<u>University of Caen</u>

University of Rennes 1



<u>Centre de Recherche en Économie et Management</u> *Center for Research in Economics and Management*



Appraising the breakdown of unequal individuals in large French cities

Pascaline VINCENT University of Rennes 1 - CREM, (UMR 6211 CNRS)

Frédéric CHANTREUIL University of Caen Basse-Normandie - CREM, (UMR 6211 CNRS)

> Benoît TARROUX University of Rennes 1 - CREM, (UMR 6211 CNRS)

April 2012 - WP 2012-20





Appraising the breakdown of unequal individuals in large French cities

Pascaline Vincent, Frédéric Chantreuil [†]and Benoît Tarroux^{‡§}

April 5, 2012

Abstract

In this paper, we examine the general properties of the Neighbourhood Sorting Index (NSI) introduced by Jargowsky (1996) and their intuitive interpretation, illustrating the inverse relation between neighbourhood's homogeneity and segregation. The use of the NSI is illustrated measuring and comparing the segregation in the 30 largest French urban areas from 2000 to 2008.

Keywords: Measure of Segregation, Residential segregation, Income, City, Neighbourhood Sorting Index

JEL Codes: D31, D63, O18

I Introduction

Sociologists as well as economists have focused more attention in recent years on important effects of segregation, arguing that this residential space crystallizes interactions that influence individual preferences, skills, children's attitude or the

^{*}CREM, University of Rennes, pascaline.vincent@univ-rennes1.fr

 $^{^{\}dagger}\mathrm{CREM},$ University of Caen, frederic.chantreuil@unicaen.fr

[‡]CREM, IDEP, University of Rennes, benoit.tarroux@univ-rennes1.fr

[§]The authors thank Stephan Bazen, Patrick Moyes, Fabien Moizeau and participants of the Journées Louis-André Gérard-Varet # 11 (Marseille, France) and the seminars of the University of Nantes and the University of La Réunion for their comments and suggestions. We are very grateful for comments from Thomas Senné who is at the origin of this paper. The financial support of the research project *The Multiple Dimensions of Inequality* (Contract No. ANR 2010 BLANC 1808) of the French National Agency for Research is gratefully acknowledged. The usual caveat applies

choice of school (see, for instance, Cutler and Glaeser (1997), Cutler et al. (2008), Echenique et al. (2006) or Goux and Maurin (2007)).

The literature dealing with measures of segregation developed many indicators in order to undertake analyses of *categorical segregation*, that is to say, the distribution of people across categories (see for instance Duncan and Duncan (1955), Massey and Denton (1998), Reardon and O'Sullivan (2004), Hutchens (1991, 2001, 2004) or Chakravarty and Silber (2007)). The occupational segregation of men and women and the residential segregation of white and black population in cities are the most popular examples of categorial segregation.

Surprisingly, residential segregation by income have not been extensively studied by the literature (notable exceptions are Harsman and Quigley (1995), Jargowsky (1996), Watson (2009), Ioannides and Seslen (2002), Hardman and Ioannides (2004) or Davidoff (2005)). By residential segregation by income, we mean the way by which individuals who can be described by their income are broken down among *local areas* of a city. Jargowsky (1996) proposed a measure of economic segregation, defined as *pure*. This measure, which Jargowsky (1996) referred to, is the Neighbourhood Sorting Index (NSI), simply defined as the square root of the ratio of the variance of neighbourhoods' mean incomes over the overall variance.

The aim of this paper is to examine theoretically and empirically the Neighbourhood Sorting Index, introduced by Jargowsky (1996), which allows us to compare cities on the basis of the breakdown of individuals who differ in terms of income among a given set of spatial areas.

We propose an approach of measuring segregation by income based on *transformations* of the breakdown of a population among local areas. These transformations capture two dimensions of the segregative phenomenon: (1) the *mean* distance (or inequality) among the spatial areas and (2) the *mean* homogeneity (or inequality) within the local areas. The intuitions captured by the transformations introduced in this paper are the following: segregation is (1) increasing with the mean distance and (2) increasing with the mean homogeneity.

We first study the sensitiveness of the NSI with respect to movements of individual(s). In particular, we argue that a segregation measure should decrease if a rich individual moves from an area - where she is richer than the average individual - to a poorer one *or* if a poor individual moves from an area - where she is poorer than the average individual - to a richer one. Moreover, we consider the impact of a switch of two individuals, which requires that a segregation measure decreases if a rich individual living in a rich area and a poor individual living in a poor area exchange their location. The second type of transformations considered leaves the breakdown of individuals unchanged but affects the income distribution among individuals. In other words, we study the sensitiveness of a segregation measure relatively to an income transfer between two individuals located in the *same* local area. Any income transfer which is *progressive* in the usual sense of Pigou-Dalton contributes to *increase* segregation. The rationale of this requirement lies to the fact that such a transfer increases the homogeneity of the area and hence lowers its social mixing. On the contrary, an income transfer from a poor to a rich individual decreases segregation because it reduces the homogeneity of the local area.

The use of the NSI is then illustrated measuring and comparing the segregation in the 30 largest French urban areas from 2000 to 2008. We perform this analysis using households' income and their distribution within the residential space from several scales: *IRIS* and *Grand Quartier*. The main result is that residential segregation by income has decreased over the period considered. Furthermore, we show that the economic segregation in French cities is scale dependent and is not related with income inequality as measured by Gini coefficient.

The remainder of the paper is structured as follows. The next section is devoted to the introduction of some notations and basic definitions. The following section explores the properties satisfied by the NSI with reference to requirements an appropriate measure of economic segregation should suit. The use of the NSI as part of the measurement of the segregation of large French cities is undertaken in section 4. The final part concludes.

II Notation and Preliminary Definitions

Our aim is to compare two or more cities on the basis of the breakdown of unequal individuals among various *areas*. We consider therefore *cities* where each individual is endowed with a certain amount of income and is located in a certain area.

We consider a city C comprising n individuals, indexed by i, taken from some finite set $N = \{1, ..., i, ..., n\}$. Let's assume also that this city is made up of m geographical areas (or, more generally, of m subgroups), indexed by j and populated by n^{j} individuals, and define M the finite set of areas of the city C, $M = \{1, ..., j, ..., m\}$.

This city can be defined by two elements: (1) a distribution of income among the population and (2) a partition of the population between the m geographical areas of \mathcal{C} . Assuming that incomes are drawn from an interval D of \mathbb{R} , the overall income distribution of the population can be described by a $n \times 1$ matrix

$$Y = (y_1, ..., y_i, ..., y_n)$$

where y_i is the income of the individual *i*. The breakdown of individuals into a set of areas is described by a partition P of the population within a set of *m* mutually exclusive and exhaustive subsets (areas). We use \mathcal{P} to represent the set of all possible partitions.

$$P = \left\{N^1, ..., N^m\right\} \text{ such that } N = \bigcup_{j=1}^m N^j \text{ , } N^j \neq \emptyset \text{ and } N^j \cap N^{j'} = \emptyset \ \forall j, j' \in M$$

where N^j is the set of individuals living in area j. The income distribution within area $j \in M$ is given by the vector $Y^j = (y_1^j, ..., y_i^j, ..., y_{n^j}^j)$ where y_i^j is the income of individual i living in area j. A city \mathcal{C} can be thus depicted by a pair $(P, Y) \in \mathcal{P} \times D^n$ Let denote μ the mean income of the city and μ^j the mean income of the area j.

An economic segregation index is a continuous function $S : \mathcal{P} \times D^n \to \mathbb{R}$. Our aim is to rank two cities thanks of a segregation index that provides a complete and transitive binary relation noted \succ_S . Given two cities \mathcal{C} and $\widetilde{\mathcal{C}}$, we will say that \mathcal{C} is *less segregated* than $\widetilde{\mathcal{C}}$, which we write $\mathcal{C} \succ_S \widetilde{\mathcal{C}}$, if $S(\mathcal{C}) < S(\widetilde{\mathcal{C}})$.

Jargowsky (1996), performing a methodological critique of the measure of segregation used by Massey and Eggers (1990), develop a measure of segregation based on the correlation ratio. Applying the correlation ratio to income, Jargowsky define the Neighborhood Sorting Index as:

$$S(\mathcal{C}) = \frac{\sqrt{V_b}}{\sqrt{V(Y)}} = \frac{\sigma_b/\mu}{\sigma/\mu} = \frac{CV(\mu^1, ..., \mu^m)}{CV(Y)}$$

where V(Y) and σ are, respectively, the variance and the standard-deviation of the income distribution Y; $V_b = V(\mu^1, ..., \mu^m)$ and $\sigma_b = \sqrt{V_b}$.

III Desirable properties and the NSI

This section proposes to explore the *desirable* properties which are respected by the NSI. In other words, we consider basic properties, that should be satisfied by a *good* measure of segregation and by properties related to the potential demographic trends of a city. We then further investigate the properties satisfied by the NSI. The proof of all propositions are in the appendix.

III.1 Basic properties

We consider first the range of values for NSI and how these values can be interpreted. A first property satisfied by the NSI is that the range of its value is bounded between 0 and 1. The following examples illustrate the case of complete integration (NSI being equal to 0) and complete segregation (NSI being equal to 1).

Consider the case of a city with 6 individuals allocated in two areas and the following configurations:

 $\begin{array}{rcl} \mathcal{C}_1: & Y &=& (10, 15, 20; 10, 15, 20) \\ \mathcal{C}_2: & Y &=& (15, 15, 15; 10, 15, 20) \\ \mathcal{C}_3: & Y &=& (10, 10, 10; 18, 18, 18) \\ \mathcal{C}_4: & Y &=& (8, 10, 12; 16, 18, 20) \end{array}$

In C_1 , the two areas' income distributions are similar and, thus, the mean income is the same in the two areas. One might expect segregation to be inexistent and in that case, the segregation measure is equal to 0. Consider now C_2 . The two areas' income distributions are now different but areas' mean incomes are equal. The NSI states that there is no segregation since there is no variability in area per capita income relative to the city mean. Hence the similarity of areas' income distributions is not necessary for complete integration.

The city C_3 is characterized by homogeneous areas and inequality among areas. In such a case, one might expect the segregation to be maximal. The city C_4 is completely stratified in the sense that the richest individual of the poorest area is poorer than the poorest individual of the richest area. Segregation is thus complete if total variability in individual income is only explained by variability between areas.

This example illustrates the complete segregation and complete integration properties satisfied by the NSI.

Proposition 1 Scale interpretability.

Complete segregation.

Consider a city C such that $y_i^j = \mu^j$ for all $j \in M$ and $i \in N^j$ and $\mu^p \neq \mu^q$ for all $p, q \in M$ and $p \neq q$. Then NSI(C) = 1.

Complete integration.

Consider a city C such that $\mu^p = \mu^q$ for all $p, q \in M$ and $p \neq q$. Then NSI(C) = 0.

The next proposition reveals the symmetry properties satisfied by the NSI. Symmetry within area demands that segregation is not affected if people within a given area trade income. For instance, (a, b, c, d; e, f, g, h) and (d, c, a, b; e, f, g, h) exhibit the same level of income segregation. The symmetry between areas states that segregation has to be invariant with respect to the permutation of areas. Segregation is for instance equivalent between the two following cities: (a, b; c, d; e, f, g, h; w, z) and (e, f, g, h; a, b; c, d; w, z).

Proposition 2 Symmetry.

Symmetry within area.

Let C = (P, Y) and $\tilde{C} = (P, \tilde{Y})$ two cities such that $\tilde{Y}^p = D^p Y^p$ $(p \in M)$ with D^p an individual permutation matrix and $\tilde{Y}^q = Y^q$ for all $q \neq p$. Then, $NSI(\mathcal{C}) = NSI(\tilde{\mathcal{C}}).$

Symmetry between areas.

Let C = (P, Y) and $\widetilde{C} = (\widetilde{P}, \widetilde{Y})$ two cities such that $\widetilde{Y} = DY$ with D an areas' permutation matrix. Then, $NSI(C) = NSI(\widetilde{C})$.

The next properties, which are related to the principle of population, allow to make comparisons between cities with different sizes. These properties simply state that the measure of the segregation should be invariant to the replication of individuals within each area and to the replication of every areas.

Proposition 3 Principles of population.

Within-area replication invariance.

Let C = (P, Y) and $\widetilde{C} = (\widetilde{P}, \widetilde{Y})$ two cities such that, for all $p \in M$ and for any $\alpha \in \mathbb{N}_+$, $\widetilde{Y}^p = (Y^p, Y^p, \dots, Y^p) = (u_1^p, u_1^p, \dots, u_p^p, \dots, u_p^p, \dots, u_p^p)$

$$\stackrel{p}{\underbrace{(Y^p,Y^p,...,Y^p)}_{\alpha+1 \ times}} = \underbrace{(y^p_1,y^p_1,...,y^p_1}_{\alpha+1 \ times},...,\underbrace{y^p_{n_p},y^p_{n_p},...,y^p_{n_p}}_{\alpha+1 \ times})$$

Then, $NSI(\mathcal{C}) = NSI(\widetilde{\mathcal{C}}).$

Area replication invariance.

Let $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$ two cities such that, each neighbourhood is replicated ℓ times as

$$\widetilde{Y} = (\underbrace{Y^1; ...; Y^1}_{\ell+1 \ times}; ...; \underbrace{Y^m; ...; Y^m}_{\ell+1 \ times})$$

Then, $NSI(\mathcal{C}) = NSI(\widetilde{\mathcal{C}}).$

The literature devoted to the measurement of income inequality distinguishes *relative* and *absolute* views of inequality (Kolm (1976a) and Kolm (1976b)). An interesting characteristic of the NSI is that it respects both views.

Proposition 4 Absolute and relative invariance.

Relative invariance.

Let $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$ two cities such that $\widetilde{y}_i^j = \lambda y_i^j$ with $\lambda \in \mathbb{N}_+$, for all $j \in M$ and $i \in N^j$. Then, $NSI(\mathcal{C}) = NSI(\widetilde{\mathcal{C}})$.

Absolute invariance.

Let $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$ two cities such that $\widetilde{y}_i^j = y_i^j + \delta$ with $\delta \in \mathbb{N}$, for all $j \in M$ and $i \in N^j$. Then, $NSI(\mathcal{C}) = NSI(\widetilde{\mathcal{C}})$.

Eventually, the NSI allows us to compare cities with different number of individuals and/or different number of areas and/or different overall mean income. Furthermore, the NSI does not depend on whether areas are labelled or whether individuals within areas are named

III.2 Key properties

We further investigate the properties satisfied by the NSI, with respect to potential impacts of movements of individuals from any geographical area comprised in an urban area to another one, or a transformation of its income distribution. The sensitivity of the NSI to the spatial partition of a given urban area is first investigated.

III.2.1 Sensitivity to the spatial partition

With the next property, we investigate the variability of the NSI with respect to the definition of area unit. This property considers aggregation of areas, which consists in merging two areas into a unique area. If merged areas have the same mean income, there is no change in segregation according to the NSI. However, if one merges two areas with different mean incomes, the aggregation increases income heterogeneity within areas and decreases the variability in mean incomes.

Definition 1 Areas aggregation.

Consider C = (P, Y) and $\widetilde{C} = (\widetilde{P}, Y)$ two cities. \widetilde{C} is obtained from C by means of an aggregation of areas p and q if for $P = \{N^1, ..., N^p, ..., N^q, ..., N^m\}$ and $\widetilde{P} = \{\widetilde{N}^1, ..., \widetilde{N}^{m-1}\}$: (a) $N^j = \widetilde{N}^j \ \forall j < q, j \neq p$ and $N^j = \widetilde{N}^{j-1} \ \forall j > q$ (b) $N^p \cup N^q = \widetilde{N}^p$

Proposition 5 Sensitivity to areas aggregation.

Consider a city C and a city \tilde{C} obtained from C by means of areas aggregation, then:

- (a) $NSI(\mathcal{C}) = NSI(\widetilde{\mathcal{C}})$ if $\mu^p = \mu^q$
- (b) $NSI(\mathcal{C}) > NSI(\widetilde{\mathcal{C}})$ if $\mu^p \neq \mu^q$

In other words, this property states that the NSI does not decrease when the number of areas increases; because the inequality between mean incomes does not decrease. As Shorrocks and Wan (2005) have showed, the expected value of between-component of any inequality measure increases with the number of areas.

One might be interested in considering the case of area division rather than areas aggregation. Obviously, if the division is such that the two new areas have the same mean income, segregation remains the same; because variability in mean incomes is the same before and after the division. But segregation becomes worse if the division of the area allows a sorting of individuals with respect to income.

Even though the NSI fails to account for spatial patterning of areas¹, these two properties might help to give a picture of spatial segregation. Indeed, it is sensitive to the definition of the boundaries of areas and assumes that each individual lives near all individuals of his area and far from individuals located in another area (even across the street from one another). Consider two sets of areas' boundaries, P and \tilde{P} , such that \tilde{P} is obtained from P by a finite set of *adjacent* areas aggregation. In other words, we consider two partitions of individuals among areas, $P = \{N^1, ..., N^m\}$ and $\tilde{P} = \{\tilde{N}^1, ..., \tilde{N}^\ell\}$, such that \tilde{N}^j is the union of two or more *adjacent* (sub)areas N^k (with $k \in \{1, ..., m\}$) for all $j \in \{1, ..., \ell\}$. If the segregation index takes the same value whatever the partition P and \tilde{P} , it means that areas are adjacent to similar areas (in term of mean income).

On the contrary, the larger difference between the two values taken by the segregation index is, the more areas are heterogeneous. This reasoning holds comparing two cities. Consider two cities with the same value of NSI for the finest areas division. If the value of NSI differs according to a different partition, then a city has more homogeneous grouping of sub-areas and is thus more spatially segregated.

¹In particular, the *checkerboard problem* and the *modifiable areal unit problem* are ignored by such indices. See for example White (1983), Reardon and O'Sullivan (2004) and Kim and Jargowsky (2005).

III.2.2 Economic segregation under movements

The simplest transformation that can be introduced is an unilateral movement of one individual from one area to another one. Two alternative definitions of a *progressive* unilateral movement are considered. The first one is to require a movement of a rich individual from a rich area to a poor one, given that he is richer than the mean individual of his initial area. The second one is to consider that a poor individual located in a poor area where he is poorer than the mean area's income moves to a rich area. For both definitions, we require that any segregation measure after such transformations should decrease. Let's now introduce the formal definition of such an elementary transformation.

Definition 2 Movement of one individual.

Let C = (P, Y) and $\tilde{C} = (\tilde{P}, \tilde{Y})$ two cities and k an individual located in area p. The city \tilde{C} is obtained from the city C by means of movement of one individual if there exists an area q such that:

- (a) $\widetilde{N}^p = N^p / \{k\}$
- (b) $\widetilde{N}^q = N^q \cup \{k\}$
- (c) $Y^{\ell} = Y^{\ell}$ for all $\ell \neq p, q$

Then,

- A movement of one individual is said to be a progressive movement of a rich individual if: y^p_k > μ^p, μ^p > μ^q and μ̃^p > μ̃^q.
- A movement of one individual is said to be a progressive movement of a poor individual if: y^p_k < μ^p, μ^p < μ^q and μ̃^p < μ̃^q.

We might rewrite conditions (a) and (b) of the previous definition in terms of areas' income distributions. Indeed, these conditions are equivalent, respectively, to:

- (a') $\widetilde{Y}^p = (y_1^p, \dots, y_{k-1}^p, y_{k+1}^p, \dots, y_{n^p}^p)$
- (b') $\widetilde{Y}^q = (y_1^q, ..., y_{n^q}^q, y_k^p)$

Another transformation is the switch of two individuals, which consists in two opposite movements of a rich individual and a poor one. Consider two areas, p and q, such that $\mu^p < \mu^q$ and two individuals k and h, initially located, respectively, in p and q, who move to, respectively, area q and p.

Such an elementary transformation is likely to decrease segregation *if* k's income is lower than h's income. Intuitively, this switch may change the income mixing within each area. The replacement of a poor individual by a rich one in the poor area and the replacement of a rich individual by a poor one in the rich area improve income diversity in each of the two areas. In the same time, the gap of mean incomes between the two areas decreases. On the contrary, segregation is expected to increase *if* k is richer than h as social diversity within each area and the gap between mean incomes deepens. Indeed, as a rich individual joins a rich area and a poor individual moves to a poor one, diversity within each area decreases and distance between the two areas in terms of mean income deepens.

Definition 3 Switch of two individuals.

Let C = (P, Y) and $\widetilde{C} = (\widetilde{P}, \widetilde{Y})$ two cities, within which two individuals k and h are located in area p and q respectively. The city \widetilde{C} is obtained from the city C by means of a switch of two individuals if:

- (a) $\widetilde{N}^p = N^p_{/\{k\}} \cup \{h\}$
- (b) $\widetilde{N}^q = N^q_{\{h\}} \cup \{k\}$
 - A switch of two individuals is said to be progressive if: y^p_k < y^q_h, μ^p < μ^q and μ̃^p < μ̃^q.
 - A switch of two individuals is said to be **regressive** if: $y_k^p < y_h^q$, $\mu^p > \mu^q$ and $\tilde{\mu}^p > \tilde{\mu}^q$.

This transformation can also be interpreted as an exchange of income between two individuals who are located in two different areas and can thus be expressed in terms of areas' income distributions. The two conditions of the definition 3 might be substituted by the following conditions:

- (a') $\widetilde{y}_k^p = y_h^q$
- (b') $\widetilde{y}_h^q = y_k^p$
- (c') $\widetilde{y}_i^j = y_i^j$ for j = p, q and $i \neq k, h$
- (d') $\widetilde{y}_i^j = y_i^j$ for all $j \neq p, q$ and $i = \{1, ..., n^j\}$

The following proposition resume the sensitivity of the NSI relatively to such movements.

Proposition 6 Segregation under movements.

Consider two cities, C and \widetilde{C} .

- If \$\tilde{C}\$ is obtained from \$\mathcal{C}\$ by a finite set of a progressive movements of a rich and/or poor individual. Then, \$NSI(\$\mathcal{C}\$) > \$NSI(\$\tilde{C}\$)\$.
- If \$\tilde{C}\$ is obtained from \$\mathcal{C}\$ by a finite set of progressive (resp. regressive) switches of two individuals. Then, \$NSI(\$\mathcal{C}\$) > \$NSI(\$\tilde{C}\$) (resp. \$NSI(\$\mathcal{C}\$) < \$NSI(\$\tilde{C}\$)]\$.

The intuition behind this proposition is quite obvious. Any emigration of an individual who is richer than the mean incomes of his initial area and of his new one reduces the gap between mean incomes. Inequality between areas is then lower. We can see by the same token that any emigration of a poor individual from a poor to a rich area reduces also between-areas inequality. Finally, any switch of two individuals is analyzed in the same way. Such transformations imply a progressive mean income transfer.

III.2.3 Economic segregation under income transfers

We consider in this section the impacts of a transformation of the income distribution on the NSI. In the measurement of income inequality, an elementary transformation is the Pigou-Dalton principle of transfer which states that a transfer of income from a rich individual to a poorer one reduces inequality. Such a principle is not so relevant in the normative assessment of the breakdown of individuals with different income levels among areas of a city.

Let us consider the simple example of a city with 6 individuals located in 2 local areas, with the following configuration:

Assuming that a policy maker transfers income between two individuals in the richest area modifies the income distribution as follows:

While inequality decreases, the richest area becomes more homogeneous in terms of income and the overlapping of areas' income distributions disappears. Indeed, the richest individual of the poorest area is now poorer than the poorest individual of the richest area.

Weigh up now the case of a regressive transfer in the richest area that leads to

the following modified income distribution:

(10, 15, 21; 12, 30, 30)

The final situation could be evaluated as better in terms of segregation since the social mixing has been increased and the overlapping of the distribution is higher.

We thus first study the case of an income transfer between two individuals located in the *same* area; namely a *within-area income transfer*. One says that such a transfer is *progressive* - in terms of income - if money is transferred from a rich individual to a poorer one and *regressive* if it is transferred from a poor individual to a richer one.

The question is to ascertain conditions for which a progressive (resp. regressive) transfer reduces or worsens segregation. A progressive transfer reduces discrepancy of income within a given area *and* in the overall city. But the *homogenization* of the population of this area (that is, the area's income distribution is more *squeezed*) tends to reduce the overlapping of the areas' income distributions.

Definition 4 Within-area income transfer.

Let C = (P, Y) and $\widetilde{C} = (P, \widetilde{Y})$ two cities and two individuals k and h located in area p. The city \widetilde{C} is obtained from the city C by means of within-area income transfer if, for any $\delta \in \mathbb{R}_+$:

- (a) $\widetilde{y}_{h}^{p} = y_{h}^{p} + \delta$ and $\widetilde{y}_{k}^{p} = y_{k}^{p} \delta$
- (b) $\widetilde{y}_i^j = y_i^j$ for j = p and $i \neq k, h$ and $\widetilde{y}_i^j = y_i^j$ for all $j \neq p$ and $i \in N^j$
 - A within-area income transfer of two individuals is said to be income-progressive if y^p_h < y^p_k and ỹ^p_h ≤ ỹ^p_k.
 - A within-area income transfer of two individuals is said to be income-regressive if: y^p_h > y^p_k.

The following proposition establishes the link between a within-area income transfer and the behavior of the NSI.

Proposition 7 Internal transfer.

Consider two cities, C and \widetilde{C} .

- (i) If \widetilde{C} is obtained from C by a finite set of within-area income-progressive transfers, then $NSI(C) < NSI(\widetilde{C})$.
- (ii) If \widetilde{C} is obtained from C by a finite set of within-area income-regressive transfers, then $NSI(C) > NSI(\widetilde{C})$.

We turn now to the *between-area income transfer*, which consists in transferring income between two individuals located in two distinct areas.² We argue that such a transfer can be interpreted as a combination of a finite set of switch(es) of individuals and a finite set of within-area income transfer(s). However, the task of determining this combination is not straightforward.

Let us take the simple case of a city with 6 individuals allocated in 2 spatial areas characterized by the following income distribution:

$$Y = (\underbrace{a, b, c}_{\text{1st area}}; \underbrace{b, c, d}_{\text{2d area}}).$$

Assuming that a ethical observer transfer an amount of ε units of income from the richest individual of the first area to the poorest individual of the second area, the distribution becomes:

$$Y = (a,b,c-\varepsilon;b+\varepsilon,c,d)$$
 with $\varepsilon < \frac{c-b}{2}$

Such transfer might be decomposed into two transformations:

1. a within-area income transfer in the first area:

$$Y = (a, b + \varepsilon, c - \varepsilon; b, c, d)$$

2. a switch of individuals:

$$Y = (a, b, c - \varepsilon; b + \varepsilon, c, d)$$

If we suppose that a < b < c < d, this between-area transfer fits with a transfer of income from a rich individual in a poor area to a poor one in a rich area. Such a transfer can be decomposed into a progressive transfer of income within the first area and a regressive switch of individuals. Hence any segregation measure should increase after such a transfer. At the contrary, if we assume that a > b > c > d, the opposite conclusion emerges. Furthermore, in the case where b > c and a < d(then, a + b + c < b + c + d) such an income transfer has an ambiguous effect on segregation.

Actually, this decomposition of the between-area income transfer is dependent to the initial distribution of income. Indeed, we need to find two individuals with the same income but located in two different areas and to switch them. In other

²see also Mussard (2006).

words, this requires that areas' income distributions are continuous and overlapped. However, if it is not the case (for instance, when the city is income-stratified), it is possible to introduce a phantom or fictitious individual as introduced by Gravel and Moyes (2012). Assume that the initial situation is the following one:

$$Y = (\underbrace{a, b, c}_{\text{1st area}}; \underbrace{d, e, f}_{\text{2d area}}).$$

A between-type income transfer occurs and the new situation is:

$$\widetilde{Y} = (a, b, c + \varepsilon; d, e, f - \varepsilon).$$

Such a transformation in such a configuration could be decomposed into two *basic* transformations:

1. A fictitious individual located in the first area and endowed with the same income than the donor is created:

$$Y_1 = (a, b, c + \varepsilon, f; d, e, f)$$

2. An income transfer within the first area between the fictitious individual and the receiver:

$$Y_1 = (a, b, c + \varepsilon, f - \varepsilon; d, e, f)$$

3. A switch of two individuals:

$$Y_2 = (a, d, c + \varepsilon, f; d, e, f - \varepsilon)$$

Of course, we do not appraise segregation in Y and Y_1 in the same way. However, the point is not to compare segregation level but rather to assess the change in segregation after a between-area transfer of income. We argue that comparing Y_1 and Y_2 might be similar to comparing Y and \tilde{Y} under a specific requirement. The presence of this fictitious individual is purely *instrumental* only if the ranking of the mean incomes are not changed, i.e. sgn(a + b + c - (d + e + f)) = $sgn\left(\frac{a+b+c+f}{4} - \frac{d+e+f}{3}\right)$.

In the case where the donor is richer and lives in a poorer area than the receiver, i.e. f > c and a + b + c > d + e + f, the income transfer within the first area leads to increase the segregation measure and the switch of two individuals participates also to increase the segregation measure. For the opposite case (the donor is poorer and lives in a richer area than the receiver), the conclusion is the opposite (the segregation decreases). For the two other cases, inconclusiveness emerges naturally.

IV Economic segregation in French urban areas

The residential segregation has discussed among the social scientists and in the public debate for a twenty years. By studying changes in social class and income composition of the close neighbourhoods, Maurin (2004) shows that income segregation has been stable over the period from 1991 to 2002. Préteceille (2006) studies the geographical breakdown of social classes in Paris urban area by using the *dissimilarity* index. The author shows that the most privileged social classes are more segregated than the *popular* categories. White-collar workers³ and executive employees⁴ tend to be less segregated in 1999 than in 1990. To contrary, segregation has increased between 1990 and 1999 for the blue-collar workers. Recently, some papers investigate the segregation of immigrants or ethnic segregation (see for instance Verdugo (2011), Safi (2009) or Pan Ké Shon (2010)). Our study complement these empirical investigations by studying how *income* segregation has changed in French urban areas during the 2000s.

Some segregation measures have been used in several papers in order to understand the segregation in the US cities (for instance, Jargowsky (1996), Kim and Jargowsky (2005), Yang and Jargowsky (2006) or Wheeler and La Jeunesse (2007)). Recently, using the census tract level family income data, Watson (2009) shows that income segregation has increased between 1970 and 2000 in 216 US urban areas.⁵ Moreover, inequality is found to be positively correlated with the NSI.

IV.1 Database description

We present first the *Revenus Fiscaux Localisés* database, provided by INSEE⁶, and used in this sub-section. This database provides, over a 8 year period (2001-2008) and for each area unit, the mean and the median income, the Gini coefficient, the quartiles and the deciles. The area unit considered by INSEE is the $IRIS^7$, defined as an area comprising between 1800 and 5000 inhabitants. *IRIS* are uniform in their habitat type and their borders are based on the large cuts in the urban area, such

³Employés and professions intermédiaires.

⁴Cadres and professions libérales.

⁵The author uses the *census tract* as the definition of a local area. As such an area is composed by roughly 4,000 people, her results could be compared with ours.

⁶Institut National de la Satistique et des Etudes Economiques

⁷IRIS: Ilots Regroupés pour l'Information Statistique.

as main roads, railways, rivers ... This spatial unit is close to the principle of Tract in the USA. Note that an important characteristic of this database is that INSEE is using exhaustive files providing by the *Direction Générale des Impôts* rather than a sample of inhabitants.

The definition of income adopted for our study is the taxable income, which is established from two different files of the income statement and property tax. INSEE estimates the taxable income for several geographical levels. The *taxable household* is an ordinary household formed by the combination of taxable families listed in the same dwelling. The taxable income is the amount of resources reported by taxpayers on the *income statement*, before any reduction (which is not equivalent to the concept of *disposable income*). Therefore this income variable accounts for wages, unemployment benefices, pensions, capital income and non salaries revenue.

The income is expressed in Consumption Unit, which accounts for the size and the structure of household into consideration. Indeed, differences in household structure between areas are sometimes such that the fact of using income per consumption unit offers a different picture of levels and differences in relation to reasoning per household or per person. This equivalence scale is commonly used by INSEE and Eurostat to study income and expressed as *equivalent adult*. For a given household, the first adult counts for one consumption unit, while the remaining persons count for 0.5 consumption unit if they are more than 14 years old, and children (less than 14 years old) count for 0.3 consumption unit.

We calculated the NSI for 7 years (2001, 2002, 2004, 2005, 2006, 2007 and 2008) for the 30 biggest urban areas and computed it for different geographical levels, testing different scales of spatial areas. Indeed we first computed the NSI for the *IRIS* level, then for the *Grand Quartier* level, which are less fine partitions of the urban areas. More precisely, a *Grand Quartier* is defined as a grouping of several adjoining *IRIS* inside a city. The size is variable but respects some population norms. A city with 20 000 inhabitants is generally divided into less than 3 *Grand Quartier* and few cities with less than 10 000 inhabitants are identified as a unique *Grand Quartier*.

Table I presents some general information related to our database, including, for each city, the population expressed in consumption unit, the number of *IRIS* and *Grand Quartier*, the absolute variation of the number of *IRIS* and *Grand Quartier*, and the variation of the population over the period considered (2001-2008). The last column presents the share of population accounted for, dropping the population belonging to *IRIS* for which at least one piece of information of the *Revenus Fiscaux Localisés* database is not available.

Table I near here.

IV.2 Results

In this section we present our empirical results, which exhibit, for the two scales considered (*Grand Quartier* scale and *IRIS* scale), a decrease in segregation over the considered period. Furthermore, we perform the inter-city comparison based on their ranking and their NSI values and we investigate the nature of the relationship between income segregation and income inequality.

A slightly decrease in segregation. Table II presents the evolution of the mean values of the NSI and some inequality index. Each city is weighted by the size of its population in 2007 in order to estimate the average segregation experienced by people. Whatever the geographical scale, income-based segregation increases between 2001 and 2004 and then decreases. In 2001, 32% of income inequality (as measured by coefficient of variation) can be explained by the inequality between IRIS, while this rate is 30% in 2008. At the same time, inequality as measured by Gini index and the coefficient of variation increases over the considered period.

Table II near here.

Inter-city comparisons. Table III gives the ranking of the 30 biggest urban areas when the IRIS and Grand Quartier scale are considered. Table IV gives the values taken by the NSI.⁸ According to the IRIS scale, the cities of Bayonne, Nice and Bordeaux are the less segregated city of the largest French cities throughout the considered period, while the most segregated urban areas are Le Havre and Lille. Analyzing the breakdown of people among Grand Quartier rather than IRIS does not change significantly the picture. Figures 1 to 4 show the correlation of ranks as well as values of NSI between 2001 and 2008. We notice a certain stability over time of the ranking based on the breakdown of people among Grands Quartier. The picture is less clear when distribution of the population between IRIS is studied. The rank can significantly differs between the two years (see for instance Paris or Nantes). However, as we can deduce from the figure 3, these changes might be explained rather close NSI's values, for which a small change in the NSI's value can cause important changes in the ranking. Finally, it should be noted that most of NSI values in 2008 are lower than 2001 ones as illustrated in figures 3 and 4.

Table III near here.

⁸As an ordinal measure, the NSI is particularly useful to classify and rank urban areas. But it cannot quantify the segregation and indicate whether segregation represents an amount x or y.

Table IV near here. Figure 1 near here. Figure 2 near here. Figure 3 near here. Figure 4 near here.

Figure 5 allows us to compare the ranking generated by *IRIS* scale and *Grand Quartier* scale for each year of the period. The two rankings seem to be strongly correlated even if this is less clear among the less segregated cities. This pattern is confirmed by a simple estimation of the coefficient of correlation : the unweighted coefficient is equal to 0.9097 (for all cities and for all years) and the null hypothesis that the ranks according to *IRIS* and the ranks according to *Grand Quartier* are independent is rejected (*p*-value < 0.001). Figure 6 depicts the relation between both scales in terms of NSI values which illustrates the proposition 5. With respect to this proposition, migrating from *IRIS* scale to *Grand Quartier* scale should not increase the segregation. Indeed, all the points are located under the first bisector. This figure highlights also the strong correlation between NSI calculated on the basis of *IRIS* and on the basis of *Grand Quartier*. The unweighted coefficient is equal to 0.9168 (for all cities and for all years) and is significantly different to zero (*p*-value < 0.001).

Figure 5 near here. Figure 6 near here.

Segregation by income and income inequality. Figures 7 and 8 represent the cities according to both scales. Whatever the scale considered, there is no clear relation between inequality (measured with Gini index) and segregation. The coefficient of correlation between the Gini ranking and the segregation ranking generated by the *IRIS*-based NSI is low (0.2805) but statistically different to zero (*p*-value < 0.001). The same conclusion emerges when we study how the Gini ranking and the ranking of the NSI based on the *Grand Quartier* (0.2866, *p*-value < 0.001). However, given that there are several observations per each city, it is likely that this estimate of the correlation does not make any sense. Consequently, we use a simple tobit model with random effects in order to estimate the link between the NSI's rank and the Gini's rank.⁹ It appears that the rank with respect to the Gini index does not

⁹The purpose of these simple econometric estimates is not to give a complete overall explanation of the segregation rankings and measures but to give an idea of the correlation between inequality and segregation.

influence the rank with respect to the NSI. More precisely, we find:¹⁰

NSI Rank_{*IRIS*} =
$$14.36^{***}$$
 + 0.04 . Gini Rank
 $N=203 \quad W=0.16 \quad (p=0.6895)$; $\rho=0.86$
 $L=-560.99$
NSI Rank_{*GQ*} = 14.68^{***} + 0.03 . Gini Rank
 $N=203 \quad W=0.10 \quad (p=0.7577)$; $\rho=0.95$
 $L=-477.87$

Interestingly, thanks to a similar tobit model with random effects, we find that the value of NSI is negatively correlated with the value of Gini index.¹¹ That is to say, more a city is equal, more segregated the city is. The econometric results are summarized as follows.

$$\begin{split} \mathrm{NSI}_{IRIS} &= 0.98^{***} - 1.84^{***} . \ \mathrm{Gini} \\ & N=203 \quad W=71.73 \quad (p=0.000) \quad ; \quad \rho=0.94 \\ & L=476.14 \\ \mathrm{NSI}_{GQ} &= 0.86^{***} - 1.64^{***} . \ \mathrm{Gini} \\ & N=203 \quad W=81.81 \quad (p=0.000) \quad ; \quad \rho=0.96 \\ & L=500.04 \end{split}$$

Figure 7 near here. Figure 8 near here.

IV.3 Beyond the results

One may argue that the NSI suffers from its lack of spatially. Nevertheless, we argue that the use of at least two levels of subdivision of the city in local areas can

 $^{^{10}}$ Three-stars means that the coefficient is significantly different to zero at 99%; two-stars means that the coefficient is significantly different to zero at 95%; W is the Chi-2 Wald statistic of test; ρ is the percent contribution to the total variance of the panel-level variance component.

¹¹The computation of the coefficient of correlation tells us that the correlation between the Gini values and the NSI values is low but significant : $Corr(Gini, NSI_{IRIS}) = 0.1706$ (*p*-value = 0.015) and $Corr(Gini, NSI_{GQ}) = 0.1907$ (*p*-value < 0.001).

reveal spatial pieces of information, even though the segregation is measured with the NSI.

Consider the three following cities populated by two groups of individuals (*rich* (**R**) and *poor* (**P**)). The three cities have the same spatial pattern, that is, each city is made up of two *Grand Quartier* (North and South) and each *Grand Quartier* is divided in two *IRIS* (East and West). The three cities differ in their breakdown of these two groups into the local areas.



Let compare cities A and B. It is obvious that the two cities have the same value of NSI calculated on the *IRIS* scale. However, the breakdown of the two groups among the two *Grand Quartier* differs significantly from a city to another. Indeed, North and South are socially heterogeneous in A and homogeneous in B, while each *IRIS* is socially uniform. This simple example could illustrate the comparison of Paris with Toulon in 2001 or of Nantes with Valenciennes in 2008.

Another interesting case is the comparison of the city A with the city C. According to the NSI calculated for *Grand Quartier* scale, the two cities are equivalent in terms of income segregation, namely 50% of rich and 50% of poor people both (North and South) areas. Indeed, the composition of the two areas is the same in both cities. But the *IRIS* are more homogenous in A than C, actually areas are socially mixed in C while they are homogeneous in A. In other words, city A and C have the same *macro*-breakdown of people, while they have a different *micro*-breakdown of people. This illustrates the comparison between Nice and Nantes in

2001 or Lyon and Montpelier in 2008.

These two examples illustrate the capacity of the NSI to exhibit pieces of information that are link to the spatial dimension of the segregation by income.

V Conclusion

The goal of this article is to lay out a set of properties satisfied by a measure of residential segregation by income: the Neighborhood Sorting Index. We considered basic properties, that should be satisfied by a *good* measure of segregation, as well as properties related to the potential demographic trends. Furthermore, properties satisfied by the NSI, with respect to impacts of potential public policy, are studied. This article concludes with an illustration as part of the measurement of the residential segregation by income in the 30 largest French cities.

As pointed out in the previous sections, the NSI is an appropriate measure of segregation for continuous variable and consequently very appealing for applied analysis. The necessary dataset for the implementation of the NSI is not prohibitive, as shown previously. Furthermore, we show that the NSI can compare cities with different number of individuals and/or different number of areas and/or different overall mean income, and does not depend on whether areas are labelled or whether individuals within areas are named. Eventually, even though the segregation is measured with the NSI., we show that the use of the NSI can reveal spatial pieces of information, despite its non-spatial dimension.

References

- Chakravarty, S. R. and Silber, J. (2007). A generalized index of employment segregation. *Mathematical Social Science*, 53:185–195.
- Cutler, D. M. and Glaeser, E. L. (1997). Are ghettos bad or good? Quarterly Journal of Economics, 112:827–872.
- Cutler, D. M., Glaeser, E. L., and Vigdor, J. L. (2008). When are ghettos bad? lessons from immigrant segregation in the United States. *Journal of Urban Economics*, 63(3):759–774.
- Davidoff, T. (2005). Income sorting: Measurement and decomposition. Journal of Urban Economics, 58:289–303.

- Duncan, O. and Duncan, B. (1955). A methodological analysis of segregation indexes. American Sociological Review, 20(2):210–217.
- Echenique, F., Fryer, R. G., and Kaufman, A. (2006). Is school segregation good or bad? American Economic Review, Papers and Proceedings, 96(2):265–269.
- Goux, D. and Maurin, E. (2007). Close neighbours matter: Neighbourgood effects on early performance at school. *Economic Journal*, 117:1–24.
- Gravel, N. and Moyes, P. (2012). Ethically robust comparisons of distributions of two attributes. *Journal of Economic Theory*, doi:10.1016/j.jet.2012.01.001.
- Hardman, A. and Ioannides, Y. M. (2004). Neighbor's income distribution: economic segregation and mixing in US urban. *Journal of Housing Economics*, 13:368–382.
- Harsman, B. and Quigley, J. M. (1995). The spatial segregation of ethnic and demographic groups: Comparative evidence from stockholm and san francico. *Journal of Urban Economics*, 37:1–16.
- Hutchens, R. M. (1991). Segregation curves, lorenz curves, and inequality in the distribution of people across occupations. *Mathematical Social Science*, 21(35-51).
- Hutchens, R. M. (2001). Numerical measures of segregation: desirable properties and their implications. *Mathematical Social Science*, 42(13-29).
- Hutchens, R. M. (2004). One measure of segregation. International Economic Review, 14(2).
- Ioannides, Y. M. and Seslen, T. N. (2002). Neighborhood wealth distributions. *Economics Letters*, 76:357–367.
- Jargowsky, P. A. (1996). Take the money and run: Economic segregation in U.S. metropolitan areas. American Sociological Review, 61:984–998.
- Kim, J. and Jargowsky, P. A. (2005). A measure of spatial segregation: the generalized neighborhood sorting index. National Poverty Center Working Paper, No. 02-05. Gerald R. Ford School of Public Policy, University of Michigan.
- Kolm, S. C. (1976a). Unequal inequalities 1. Journal of Economic Theory, 12:416– 442.

- Kolm, S. C. (1976b). Unequal inequalities 2. Journal of Economic Theory, 13:82– 111.
- Massey, D. and Denton, N. (1998). The dimensions of residential segregation. Social Forces, 67:218–315.
- Massey, D. S. and Eggers, M. L. (1990). The Ecology of Inequality: Minorities and the Concentration of Poverty, 1970-1980. *American Journal of Sociology*, 95:1153–88.
- Maurin, E. (2004). Le ghetto français. Seuil.
- Mussard, S. (2006). Between-Group Pigou-Dalton Transfers. Working Paper GREDI 07-06.
- Pan Ké Shon, J.-L. (2010). The ambivalent nature of ethnic segregation in france's disadvantaged neighborhoods. Urban Studies, 47.
- Préteceille, E. (2006). La ségrégation sociale a t-elle augmenté ? la métropole parisienne entre polarisation et mixité. *Sociétés Contemporaines*, 62:69–93.
- Reardon, S. F. and O'Sullivan, D. (2004). Measures of spatial segregation. Sociological Methodology, 34:121–162.
- Safi, M. (2009). La dimension spatiale de l'intégration : évolution de la ségrégation des populations immigrée en france entre 1968 et 1999. Revue Française de Sociologie, 50(3).
- Shorrocks, A. F. and Wan, G. (2005). Spatial decomposition of inequality. *Journal of Economic Geography*, 4:59–81.
- Verdugo, G. (2011). Public housing and residential segregation of immigrants in france, 1968-199. *Population*. mimeo Banque de France.
- Watson, T. (2009). Inequality and the measurement of residential segregation by income in American neighborhoods. *Review of Income and Wealth*, 55(3):820– 844.
- Wheeler, C. H. and La Jeunesse, E. A. (2007). Neighborhood income inequality. WP 2006-039, Federal Reserve Bank of StLouis.
- White, M. J. (1983). The measurement of spatial segregation. American Journal of Sociology, 88:1008–1018.

Yang, R. and Jargowsky, P. A. (2006). Suburban development and economic segregation in the 1980s. Journal of Urban Affairs, 28(3):253–273.

A Proofs of propositions

A.1 Proof of proposition 1 (Scale interpretability)

The proof of this proposition is straightforward.

If $y_i^j = \mu^j$ for all $j \in M$, then $V(Y^j) = 0$ for all $j \in M$ and $V_w = \sum_{j \in M} \frac{n^j}{n} V(Y^j) = 0$. Obviously, $V(Y) = V_b(Y)$ and NSI = 1 if $\mu^j \neq \mu^{j'}$ for two at least areas j and j' such that $j \neq j' \in M$. If $\mu^j = \mu^{j'}$ for all j and j' such that $j \neq j' \in M$, $V_b(Y) = 0$. Then NSI = 0.

A.2 Proof of proposition 2 (Symmetry)

Symmetry within areas :

Consider two cities $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$ such that $\widetilde{Y}^j = Y^j \forall j \in M_{/\{p\}}$ and $\widetilde{Y}^p = D^p Y^p$ for one $p \in M$ with D^p a permutation matrix. It is straightforward to show that $V(\widetilde{Y}^p) = V(Y^p)$. Then, $V_b(\widetilde{Y}) = V_b(Y)$ and $NSI(Y) = NSI(\widetilde{Y})$.

Symmetry between areas :

Consider two cities C = (P, Y) and $\widetilde{C} = (\widetilde{P}, \widetilde{Y})$ such that $\widetilde{Y} = Y$ with D a permutation matrix of areas. It is straightforward to see that $V(\widetilde{Y}) = V(Y)$ and $V_b(\widetilde{Y}) = V_b(Y)$. Hence, $NSI(Y) = NSI(\widetilde{Y})$.

A.3 Proof of proposition 3 (Principles of Population)

Within-area replication invariance :

Consider two cities $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$. The latter is obtained from the former by a transformation which states that the population **within each** area is replicated α times.

By definition, $\tilde{n}^j = \alpha n^j$ and $\tilde{n} = \alpha n$. Areas' mean incomes stay unchanged: $\mu^j = \tilde{\mu}^j$; then, variance of areas' mean incomes does not changed: $V_b(Y) = V_b(\tilde{Y})$. It is also obvious that overall variance of incomes is stable:

$$V(\tilde{Y}) = \frac{1}{\tilde{n}} \sum_{j \in M} \sum_{i=1}^{\tilde{n}^{j}} (\tilde{y}_{i}^{j})^{2} - \tilde{\mu}^{2} = \frac{1}{\alpha n} \sum_{j \in M} \sum_{i=1}^{n^{j}} \alpha(y_{i}^{j})^{2} - \mu^{2} = V(Y)$$

Hence, $NSI(Y) = NSI(\widetilde{Y})$.

Area replication invariance .

Consider two cities $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$ two cities. The latter is obtained

from the former by a transformation, which states that all areas are replicated ℓ times. Obviously: $|\widetilde{M}| = \ell |M|$; $\widetilde{n} = \sum_{j \in \widetilde{M}} \widetilde{n}^j = \sum_{j \in M} \ell n^j = \ell n$; $\mu = \widetilde{\mu}$. The overall variance and the variance of mean incomes are unchanged:

$$V(\widetilde{Y}) = \frac{1}{\widetilde{n}} \sum_{j \in \widetilde{M}} \sum_{i=1}^{\widetilde{n}^j} (\widetilde{y}_i^j)^2 - \widetilde{\mu}^2 = \frac{1}{\ell n} \sum_{j \in M} \ell \sum_{i=1}^{n^j} (y_i^j)^2 - \mu^2 = V(Y)$$
$$V_b(\widetilde{Y}) = \frac{1}{\widetilde{n}} \sum_{j \in \widetilde{M}} \widetilde{n}^j (\widetilde{\mu}^j)^2 - \widetilde{\mu}^2 = \frac{1}{\ell n} \sum_{j \in M} \ell n^j (\mu^j)^2 - \mu^2 = V_b(Y)$$

Hence, $NSI(Y) = NSI(\tilde{Y})$.

A.4 Proof of proposition 4 (Absolute and relative invariance)

Relative invariance :

Consider two cities $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (P, \widetilde{Y})$ such that there exists $\lambda \in \mathbb{R}_+$: $\widetilde{y}_i^j = \lambda y_i^j$. It is straightforward to show that:

$$V(\widetilde{Y}) = \frac{1}{n} \sum_{j \in M} \sum_{i \in N^j} (\lambda y_i^j)^2 - (\lambda \mu)^2 = \lambda^2 V(Y)$$
$$V_b(\widetilde{Y}) = \frac{1}{n} \sum_{j \in M} n^j (\lambda \mu^j)^2 - (\lambda \mu)^2 = \lambda^2 V_b(Y)$$

Hence, $NSI(Y) = NSI(\widetilde{Y})$.

Absolute invariance :

Consider two cities $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (P, \widetilde{Y})$ such that there exists $\gamma \in \mathbb{R}_+$: $\widetilde{y}_i^j = y_i^j + \gamma$. It is straightforward to show that:

$$V(\widetilde{Y}) = \frac{1}{n} \sum_{j \in M} \sum_{i \in N^j} (y_i^j + \gamma - \mu - \gamma)^2 = V(Y)$$
$$V_b(\widetilde{Y}) = \frac{1}{n} \sum_{j \in M} n^j (\mu^j + \gamma - \mu - \gamma)^2 = V_b(Y)$$

Hence, $NSI(Y) = NSI(\tilde{Y})$.

A.5 Proof of proposition 5 (Sensitivity to areas aggregation)

Consider C = (P, Y) and $\widetilde{C} = (\widetilde{P}, Y)$ two cities. By definition, if \widetilde{C} is obtained from C by means of an aggregation of areas p and q if for $P = \{N^1, ..., N^p, ..., N^q, ..., N^m\}$ and $\widetilde{P} = \{\widetilde{N}^1, ..., \widetilde{N}^{m-1}\}$, then : $N^j = \widetilde{N}^j, \forall j < q, j \neq p$ and $N^j = \widetilde{N}^{j-1}, \forall j > q$ $N^p \cup N^q = \widetilde{N}^p$ $n^p + n^q = \widetilde{n}^p$ $\mu = \widetilde{\mu}$ $n^p \mu^p + n^q \mu^q = \widetilde{n}^p \widetilde{\mu}^p \Leftrightarrow (\widetilde{\mu}^p)^2 = [\delta \mu^p + (1 - \delta) \mu^q]^2$ (6.1) with $\delta = \frac{n^p}{\widetilde{n}^p}$ and $(1 - \delta) = \frac{n^q}{\widetilde{n}^p}$ Hence, $S(C) \ge S(\widetilde{C}) \Leftrightarrow \sum_{j=1}^m n^j (\mu_j - \mu)^2 \ge \sum_{j=1}^m \widetilde{n}^j (\widetilde{\mu}_j - \widetilde{\mu})^2$ $\Leftrightarrow n^p (\mu^p)^2 + n^q (\mu^q)^2 = \widetilde{n}^p (\widetilde{\mu}^p)^2$ (6.2) From (6.1) and (6.2), it follows that $S(C) \ge S(\widetilde{C}) \Leftrightarrow \delta(\mu^p)^2 + (1 - \delta) (\mu^q)^2 \ge [\delta \mu^p + (1 - \delta) \mu^q]^2$ $\Leftrightarrow (\mu^q - \mu^p)^2 \ge 0$

A.6 Proof of proposition 6 (Segregation under movements)

A.6.1 Unilateral movements

Let $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$ two cities. By definition if city $\widetilde{\mathcal{C}}$ is obtained from the city \mathcal{C} by means of a movement of an individual from p to h. By simplicity, we consider that individual n^p moves from p to q. Then:

$$\begin{split} \widetilde{y}_{n^{q}+1}^{q} &= y_{n^{p}}^{p} \\ \widetilde{y}_{i}^{j} &= y_{i}^{j} \text{ for } j = p \text{ and } i \neq n^{p} \\ \widetilde{y}_{i}^{j} &= y_{i}^{j} \text{ for all } j \neq p, q \text{ and } i = \left\{1, ..., n^{j}\right\} \\ \mu^{p} &< \mu^{q} \text{ and } \widetilde{\mu}^{p} < \widetilde{\mu}^{q}. \\ \mu &= \widetilde{\mu} \text{ and } \mu^{j} = \widetilde{\mu}^{j}, \text{ for all } j \neq p, q \end{split}$$

Hence,

$$\begin{split} S(C) > S(\widetilde{C}) &\Leftrightarrow n^p (\mu^p - \mu)^2 + n^q (\mu^q - \mu)^2 > (n^p - 1)(\widetilde{\mu}^p - \mu)^2 + (n^q + 1)(\widetilde{\mu}^q - \mu)^2 \\ &\Leftrightarrow n^p (\mu^p - \mu)^2 + n^q (\mu^q - \mu)^2 > n^p (\widetilde{\mu}^p - \mu)^2 + n^q (\widetilde{\mu}^q - \mu)^2 - (\widetilde{\mu}^p - \mu)^2 + (\widetilde{\mu}^q - \mu)^2 \\ &\Leftrightarrow n^p [(\mu^p - \mu)^2 - (\widetilde{\mu}^p - \mu)^2] + n^q [(\mu^q - \mu)^2 - (\widetilde{\mu}^q - \mu)^2] > (\widetilde{\mu}^q - \mu)^2 - (\widetilde{\mu}^p - \mu)^2 \\ \end{split}$$

Progressive movements of a rich individual

By definition, such a transformation increases mean income of area q and decreases the one area p. Thus between-variance component decreases whereas overall variance is unchanged.

Progressive movements of a poor individual

By definition, such a transformation increases mean income of area p and decreases the one of area q. Thus between-variance component decreases whereas overall variance is unchanged.

A.6.2 Switch

 $\Leftrightarrow \mu^q - \mu^p > \widetilde{\mu}^p - \widetilde{\mu}^q$

Let $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (\widetilde{P}, \widetilde{Y})$ two cities, within which two individuals k and h are located in area p and q respectively. By definition if city $\widetilde{\mathcal{C}}$ is obtained from the city \mathcal{C} by means of a progressive switch of individuals k and h, then:

$$\begin{split} \widetilde{y}_{k}^{p} &= y_{h}^{q} \\ \widetilde{y}_{i}^{h} &= y_{k}^{p} \\ \widetilde{y}_{i}^{j} &= y_{i}^{j} \text{ for } j = p, q \text{ and } i \neq k, h \\ \widetilde{y}_{i}^{j} &= y_{i}^{j} \text{ for all } j \neq p, q \text{ and } i = \left\{1, ..., n^{j}\right\} \\ y_{k}^{p} &< y_{h}^{q}, \mu^{p} < \mu^{q} \text{ and } \widetilde{\mu}^{p} < \widetilde{\mu}^{q}. \\ \mu &= \widetilde{\mu} \text{ and } \mu^{j} = \widetilde{\mu}^{j}, \text{ for all } j \neq p, q \\ \end{split}$$
Hence,

$$\begin{split} S(C) &> S(\widetilde{C}) \Leftrightarrow n^{p}(\mu^{p})^{2} + n^{q}(\mu^{q})^{2} > n^{p}(\widetilde{\mu}^{p})^{2} + n^{q}(\widetilde{\mu}^{q})^{2} \\ \Leftrightarrow n^{p} \left[(\mu^{p})^{2} - (\mu^{p})^{2} - \frac{2\mu^{p}(y_{h}^{q} - y_{k}^{p})}{n^{p}} - \frac{(y_{h}^{q} - y_{k}^{p})^{2}}{(n^{p})^{2}} \right] > n^{q} \left[(\mu^{q})^{2} - (\mu^{q})^{2} + \frac{2\mu^{q}(y_{k}^{p} - y_{h}^{q})}{n^{q}} - \frac{(y_{k}^{p} - y_{h}^{q})^{2}}{(n^{q})^{2}} \right] \\ \Leftrightarrow -2\mu^{p}(y_{h}^{q} - y_{k}^{p}) - \frac{(y_{h}^{q} - y_{k}^{p})^{2}}{n^{p}} > 2\mu^{q}(y_{k}^{p} - y_{h}^{q}) + \frac{(y_{k}^{p} - y_{h}^{q})^{2}}{n^{q}} \end{split}$$

For a regressive switch of individuals k and h, the proof is similar.

A.7 Proof of proposition 7 (Internal transfer)

Let $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (P, \widetilde{Y})$ two cities and two individuals k and h located in area p. If the city $\widetilde{\mathcal{C}}$ is obtained from the city \mathcal{C} by means of a within-area income regressive transfer then, for any $\delta \in \mathbb{R}_+$:

$$\begin{split} \widetilde{y}_h^p &= y_h^p + \delta \text{ and } \widetilde{y}_k^p = y_k^p - \delta \\ \widetilde{y}_i^j &= y_i^j \text{ for } j = p \text{ and } i \neq k, h \text{ and } \widetilde{y}_i^j = y_i^j \text{ for all } j \neq p \text{ and } i \in N^j \end{split}$$

$$\begin{split} \mu^{j} &= \widetilde{\mu}^{j} \text{ for all } j \\ y_{h}^{p} &> y_{k}^{p} \\ \text{Hence,} \end{split}$$

$$\begin{split} S(C) &> S(\widetilde{C}) \Leftrightarrow (y_h^p - \mu)^2 + (y_k^p - \mu)^2 < (\widetilde{y}_h^p - \widetilde{\mu})^2 + (\widetilde{y}_k^p - \widetilde{\mu})^2 \\ \Leftrightarrow (y_h^p - \mu)^2 + (y_k^p - \mu)^2 < (y_h^p + \delta - \mu)^2 + (y_k^p - \delta - \mu)^2 \\ \Leftrightarrow y_k^p - \delta - y_h^p < 0 \end{split}$$

Let $\mathcal{C} = (P, Y)$ and $\widetilde{\mathcal{C}} = (P, \widetilde{Y})$ two cities and two individuals k and h located in area p. If the city $\widetilde{\mathcal{C}}$ is obtained from the city \mathcal{C} by means of a within-area income progressive transfer then, for any $\delta \in \mathbb{R}_+$:

$$\begin{split} \widetilde{y}_{h}^{p} &= y_{h}^{p} + \delta \text{ and } \widetilde{y}_{k}^{p} = y_{k}^{p} - \delta \\ \widetilde{y}_{i}^{j} &= y_{i}^{j} \text{ for } j = p \text{ and } i \neq k, h \text{ and } \widetilde{y}_{i}^{j} = y_{i}^{j} \text{ for all } j \neq p \text{ and } i \in N^{j} \\ \mu^{j} &= \widetilde{\mu}^{j} \text{ for all } j \\ y_{h}^{p} &< y_{k}^{p} \\ \text{Hence,} \\ S(C) &< S(\widetilde{C}) \Leftrightarrow (y_{h}^{p} - \mu)^{2} + (y_{k}^{p} - \mu)^{2} > (\widetilde{y}_{h}^{p} - \widetilde{\mu})^{2} + (\widetilde{y}_{k}^{p} - \widetilde{\mu})^{2} \\ \Leftrightarrow (y_{h}^{p} - \mu)^{2} + (y_{k}^{p} - \mu)^{2} > (y_{h}^{p} + \delta - \mu)^{2} + (y_{k}^{p} - \delta - \mu)^{2} \end{split}$$

$$\Rightarrow y_k^p - \delta - y_h^p > 0 \Rightarrow y_k^p - \tilde{y}_h^p > 0$$

B Database description and results

City	Population	Nb of	Δ IRIS	Nb of	Δ quartier	Δ pop.	Pop. included
City	(U.C.)	IRIS		quartier		(%)	(%)
Paris	6494105,47	3793,17	139	685,00	42	3,36	95,85
M.A.P.	892095, 88	495,00	17	139,50	2	3,03	87,62
Lyon	889617, 18	451,83	13	195, 33	4	4,54	87, 34
Nice	641328, 63	316,00	0	67,00	0	4,51	89,81
\mathbf{Lille}	621840, 98	382,50	10	128,67	2	2,17	90,62
Toulouse	517755, 17	234,00	0	95,00	0	8,17	80, 31
Bordeaux	502811, 87	270,50	11	64,83	4	4,77	92,58
Toulon	367608,95	186,17	3	32,00	0	3,96	86,44
Nantes	355548, 52	169,00	0	38,00	0	2,88	85, 32
Grenoble	267684, 30	140, 17	2	34,50	2	2,33	81, 48
Strasbourg	267075, 87	145,67	2	51,83	1	1,86	89,92
Rouen	246341,53	147,00	28	51,83	13	0,93	92,50
Valenciennes	222417,95	93,17	ų	39,83	1	1,81	72,88
Nancy	204192,07	101,33	10	35,50	3	0,45	82,69
Metz	204154, 15	85,50	7	32,67	2	0,79	74,89
Tours	193564, 20	115,00	0	47,00	0	3,12	94,05
Saint-Etienne	186481, 27	100,67	9	33,50	2	-1,47	81,49
Montpellier	188314,73	78,00	0	28,00	0	7,01	74,22
Avignon	174692, 27	71,00	0	14,00	0	5,81	75,60
Orleans	170438,23	82,67	1	38,00	0	1,98	79,77
Rennes	166934, 38	106,00	0	23,00	0	1,38	97,54
Le Havre	155616, 93	98, 83	13	31, 33	4	-1,86	93,65
Mulhouse	151921, 17	82,00	0	31,00	0	1,33	87,65
Dijon	149471, 37	90,50	15	29,17	IJ	1,25	91,46
Angers	138896, 10	73, 33	4	20,00	0	-0,83	84,59
Reims	130118,97	78,00	0	45,00	0	-0,83	92, 36
Brest	129505,97	74,00	0	8,00	0	0,27	90,50
Le Mans	125271,28	73,00	0	31,00	0	0,27	88, 34
Bayonne	126977,00	63,00	0	21,00	0	5,55	89,11

Table I: The data

	NI		т	• •
	IN:	51 based on:	Incor	ne inequality
	IRIS	Grand Quartier	Gini index	Coeff. of variation
2001	0.3230	0.2796	0.3819	1.0704
2002	0.3243	0.2819	0.3827	1.0612
2004	0.3250	0.2830	0.3888	1.0820
2005	0.3168	0.2761	0.3904	1.1267
2006	0.3069	0.2675	9.3948	1.2018
2007	0.3024	0.2637	0.3972	1.2441
2008	0.2972	0.2596	0.3993	1.2518

Table II: Income segregation and income inequality (weighted mean)

	20	01	20	02	20	04	20	05	20	06	20	07	20	08
City	IRIS	Quart	IRIS	Quart	IRIS	Quart	IRIS	Quart	IRIS	Quart	IRIS	Quart	IRIS	Quart
	2001		2002		2004		2005		2006		2007		2008	
Paris	12	16	14	17	19	20	19	19	20	20	21	23	24	26
MAP	26	25	27	26	26	26	27	27	26	24	25	25	25	23
Lyon	17	20	18	22	18	22	21	22	14	19	∞	15	14	20
Nice	က	4	က	Ŋ	2	2	2	က	2	က	2	2	2	2
Lille	28	27	26	25	29	27	28	28	28	28	28	28	29	28
Toulouse	9	12	∞	12	ъ	12	ល	12	9	11	ъ	12	9	11
Bordeaux	2	9	ъ	∞	4	9	4	9	IJ	7	9	6	ъ	7
Toulon	11	c,	10	က	6	က	13	ъ	11	4	14	က	12	4
Nantes	14	ъ	2	4	12	ъ	က	2	4	2	11	ŋ	∞	റ
Grenoble	25	23	22	20	24	23	23	21	22	22	10	14	16	17
Strasbourg	21	22	23	23	16	19	24	23	21	23	18	21	11	15
Rouen	27	29	28	28	27	28	25	25	27	27	27	27	23	24
Valenciennes	7	11	9	11	က	×	9	10	x	12	က	7	7	13
Nancy	6	7	6	7	2	7	2	7	က	9	6	x	4	ഹ
Metz	18	15	16	14	13	14	16	16	18	15	19	16	6	12
Tours	13	14	15	15	17	17	12	14	17	16	20	19	20	19
Saint-Etienne	ю	6	2	9	9	11	∞	11	2	6	7	11	က	×
Montpellier	22	17	21	16	21	16	11	13	6	13	22	17	26	21
Avignon	10	2	4	2	×	4	10	4	12	ю	12	4	13	9
Orléans	16	19	13	18	11	15	15	17	16	17	16	18	17	18
Rennes	×	10	11	10	10	10	6	×	10	10	4	9	10	6
Le Havre	29	28	29	29	28	29	29	29	29	29	29	29	28	29
Mulhouse	24	24	25	24	25	24	26	26	25	25	26	26	27	27
Dijon	4	8	17	13	14	13	18	15	19	14	17	13	15	14
Angers	15	13	12	6	15	6	14	6	13	∞	15	10	22	10
Reims	23	26	24	27	23	25	22	24	24	26	13	20	19	25
Le Mans	19	21	19	21	20	21	17	20	15	18	23	24	21	22
Bayonne	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Caen	20	18	20	19	22	18	20	18	23	21	24	22	18	16

Table III: Ranking per year

	20	01	20	02	20	04	20	05	20	06	20	07	20	08
City	IRIS	Quart												
Paris	0.315	0.284	0.322	0.290	0.330	0.298	0.322	0.290	0.312	0.283	0.311	0.282	0.315	0.286
MAP	0.387	0.342	0.388	0.344	0.383	0.339	0.372	0.330	0.363	0.321	0.346	0.305	0.316	0.277
Lyon	0.333	0.306	0.335	0.308	0.326	0.301	0.331	0.307	0.296	0.273	0.268	0.248	0.287	0.266
Nice	0.269	0.199	0.260	0.193	0.239	0.177	0.242	0.177	0.233	0.172	0.229	0.168	0.216	0.161
Lille	0.403	0.360	0.380	0.340	0.402	0.360	0.399	0.357	0.381	0.341	0.379	0.339	0.371	0.331
Toulouse	0.275	0.244	0.285	0.254	0.278	0.246	0.265	0.234	0.257	0.227	0.258	0.229	0.247	0.218
Bordeaux	0.265	0.211	0.277	0.222	0.266	0.214	0.245	0.197	0.257	0.204	0.259	0.207	0.247	0.194
Toulon	0.308	0.177	0.296	0.173	0.294	0.183	0.297	0.184	0.286	0.179	0.285	0.179	0.274	0.172
Nantes	0.321	0.205	0.285	0.184	0.308	0.198	0.245	0.158	0.253	0.165	0.277	0.183	0.257	0.164
Grenoble	0.383	0.330	0.353	0.305	0.349	0.305	0.348	0.304	0.322	0.285	0.276	0.246	0.290	0.258
Strasbourg	0.344	0.317	0.355	0.327	0.320	0.294	0.349	0.321	0.320	0.295	0.302	0.280	0.267	0.248
Rouen	0.401	0.374	0.389	0.359	0.394	0.360	0.356	0.325	0.373	0.341	0.370	0.339	0.311	0.284
Valenciennes	0.279	0.239	0.283	0.243	0.259	0.223	0.271	0.234	0.268	0.228	0.241	0.204	0.256	0.220
Nancy	0.292	0.216	0.294	0.219	0.290	0.219	0.279	0.211	0.252	0.191	0.272	0.205	0.241	0.182
Metz	0.336	0.283	0.332	0.281	0.309	0.262	0.316	0.264	0.310	0.256	0.304	0.252	0.261	0.219
Tours	0.318	0.279	0.327	0.287	0.323	0.284	0.294	0.259	0.301	0.266	0.307	0.273	0.296	0.263
Saint-Etienne	0.273	0.224	0.260	0.214	0.290	0.243	0.280	0.234	0.259	0.216	0.264	0.221	0.238	0.204
Montpellier	0.348	0.289	0.344	0.288	0.335	0.279	0.288	0.241	0.272	0.229	0.312	0.263	0.316	0.266
Avignon	0.297	0.175	0.268	0.162	0.292	0.189	0.285	0.182	0.287	0.185	0.281	0.181	0.274	0.184
Orléans	0.328	0.299	0.320	0.290	0.306	0.276	0.303	0.273	0.300	0.271	0.298	0.268	0.290	0.261
Rennes	0.291	0.230	0.304	0.240	0.303	0.239	0.284	0.222	0.281	0.219	0.255	0.201	0.263	0.207
Le Havre	0.407	0.371	0.434	0.399	0.400	0.368	0.405	0.371	0.401	0.367	0.395	0.361	0.365	0.332
Mulhouse	0.374	0.331	0.372	0.330	0.369	0.328	0.370	0.328	0.362	0.323	0.363	0.324	0.349	0.313
Dijon	0.273	0.219	0.334	0.270	0.310	0.251	0.321	0.262	0.312	0.251	0.301	0.243	0.289	0.235
Angers	0.325	0.246	0.317	0.239	0.311	0.234	0.301	0.227	0.295	0.214	0.287	0.209	0.300	0.218
Reims	0.362	0.350	0.356	0.345	0.347	0.335	0.336	0.325	0.337	0.325	0.284	0.273	0.293	0.286
Le Mans	0.337	0.309	0.336	0.306	0.331	0.301	0.320	0.292	0.298	0.272	0.315	0.286	0.297	0.274
Bayonne	0.181	0.139	0.192	0.148	0.208	0.158	0.189	0.146	0.203	0.159	0.200	0.154	0.153	0.121
Caen	0.337	0.291	0.340	0.293	0.337	0.291	0.326	0.281	0.331	0.285	0.328	0.281	0.292	0.249

Table IV: Values of NSI per year

C Figures



Figure 1: Rankings according to the NSI based on IRIS in 2001 and 2008



Figure 2: Rankings according to the NSI based on *Grand Quartier* in 2001 and 2008



Figure 3: The values of the NSI based on IRIS in 2001 and 2008



Figure 4: The values of the NSI based on $\mathit{Grand}\ \mathit{Quartier}$ in 2001 and 2008



Figure 5: Annual ranking according to the scale of area unit



Figure 6: The value of the NSI according to the scale of area unit



Figure 7: Correlation between Gini index and NSI based on IRIS



Figure 8: Correlation between Gini index and NSI based on $Grand \ Quartier$