\alpha-\lambda(1-\alpha)}, \frac{2v((1-\alpha-(2-\lambda(1-\alpha))\sqrt{Y}))}{4-(5-\alpha)(1-\alpha)\lambda+(1-\alpha)^2\lambda^2} \right\}$$

or

$$\delta > \max \left\{ \frac{v(1-\alpha+(1+\alpha)\sqrt{X})}{1+\alpha-\lambda(1-\alpha)}, \frac{2v((1-\alpha+(2-\lambda(1-\alpha))\sqrt{Y}))}{4-(5-\alpha)(1-\alpha)\lambda+(1-\alpha)^2\lambda^2} \right\},$$

with:

$$\begin{aligned} X &= \frac{(1+\alpha)(2-\lambda(1-\alpha))(-1-2\alpha+2\lambda+\alpha^2\lambda^2)}{\lambda}, \\ Y &= \frac{(1-\alpha)^2\lambda(2-\lambda)-(3-\alpha)(1-\lambda)}{(1+\alpha)\lambda(1+\alpha\lambda)}. \end{aligned}$$

This second condition concerns only cases in which  $X > 0$  or  $Y > 0$ . If only  $X$  or  $Y$  is positive, for instance  $X$ , then the condition is reduced to  $\delta < \frac{v(1-\alpha-(1+\alpha)\sqrt{X})}{1+\alpha-\lambda(1-\alpha)}$  or  $\delta > \frac{v(1-\alpha+(1+\alpha)\sqrt{X})}{1+\alpha-\lambda(1-\alpha)}$  (respectively for  $Y > 0$  and  $X < 0$ ). Note that we have:

- if  $\lambda < 1/2$ , then  $X < 0$  and  $Y < 0$ .
- if  $\lambda \in [1/2, \frac{1}{2}(5 - \sqrt{13})]$ , then  $X > 0$  if and only if  $0 < \alpha < \frac{1 - \sqrt{(1-\lambda)(1+\lambda+2\lambda^2)}}{\lambda^2}$ , and  $Y < 0$  for all  $\alpha$ .
- if  $\lambda \in [\frac{1}{2}(5 - \sqrt{13}), 0.906]$ , then  $X > 0$  and  $Y > 0$  if  $0 < \alpha < \frac{1-5\lambda+2\lambda^2+\sqrt{(1-\lambda)(1+15\lambda-8\lambda^2)}}{2(-2+\lambda)\lambda}$ .  
If  $\frac{1-5\lambda+2\lambda^2+\sqrt{(1-\lambda)(1+15\lambda-8\lambda^2)}}{2(-2+\lambda)\lambda} \leq \alpha < \frac{1}{\lambda^2} - \sqrt{\frac{1+\lambda^2-2\lambda^3}{\lambda^4}}$ , then  $X > 0$  and  $Y < 0$ . Otherwise,  $X < 0$  and  $Y < 0$ .
- If  $\lambda \in [0.906, 1]$ , then if  $0 < \alpha < \frac{1}{\lambda^2} - \sqrt{\frac{1+\lambda^2-2\lambda^3}{\lambda^4}}$ ,  $X > 0$  and  $Y > 0$ . If  $\frac{1}{\lambda^2} - \sqrt{\frac{1+\lambda^2-2\lambda^3}{\lambda^4}} \leq \alpha < \frac{1-5\lambda+2\lambda^2+\sqrt{(1-\lambda)(1+15\lambda-8\lambda^2)}}{2(-2+\lambda)\lambda}$ , then  $Y > 0$  and  $X < 0$ . Otherwise  $X < 0$  and  $Y < 0$ .

In the following, we explain how we obtain these thresholds.

- First, this equilibrium may exist if and only if  $k_B^{BL} < \sqrt{(k_L^{BL})^2 + 2k_L^{BL}v + v^2 + \delta^2\lambda} - v$ , which is true in the area where there is no profitable deviation from (BL).
- Potential deviation of P towards (B) by setting  $k_B^B$  is profitable if  $\Pi_P^{BL}(k_L^{BL}, k_B^{BL}) < \Pi_P^B(k_L^{BL}, k_B^B) = \frac{(1-\alpha)(2v\delta\lambda(1+\alpha\lambda)^2 + \delta^2\lambda^2(1+\alpha\lambda)^2 - v^2(1+\alpha(2-\lambda(2+\alpha\lambda))))}{2(1+\alpha)(1+\alpha\lambda)^2}$ . Thus, there is no profitable deviation for P towards (B) as long as:

$$\delta < \frac{v(1-\alpha)}{1+\alpha-\lambda+\alpha\lambda} \left( 1 - \alpha - \sqrt{\frac{(\lambda-\alpha\lambda-2)(2(1-2\lambda)+\alpha^2(1+\lambda)(\alpha(1-\lambda)^2-\lambda^2)+4\alpha(1-\lambda(1-\lambda))+3\alpha^2(1-\lambda))}{2(1+\alpha)\lambda}} \right),$$

or

$$\delta > \frac{v(1-\alpha)}{1+\alpha-\lambda+\alpha\lambda} \left( 1 - \alpha + \sqrt{\frac{(\lambda-\alpha\lambda-2)(2(1-2\lambda)+\alpha^2(1+\lambda)(\alpha(1-\lambda)^2-\lambda^2)+4\alpha(1-\lambda(1-\lambda))+3\alpha^2(1-\lambda))}{2(1+\alpha)\lambda}} \right).$$

- Potential deviation of P towards (L) by setting  $k_L^L$  is profitable whenever  $\Pi_P^{BL}(k_L^{BL}, k_B^{BL}) < 0$ . Thus when  $\delta > v \left( \frac{\sqrt{2(2-(1-\alpha)\lambda)}}{(1+\alpha\lambda)} - 1 \right)$  there is no profitable deviation for P towards (L).
- Potential deviation of R towards (B) by setting  $k_L^B$  is profitable if and only if  $\Pi_R^B(k_L^B, k_B^{BL}) > \Pi_R^{BL}(k_L^{BL}, k_B^{BL})$ , which is true when:

$$\delta > \frac{2v}{4-(5-\alpha)(1-\alpha)\lambda+(1-\alpha)^2\lambda^2} \left( 1 - \alpha - (2 - \lambda(1 - \alpha)) \sqrt{\frac{(1-\alpha)^2\lambda(2-\lambda)-(3-\alpha)(1-\lambda)}{(1+\alpha)\lambda(1+\alpha\lambda)}} \right),$$

and

$$\delta < \frac{2v}{4-(5-\alpha)(1-\alpha)\lambda+(1-\alpha)^2\lambda^2} \left( 1 - \alpha + (2 - \lambda(1 - \alpha)) \sqrt{\frac{(1-\alpha)^2\lambda(2-\lambda)-(3-\alpha)(1-\lambda)}{(1+\alpha)\lambda(1+\alpha\lambda)}} \right)$$

For any  $\delta$  satisfying these conditions, the deviation also exists.

- Potential deviation of R towards (L) by setting  $k_L^L$  is possible if  $\delta < v(1 - \lambda + \alpha\lambda)$  and such a deviation is profitable if and only  $\Pi_R^{BL}(k_L^{BL}, k_B^{BL}) < \Pi_R^L(k_L^L, k_B^{BL}) = \frac{v^2}{2}$ . This arises if  $\delta < v \left( \frac{(2-(1-\alpha)\lambda)}{\sqrt{1+\alpha\lambda}} - 1 \right)$ . Because we have  $v \left( \frac{(2-(1-\alpha)\lambda)}{\sqrt{1+\alpha\lambda}} - 1 \right) < v(1 - \lambda + \alpha\lambda)$ , as long as  $\delta \geq v \left( \frac{(2-(1-\alpha)\lambda)}{\sqrt{1+\alpha\lambda}} - 1 \right)$ , there is no profitable deviation by R towards (L).

**Existence of (B):** Whenever  $\lambda > \frac{\sqrt{17}-3}{2}$ ,  $\alpha < 2\lambda - 1$  and  $\delta \in [\delta_3, \delta_4]$ , with:

$$\begin{aligned} \delta_3 &= v \max \left\{ \frac{1}{1+\alpha-\lambda(1-\alpha)} \left( 1 - \alpha - \sqrt{\frac{(\alpha-1)(1+\alpha-2\lambda)(2-\lambda+\alpha\lambda)}{(1+\alpha)\lambda}} \right), \frac{1}{\lambda} \left( \sqrt{\frac{2}{1+\alpha}} - 1 \right), \right. \\ &\quad \left. \frac{2}{(1+\alpha)^2-(1-\alpha)^2\lambda} \left( 1 - \alpha - \sqrt{\frac{(1+\alpha)(-\alpha-\alpha^2+\lambda-\alpha\lambda+2\alpha^2\lambda)}{\lambda(1+\alpha\lambda)}} \right), \min \left\{ \frac{2\alpha}{1+\alpha+\lambda-\alpha\lambda}, \frac{\sqrt{1+\alpha}-1}{\lambda} \right\} \right\}, \\ \delta_4 &= v \min \left\{ \frac{1}{1+\alpha-\lambda(1-\alpha)} \left( 1 - \alpha + \sqrt{\frac{(\alpha-1)(1+\alpha-2\lambda)(2-\lambda+\alpha\lambda)}{(1+\alpha)\lambda}} \right), \right. \\ &\quad \left. \frac{2}{(1+\alpha)^2-(1-\alpha)^2\lambda} \left( 1 - \alpha + \sqrt{\frac{(1+\alpha)(-\alpha-\alpha^2+\lambda-\alpha\lambda+2\alpha^2\lambda)}{\lambda(1+\alpha\lambda)}} \right) \right\} \end{aligned}$$

- First this equilibrium may exist if and only if:  $k_B^B > \sqrt{(k_L^B)^2 + 2k_L^B v + v^2 + \delta^2 \lambda} - v$ , or equivalently  $\delta < \frac{4(1-\alpha)v}{(1+\alpha)^2-(1-\alpha)^2\lambda}$ . This condition is always verified when  $\delta \in [\delta_3, \delta_4]$ ; therefore, this equilibrium always exists for  $\delta \in [\delta_3, \delta_4]$ .
- Potential deviation of P towards (BL) by setting  $k_B^{BL}$  is possible when  $k_L^B - \delta \leq k_B^{BL} \leq \sqrt{(k_L^B + v)^2 + \lambda\delta^2} - v$ , which is never binding. It is profitable if  $\Pi_P^B(k_L^B, k_B^B) < \Pi_P^{BL}(k_L^B, k_B^{BL}) = \frac{(1-\alpha)\lambda((1+\alpha)^2\delta^2+2(1+\alpha)^2\delta v-(1-\alpha)(3+\alpha-2\lambda)v^2)}{2(1+\alpha)^2(2-(1-\alpha)\lambda)}$ . Thus, there is no profitable deviation for P towards (BL) if  $\frac{1}{2} < \lambda < 1$ ,  $0 < \alpha < -1 + 2\lambda$  and:

$$\begin{aligned} \delta \in \left[ \min \left\{ v, \frac{v}{1+\alpha-\lambda(1-\alpha)} \left( 1 - \alpha - \sqrt{\frac{(\alpha-1)(1+\alpha-2\lambda)(2-\lambda+\alpha\lambda)}{(1+\alpha)\lambda}} \right) \right\}, \right. \\ \left. \min \left\{ v, \frac{v}{1+\alpha-\lambda(1-\alpha)} \left( 1 - \alpha + \sqrt{\frac{(\alpha-1)(1+\alpha-2\lambda)(2-\lambda+\alpha\lambda)}{(1+\alpha)\lambda}} \right) \right\} \right]. \end{aligned}$$

For sufficiently low values of  $\lambda$ ,  $\frac{1}{1+\alpha-\lambda(1-\alpha)} \left(1 - \alpha - \sqrt{\frac{(\alpha-1)(1+\alpha-2\lambda)(2-\lambda+\alpha\lambda)}{(1+\alpha)\lambda}}\right)$  is increasing in  $\alpha$  for all  $\alpha < -1 + 2\lambda$ . When  $\alpha = 0$ , it is equal to  $\frac{1}{1-\lambda} \left(1 - \sqrt{-\frac{(1-2\lambda)(2-\lambda)}{\lambda}}\right)$ , which is larger than 1 as long as  $\lambda < \frac{\sqrt{17}-3}{2}$ . Therefore, the former condition can be summarized as  $\frac{\sqrt{17}-3}{2} < \lambda < 1$ ,  $\alpha < -1 + 2\lambda$  and:

$$\frac{v}{1+\alpha-\lambda(1-\alpha)} \left(1 - \alpha - \sqrt{\frac{(\alpha-1)(1+\alpha-2\lambda)(2-\lambda+\alpha\lambda)}{(1+\alpha)\lambda}}\right) < \delta < \frac{v}{1+\alpha-\lambda(1-\alpha)} \left(1 - \alpha + \sqrt{\frac{(\alpha-1)(1+\alpha-2\lambda)(2-\lambda+\alpha\lambda)}{(1+\alpha)\lambda}}\right).$$

- Potential deviation of P towards (L) by setting  $k_B^L$  is possible whenever  $k_L^B - \delta > k_B^L = 0$ . It is profitable whenever  $\Pi_P^B(k_L^B, k_B^B) < 0$ . Thus when  $\delta > \frac{v}{\lambda} \left(\sqrt{\frac{2}{(1+\alpha)}} - 1\right)$  there is no profitable deviation for P towards (L).
- Potential deviation of R towards (BL) by setting  $k_L^{BL}$  is possible if  $k_L^{BL} - \delta \leq k_B^B \leq \sqrt{(k_L^{BL} + v)^2 + \lambda\delta^2} - v$ . This condition is never binding. It is profitable whenever  $\Pi_R^B(k_L^B, k_B^B) < \Pi_R^{BL}(k_L^{BL}, k_B^B)$ , where:

$$\begin{aligned} \Pi_R^{BL}(k_L^{BL}, k_B^B) &= \frac{\alpha\lambda}{4} \left( \delta + \frac{(1-\alpha)(v+\delta\lambda)}{1+\alpha} - \frac{v(1-\alpha\lambda)}{1+\alpha\lambda} \right) \left( 2v + \delta + \frac{(1-\alpha)(v+\delta\lambda)}{1+\alpha} + \frac{v(1-\alpha\lambda)}{1+\alpha\lambda} \right) \\ &+ \frac{v^2(2-(1-\alpha\lambda)^2)}{2(1+\alpha\lambda)^2} \end{aligned}$$

Thus, there is no profitable deviation for R towards (BL) if  $0 < \alpha < \frac{1+\lambda-\sqrt{1+6\lambda-7\lambda^2}}{2(-1+2\lambda)}$  and:

$$\delta \in \left[ \min \left\{ v, \frac{2v}{(1+\alpha)^2-(1-\alpha)^2\lambda} \left( 1 - \alpha - \sqrt{\frac{(1+\alpha)(-\alpha-\alpha^2+\lambda-\alpha\lambda+2\alpha^2\lambda)}{\lambda(1+\alpha\lambda)}} \right) \right\}, \right. \\ \left. \min \left\{ v, \frac{2v}{(1+\alpha)^2-(1-\alpha)^2\lambda} \left( 1 - \alpha + \sqrt{\frac{(1+\alpha)(-\alpha-\alpha^2+\lambda-\alpha\lambda+2\alpha^2\lambda)}{\lambda(1+\alpha\lambda)}} \right) \right\} \right].$$

- Potential deviation of R towards (L) by setting  $k_L^L$  is possible as long as  $k_L^L - \delta > k_B^B$ , or equivalently  $\delta < \frac{2\alpha v}{1+\alpha+\lambda(1-\alpha)}$ . It is profitable if  $\Pi_R^B(k_L^B, k_B^B) < \Pi_R^L(k_L^L, k_B^B) = \frac{v^2}{2}$ . Thus, when  $\delta > \min \left\{ \frac{2v\alpha}{1+\alpha+\lambda-\alpha\lambda}, \frac{v(\sqrt{1+\alpha}-1)}{\lambda} \right\}$  there is no profitable deviation for R towards (L).

## A.5 Proof of Proposition 3

We obtain the thresholds  $\delta^*$  and  $\delta^{**}$  by comparing total industry profits in Channel f and Channel P. First,  $\Pi^{P*}$ , the total industry profit in Channel P is:

$$\Pi^{P*} = \Pi_R^P(k_L^P, k_B^P) + \Pi_P^P(k_L^P, k_B^P) = \frac{(2+2(2-\alpha)\alpha)v^2+4(1+(2-\alpha)\alpha)v\delta\lambda+\delta^2\lambda(1+\lambda+\alpha(2+\alpha+2\lambda-3\alpha\lambda))}{4(1+\alpha)^2}.$$

Then  $\Pi^{f*}$ , the total industry profit in Channel f depends on the equilibrium. If the equilibrium is (L), Channel P is chosen over Channel f if:

$$\Pi^{f*} = \Pi_R^L(k_L^L, k_B^L) < \Pi^{P*}.$$

This gives the first threshold:

$$\delta^* = \frac{2v\alpha^2}{(1+(2-\alpha)\alpha)\lambda + \sqrt{(1+\alpha)^2\lambda(\alpha^2 - (1+2(1-\alpha)\alpha)\lambda)}}.$$

When the equilibrium in Channel f is (BL), Channel P is chosen over Channel f if:

$$\Pi^{f*} = \Pi_R^{BL}(k_L^{BL}, k_B^{BL}) + \Pi_P^{BL}(k_L^{BL}, k_B^{BL}) < \Pi^{P*}.$$

This gives the second threshold:

$$\delta^{**} = \frac{-2(1-\alpha)(1-\lambda)\lambda(1+\alpha\lambda)^2(2-\lambda+\alpha(6-(1-(3-\alpha)\alpha)\lambda)) + (1-\alpha^2)\lambda(2-(1-\alpha)\lambda)(1+\alpha\lambda)\sqrt{2A}}{(1-\alpha)^2(1-\lambda)\lambda^2(1+\alpha\lambda)^2(2-\lambda+\alpha(8-2\lambda+\alpha(2+3\lambda)))}$$

where:

$$A = (1-\lambda) [2 - (5-2\lambda)\lambda - 5\alpha^4\lambda^3 + 2\alpha(4 - (7-\lambda)\lambda) + 2\alpha^3\lambda(4 - (3-\lambda)\lambda) + \alpha^2(4 + \lambda(13 - (12-\lambda)\lambda))] .$$

## A.6 Proof of Proposition 6

Consider the case in which there is still duplication of costs even when the retailer entrusts the brand producer with the production of the private label. As for the case without cost duplication, we first determine the equilibria of subgames “Channel f” and “Channel  $\tilde{P}$ ” (we



use the tilde to differentiate this case from the case without cost duplication), and then compare the profits earned by the industry in these two channels. An equilibrium of the game is then such that the profit of the entire vertical structure is maximized.

Note that in Stage 4, prices are set as in the former case, and that for given levels of quality, the total profit of the industry before investment costs is still  $\pi(k_L, k_B)$ .

**Channel f.** The equilibria in Channel f are unchanged compared with the case without cost duplication because the cost structure in Channel f is unchanged.

**Channel  $\tilde{P}$ .** When the retailer chooses Channel P, the sharing of the joint profit is given by:

$$\tilde{\Pi}_P^P(k_L, k_B) = (1 - \alpha) [\pi(k_L, k_B) - \bar{\pi}(0)] - C(k_B) - C(k_L) \quad (13)$$

$$\tilde{\Pi}_R^P(k_L, k_B) = \bar{\pi}(0) + \alpha [\pi(k_L, k_B) - \bar{\pi}(0)] \quad (14)$$

In Stage 3, profits derive from equations (13) and (14). There are three possible local maxima:

1. **One candidate, denoted (PL), is such that the private label only is sold.**

Equilibrium investments of the producer are thus  $k_B^{PL} = 0$  and  $k_L^{PL} = \frac{(1-\alpha)v}{1+\alpha}$ . Profits are thus given by:

$$\tilde{\Pi}_P^{PL} = \frac{(1-\alpha)^2 v^2}{4(1+\alpha)}, \quad \tilde{\Pi}_R^{PL} = \frac{(1+5\alpha-\alpha^2(1-\alpha))v^2}{4(1+\alpha)^2}.$$

2. **One candidate denoted (PBL) is such that both the private label and the brand are sold.** Equilibrium investments of the producer are thus:

$$\tilde{k}_B^{PBL} = \frac{\lambda(1-\alpha)(v+\delta)}{2-\lambda(1-\alpha)}, \quad \tilde{k}_L^{PBL} = \frac{v(1-\alpha)(1-\lambda)}{1+\alpha+\lambda(1-\alpha)}.$$

Profits are then:

$$\begin{aligned}\tilde{\Pi}_P^{PBL} &= (1-\alpha) \left( \frac{(1-\lambda)v^2}{2(1+\alpha+\lambda(1-\alpha))} + \frac{\lambda(\delta+v)^2}{2(2-\lambda(1-\alpha))} - \frac{v^2}{4} \right), \\ \tilde{\Pi}_R^{PBL} &= (1-\alpha) \frac{v^2}{4} + \alpha \left( \frac{(1-\lambda)v^2}{(1+\alpha+\lambda(1-\alpha))^2} + \frac{\lambda(\delta+v)^2}{(2-(1-\alpha)\lambda)^2} \right).\end{aligned}$$

### 3. One candidate denoted (PB) is such that only the national brand is sold.

Equilibrium investments of the producer are thus  $\tilde{k}_L^{PB} = 0$  and  $\tilde{k}_B^{PB} = \frac{(1-\alpha)(v+\delta\lambda)}{1+\alpha}$ , and profits are given by:

$$\tilde{\Pi}_P^{PB} = \frac{(1-\alpha)(2\delta\lambda(\lambda\delta+2v)+(1-\alpha)v^2)}{4(1+\alpha)}, \quad \tilde{\Pi}_R^{PB} = (1-\alpha) \frac{v^2}{4} + \alpha \frac{(\delta\lambda+v)^2}{(1+\alpha)^2}.$$

Each of these local maxima exists only if the relevant conditions for  $k_B$  and  $k_L$  are satisfied: (PL) exists if qualities are such that  $\tilde{k}_B^{PL} > \sqrt{v^2 + \delta^2\lambda + (\tilde{k}_L^{PL})^2 + 2\tilde{k}_L^{PL}v} - v$ ; (PBL) exists if qualities satisfy  $\tilde{k}_B^{PBL} \in (\tilde{k}_L - \delta, \sqrt{(\tilde{k}_L^{PBL})^2 + 2\tilde{k}_L^{PBL}v + v^2 + \delta^2\lambda} - v]$ ; and finally, (PB) exists if qualities satisfy  $\tilde{k}_B^{PB} \in [0, \tilde{k}_L^{PB} - \delta]$ . In the following, we determine the relevant areas.

Candidate (PL) exists as long as  $\delta \in [0, \frac{v(1-\alpha)}{1+\alpha}]$ . Candidate (PBL) exists under the following conditions:

- $\lambda \in [0, 3/7]$  and  $\delta > \frac{(1-\alpha)(1-2\lambda)v}{1+\alpha+\lambda(1-\alpha)}$ ,
- $\lambda \in [3/7, 1/2]$  and:

$$\begin{aligned}\delta &\in \left[ \frac{(1-\alpha)(1-2\lambda)v}{1+\alpha+\lambda(1-\alpha)}, \frac{2v}{4-\lambda(1-\alpha)((5-\alpha)-\lambda(1-\alpha))} \left( 1 - \alpha - \frac{2-\lambda(1-\alpha)}{1+\alpha+(1-\alpha)\lambda} \sqrt{\frac{(1-\alpha)(-3-\alpha+7\lambda+\alpha\lambda)}{\lambda}} \right) \right] \\ \text{or } \delta &> \frac{2v}{4-\lambda(1-\alpha)((5-\alpha)-\lambda(1-\alpha))} \left( 1 - \alpha + \frac{2-\lambda(1-\alpha)}{1+\alpha+(1-\alpha)\lambda} \sqrt{\frac{(1-\alpha)(-3-\alpha+7\lambda+\alpha\lambda)}{\lambda}} \right).\end{aligned}$$

- $\lambda \in [1/2, 1]$  and:

$$\delta > \frac{2v}{4-\lambda(1-\alpha)((5-\alpha)-\lambda(1-\alpha))} \left( 1 - \alpha + \frac{2-\lambda(1-\alpha)}{1+\alpha+(1-\alpha)\lambda} \sqrt{\frac{(1-\alpha)(-3-\alpha+7\lambda+\alpha\lambda)}{\lambda}} \right).$$

Finally, candidate (PB) exists as long as:

$$\delta < \frac{v}{(1+\alpha)^2 - (1-\alpha)^2 \lambda} \left( 2(1-\alpha) + (1+\alpha) \sqrt{\frac{(1-\alpha)(3+\alpha+\lambda(1-\alpha))}{\lambda}} \right).$$

The three resulting areas may overlap. In particular, we find that the area in which the candidate (PL) is possible is entirely included in the area in which the candidate (PB) is possible. In addition, there exists an area within which both (PBL) and (PB) are possible. There is no area, however, in which both (PBL) and (PL) are possible. To choose its strategy, the producer thus must compare the profit it would earn in all three cases. First, we find that  $\tilde{\Pi}_P^{PB} > \tilde{\Pi}_P^{PL}$  for all values of  $\alpha$ ,  $\lambda$  and  $\delta$ ; thus it is never optimal for the producer to sell only the private label. We now compare  $\tilde{\Pi}_P^{PBL}$  and  $\tilde{\Pi}_P^{PB}$ , and find the following condition:

$$\tilde{\Pi}_P^{PBL} > \tilde{\Pi}_P^{PB} \Leftrightarrow \delta > \tilde{\delta} = \frac{v}{1+\alpha-\lambda(1-\alpha)} \left( 1 - \alpha + \sqrt{\frac{2(1-\alpha^2)(2-\lambda(1-\alpha))}{1+\alpha+\lambda(1-\alpha)}} \right).$$

This threshold is always lower than the threshold under which (PB) may occur and larger than the threshold above which (PBL) may occur.

We summarize the results as follows. Two equilibria may arise:

- Whenever  $\delta < \tilde{\delta}$ , the equilibrium is (PB).
- If, on the contrary,  $\delta \geq \tilde{\delta}$ , the equilibrium is (PBL).

Comparing Channel P to Channel  $\tilde{P}$ , we find qualitatively the same results as in the case without the duplication of costs. The primary difference between the two frameworks is that, in Channel  $\tilde{P}$ , the equilibrium in which the retailer ends up selling only the national brand occurs much more often than in Channel P.

**Choice of the private label production channel.** As previously, R chooses Channel  $\tilde{P}$  if and only if the joint profit of the industry is larger in this structure than in Channel f. We make the following six comparisons:

1. When the equilibrium in Channel f is (BL), that is  $\delta > \delta_2$ , and the equilibrium in Channel  $\tilde{P}$  is (PBL), that is  $\delta > \tilde{\delta}$ . In both cases, the quality of the national brand

is equal to  $\frac{\lambda(1-\alpha)(v+\delta)}{2-\lambda(1-\alpha)}$ , and only the quality of the private label changes. Furthermore, in both cases the industry revenue is  $\pi(k_L, k_B)$ . Comparing the profits in Channel  $\tilde{P}$  and Channel f thus amounts to comparing the value of  $\frac{1-\lambda}{4}(v+k_L)^2 - \frac{k_L^2}{2}$  for the two different values of  $k_L$ :  $k_L^{BL} = \frac{v(1-\alpha\lambda)}{1+\alpha\lambda}$  and  $\tilde{k}_L^{PBL} = \frac{v(1-\alpha)(1-\lambda)}{1+\alpha+\lambda(1-\alpha)}$ . We obtain the following result:

$$\Pi^{BL} < \tilde{\Pi}^{PBL} \Leftrightarrow \alpha < \frac{\sqrt{(1-\lambda)^2 + 4\lambda^2(3-\lambda-\lambda^2)} - (1-\lambda-\lambda^2)}{4\lambda(1-\lambda)}.$$

2. When the equilibrium in Channel f is (BL), and the equilibrium in Channel  $\tilde{P}$  is (PB), that is  $\delta < \tilde{\delta}$ . Here, all quality decisions change in Channel  $\tilde{P}$  compared with Channel f. We thus compare the two joint profits:

$$\Pi^{BL} = \frac{1}{4} \left( \frac{v^2(2(1-\lambda) - (1-\alpha\lambda)^2)}{(1+\alpha\lambda)^2} + \frac{\lambda(\delta+v)^2(2 - (1-\alpha)^2\lambda)}{2 - (1-\alpha)\lambda^2} \right), \quad \tilde{\Pi}^{PB} = \frac{(1+2\alpha-\alpha^2)(\delta\lambda+v)^2}{2(1+\alpha)^2}.$$

We find that  $\Pi^{BL} < \tilde{\Pi}^{PB}$  if and only if:

$$\delta < -\frac{(1-\alpha)(-2-6\alpha+\lambda+\alpha\lambda-3\alpha^2\lambda+\alpha^3\lambda)v}{2(1+\alpha)^2-(1-\alpha)(1+2\alpha-\alpha^2)\lambda(3+\alpha-\lambda+\alpha\lambda)} + \frac{(1+\alpha)(2-\lambda(1-\alpha))v}{(1+\alpha\lambda)(2(1+\alpha)^2-(1-\alpha)(1+2\alpha-\alpha^2)\lambda(3+\alpha-\lambda+\alpha\lambda))} \sqrt{\frac{2D}{(-1+\lambda)\lambda}},$$

where:

$$D = (1 - 8\alpha^2 - 2\alpha^3 + 5\alpha^4 - 2\alpha^5) \lambda^2 + \alpha^2 (2 - (3(1 - 2\alpha) + \alpha^2) \lambda) - (1 + 2\alpha - \alpha^2) \lambda (2(1 - \lambda) + \lambda^2 (1 - 2\alpha - (1 - \alpha)\alpha^2(2 - \lambda + \alpha\lambda))).$$

3. When the equilibrium in Channel f is (L) and the equilibrium in Channel  $\tilde{P}$  is (PBL).

The retailer must compare  $\Pi^L = \frac{v^2}{2}$  to  $\tilde{\Pi}^{PBL}$ . We find the following condition:

$$\Pi^L > \tilde{\Pi}^{PBL} \Leftrightarrow \delta < v \left( \frac{2-\lambda(1-\alpha)}{1+\alpha+\lambda(1-\alpha)} \sqrt{\frac{2(\alpha(1-\lambda)(\alpha+\lambda(2-\alpha))+\lambda(1+\lambda))}{\lambda(2-\lambda(1-\alpha(2-\alpha)))}} - 1 \right).$$

4. When the equilibrium in Channel f is (L) and the equilibrium in Channel  $\tilde{P}$  is (PB).

The retailer must compare  $\Pi^L = \frac{v^2}{2}$  to  $\tilde{\Pi}^{PB} = \frac{(1+2\alpha-\alpha^2)(\delta\lambda+v)^2}{2(1+\alpha)^2}$ . We find the following condition:

$$\Pi^L < \tilde{\Pi}^{PB} \Leftrightarrow \delta > \frac{v}{\lambda} \left( \frac{1+\alpha}{\sqrt{1+2\alpha-\alpha^2}} - 1 \right).$$

5. When the equilibrium in both channels is such that only the national brand is sold. As far as the joint profit of the industry is concerned, this case is actually equivalent to the case in which the retailer chooses Channel  $\tilde{P}$  and ends up selling only the national brand on the final market. Indeed, in both cases, the quality of the national brand is  $k_B^B = \tilde{k}_B^{PB} = \frac{(1-\alpha)(v+\delta\lambda)}{1+\alpha}$ . However, in contrast to Channel f, where the retailer invests  $k_L^B$  for the private label and thus pays a cost  $\frac{(k_L^B)^2}{2}$ , in Channel  $\tilde{P}$ , P invests 0 for the private label and the investment cost is thus saved. Therefore, the total profit of the industry is always larger in Channel  $\tilde{P}$  than in Channel f.
6. Finally, there is no case in which the equilibrium in Channel f is (B) and the equilibrium in Channel P is (PBL). Indeed, the former does not exist if  $\delta > \delta_4$ , while the latter does not exist if  $\delta < \tilde{\delta}$ . Moreover,  $\tilde{\delta} > \delta_4$  for all relevant values of  $\alpha$ ,  $\lambda$  and  $v$ ; therefore, the two equilibria cannot coexist.

Finally, we can summarize the results as follows. Whenever  $\delta > \max\{\delta_2, \tilde{\delta}\}$  and  $\alpha < \bar{\alpha}$  or  $\delta \in [\max\{\tilde{\delta}, \tilde{\delta}^*\}, \delta_2]$ , R chooses Channel  $\tilde{P}$  and the two goods are sold on the final market. Whenever  $\delta \in [\delta_2, \min\{\tilde{\delta}, \tilde{\delta}^{**}\}]$  or  $\delta \in [\max\{\delta_2, \tilde{\delta}^{***}\}, \tilde{\delta}]$ , R chooses Channel  $\tilde{P}$  and only the national brand is sold on the final market.

The expressions of the thresholds are given by:

$$\begin{aligned}\bar{\alpha} &= \frac{\sqrt{(1-\lambda)^2 + 4\lambda^2(3-\lambda-\lambda^2)} - (1-\lambda-\lambda^2)}{4\lambda(1-\lambda)}, \\ \tilde{\delta}^* &= v \left( \frac{2-\lambda(1-\alpha)}{1+\alpha+\lambda(1-\alpha)} \sqrt{\frac{2(\alpha(1-\lambda)(\alpha+\lambda(2-\alpha))+\lambda(1+\lambda))}{\lambda(2-\lambda(1-\alpha(2-\alpha)))}} - 1 \right), \\ \tilde{\delta}^{**} &= \frac{(1-\alpha)(2+6\alpha-\lambda-\alpha\lambda+3\alpha^2\lambda-\alpha^3\lambda)v}{2(1+\alpha)^2-(1-\alpha)(1+2\alpha-\alpha^2)\lambda(3+\alpha-\lambda+\alpha\lambda)} + \frac{(1+\alpha)(2-\lambda(1-\alpha))v}{(1+\alpha\lambda)(2(1+\alpha)^2-(1-\alpha)(1+2\alpha-\alpha^2)\lambda(3+\alpha-\lambda+\alpha\lambda))} \sqrt{\frac{2B}{(-1+\lambda)\lambda}}, \\ \tilde{\delta}^{***} &= \frac{v}{\lambda} \left( \frac{1+\alpha}{\sqrt{1+2\alpha-\alpha^2}} - 1 \right),\end{aligned}$$

with:

$$\begin{aligned}B &= (1 - 8\alpha^2 - 2\alpha^3 + 5\alpha^4 - 2\alpha^5) \lambda^2 + \alpha^2 (2 - (3(1 - 2\alpha) + \alpha^2) \lambda) \\ &\quad - (1 + 2\alpha - \alpha^2) \lambda (2(1 - \lambda) + \lambda^2 (1 - 2\alpha - (1 - \alpha)\alpha^2(2 - \lambda + \alpha\lambda))).\end{aligned}$$

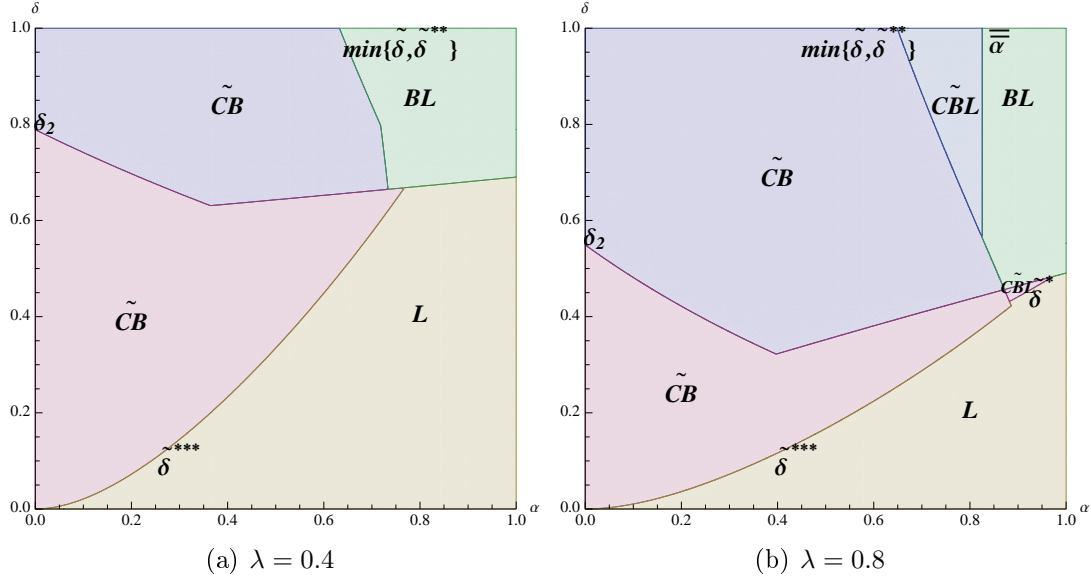


Figure 4: Equilibrium of the game with cost duplication.

The figure below represents the aforementioned thresholds together with the frontier between the equilibria (L) and (BL) in Channel f.

## A.7 Proof of the managerial implications

**Proof of Result 1** We show that compared with any equilibrium of the subgame Channel f, Channel P yields a larger profit for the producer, thus that  $\Pi_P^f < \Pi_P^P$ .

Consider first the case in which the equilibrium in Channel f is (B). Then, noting that  $k_L^B < k_B^B = k^P$ , the profit of P in the two channels is given by:

$$\begin{aligned}\Pi_P^B &= (1 - \alpha)(\pi(k_L^B, k^P) - \bar{\pi}(k_L^B)) - C(k^P), \\ \Pi_P^P &= (1 - \alpha)(\pi(k^P, k^P) - \bar{\pi}(0)) - C(k^P),\end{aligned}$$

from which we deduce:

$$\Pi_P^P - \Pi_P^B = (1 - \alpha) [(\pi(k^P, k^P) - \pi(k_L^B, k^P)) + (\bar{\pi}(k_L^B) - \bar{\pi}(0))].$$

From lemma 1, the joint revenue  $\pi(k^P, k^P)$  is increasing in both qualities and therefore  $\pi(k^P, k^P) > \pi(k_L^f, k^P)$ . Similarly, the outside option of the retailer is increasing in the

quality  $k_L$ , and thus  $\bar{\pi}(k_L^f) - \bar{\pi}(0) > 0$ . As a consequence, it is always true in that case that  $\Pi_P^P > \Pi_P^B$ .

Now consider the case for which the equilibrium in Channel f is (BL). In that case, it is immediate that  $\Pi_P^P > \Pi_P^{BL}$  for any values of  $\alpha$ ,  $\lambda$  and  $\delta$ . In the case for which the equilibrium in Channel f is (L), the producer's profit is zero, and again we show that  $\Pi_P^P > 0$  for any values of  $\alpha$ ,  $\lambda$  and  $\delta$ .

**Proof of Result 2** We compare equilibrium (B) in channel f and the only equilibrium (BL) in channel P, and show that whenever channel P is chosen over channel f in that case, the retailer would lose profit without the transfer.

We can write the profits in both cases as follows. Noting first that in this particular equilibrium of Channel f, we have  $k_B^f = k^P$ , the profit of R in Channel f is given by:

$$\Pi_R^f = \bar{\pi}(k_L^f) + \alpha(\pi(k_L^f, k^P) - \bar{\pi}(k_L^f)) - C(k_L^f).$$

In channel P, it is given by:

$$\Pi_R^P = \bar{\pi}(0) + \alpha(\pi(k^P, k^P) - \bar{\pi}(0)).$$

The retailer is thus better off in Channel P if:

$$\Pi_R^P - \Pi_R^f = (1 - \alpha)(\bar{\pi}(0) - \bar{\pi}(k_L^f)) + C(k_L^f) + \alpha(\pi(k^P, k^P) - \pi(k_L^f, k^P)) > 0.$$

One condition for (B) to be the Pareto-optimal equilibrium in Channel f is that the profit of the retailer must be higher in this situation than in any other situation in Channel f; in particular the profit must be higher than when the retailer sets  $k_L = k^P$  and the producer

sets  $k_B^f$ . This condition can be written as follows:

$$\begin{aligned}
& \bar{\pi}(k_L^B) + \alpha(\pi(k_L^B, k^P) - \bar{\pi}(k_L^B)) - C(k_L^B) > \bar{\pi}(k^P) + \alpha(\pi(k^P, k^P) - \bar{\pi}(k^P)) - C(k^P) \\
& \Leftrightarrow \bar{\pi}(0) + \alpha(\pi(k^P, k^P) - \bar{\pi}(0)) < \bar{\pi}(k_L^B) + \alpha(\pi(k_L^B, k^P) - \bar{\pi}(k_L^B)) - C(k_L^B) \\
& \quad + (1 - \alpha)(\bar{\pi}(0) - \bar{\pi}(k^P)) + C(k^P), \\
& \Leftrightarrow \Pi_R^P < \Pi_R^B + (1 - \alpha)(\bar{\pi}(0) - \bar{\pi}(k^P)) + C(k^P).
\end{aligned}$$

We show that for any values of  $\alpha$ ,  $\lambda$  and  $\delta$ ,  $(1 - \alpha)(\bar{\pi}(0) - \bar{\pi}(k^P)) + C(k^P) < 0$ . Therefore  $\Pi_R^P < \Pi_R^B$  and if (B) is an equilibrium in Channel f, then the retailer is better off in this equilibrium than in Channel P.

**Proof of Result 3** We now compare equilibria (BL) and (L) to the equilibrium in Channel P. We have  $\Pi_R^{BL} < \Pi_R^P$  if:

$$\delta < \min \left\{ v, \frac{v(1+\alpha)(2-\lambda+\alpha\lambda)}{\lambda(1-\alpha)(8+4\alpha-3\lambda+2\alpha\lambda+\alpha^2\lambda)} \sqrt{\frac{4(2+\alpha)^2-8-3\lambda-29\alpha\lambda+3\alpha^2\lambda+5\alpha^3\lambda+9\alpha\lambda^2-13\alpha^2\lambda^2-(1-\alpha)\alpha^3\lambda^2}{\alpha(1-\lambda)(1+\alpha\lambda)}} \right\}.$$

If the equilibrium in Channel f is (L), we have  $\Pi_R^L < \Pi_R^P$  if:

$$\delta < \min \left\{ v, \frac{v}{(1+\alpha)^2+(1-\alpha)(3+\alpha)\lambda} \left( (1 + \alpha) \sqrt{\frac{1-\alpha+3\alpha^2+\alpha^3+3\lambda}{\alpha\lambda}} + 5 - 3\alpha - \alpha^2 - 4 \right) \right\}.$$