Centre de Recherche en Économie et Management

Center for Research in Economics and Management

University of Caen

University of Rennes 1

Heterogeneity and the Formation of Risk-Sharing Coalitions

Fabien Moizeau

University of Rennes 1, CREM-CNRS

Fernando Jaramillo

Universidad del Rosario, Bogota (Colombia)

Hubert Kempf

École Normale Supérieure de Cachan et Paris School of Economics

September 2011 - WP 2011-11

Working Paper

Heterogeneity and the Formation of Risk-Sharing Coalitions.*

Fernando JARAMILLO[†], Hubert KEMPF[‡] and Fabien MOIZEAU[§]

September 2011.

Abstract

We study the relationship between the distribution of individuals' attributes over the population and the extent of risk sharing in a risky environment. We consider a society where individuals differing with respect to risk or their degree of risk aversion form risk-sharing coalitions in the absence of financial markets. We obtain a partition belonging to the core of the membership game. It is homophily-based: the less risky (or the more risk tolerant) agents congregate together and reject more risky ones (or less risk tolerant ones) into other coalitions. The distribution of risk or risk aversion affects the number and the size of these coalitions. It turns out that individuals may pay a lower risk premium in more risky societies. We also show that a higher heterogeneity in risk or risk aversion leads to a lower degree of partial risk-sharing. The empirical evidence on partial risk sharing can be understood when the endogenous partition of society into risk-sharing coalitions is taken into account.

Keywords: Risk Sharing, Group Membership, Social Segmentation.

JEL Classification: C71, D3, D71, D81.

*We are very grateful to David Bardey, Max Engers, Laszlo Koczy, Sophie Larribeau, Thierry Magnac as well as to conference participants at Coalition Theory Network Workshop 2009, the PET 09 Galway, the Journées Louis-André Gérard-Varet #9 and the EEA Glasgow meeting 2010, and seminar participants at Université de Lille 1, Tinbergen Institute, Banque de France, Université du Maine, Université de Strasbourg, Université de Clermont 1, Université de Montpellier 1, SMART (INRA, Rennes), Université de Cergy-Pontoise and Universidad del Rosario (Bogota).

[†]Universidad del Rosario, Bogota (Colombia), email: fernando.jaramillo@urosario.edu.co.

[‡]Ecole Normale Supérieure de Cachan and Paris School of Economics, email: hubert.kempf@univ-paris1.fr.

[§]CREM, Université de Rennes 1, email: fabien.moizeau@univ-rennes1.fr.

1 Introduction.

In developing economies where financial markets are lacking, individual incomes vary widely (see, in particular, Townsend, 1994, for ICRISAT villages in India or Dubois, Jullien and Magnac, 2008, for Pakistan villages). Nonetheless, the idiosyncratic part of income risk is relatively large, suggesting that insurance against shocks is desirable (Townsend, 1995, Dercon, 2004). Thus we should expect risk-averse households to pool risk in order to smooth consumption. If risk is fully insured, the theory tells us that individual consumption is determined by aggregate consumption (see Borch, 1962, Arrow, 1964, Wilson, 1968). However, this proposition has been subject to many empirical rebuttals. In developing economies, the empirical evidence supports partial risk sharing. Households are able to protect consumption against adverse income shocks but full insurance is not achieved (see, among many others, Townsend, 1994, Kazianga and Udry, 2005).[1] Moreover, empirical works identify risk-sharing groups and networks smaller than the entire society. For instance, Fafchamps and Lund (2003) show that mutual insurance is implemented within confined networks of families and friends. Mazzocco and Saini (2009), using ICRISAT data, show that the relevant unit to test for efficient risk sharing is the caste and not the village. Other individual characteristics also appear to be key determinants of membership in risk-sharing groups or networks. Geographic proximity as well as age and wealth differences also play a role in the formation of networks (see Fafchamps and Gubert, 2007). Using data on group-based funeral insurance in Ethiopia and Tanzania, Dercon *et alii* (2006) provide evidence of assortative matching according to physical distance, kinship, household size and the age of the member. Arcand and Fafchamps (forthcoming) find robust evidence of individuals' sorting with respect to physical or ethnic proximity as well as wealth and household size for community-based organizations in Senegal and Burkina Faso.[2] It turns out that the distribution of individuals' attributes over the population plays a key role in group memberships and the extent of risk sharing.

In the present paper, we develop a model of endogenous formation of risk-sharing coalitions that allows us to characterize the relationship between *ex ante* heterogeneity among individuals with respect to their exposure to risk or their risk aversion and the equilibrium size of risk-sharing groups. Our analysis stresses that the relationship between the distribution of risk and the pattern of risk-sharing coalitions is key to understand partial risk sharing.

Formally, we first study a society comprised of many individuals, each one characterized by

[1] Townsend (1995), Ray (1998), Dubois (2002), or Dercon (2004) are excellent surveys of the literature. For developped economies also, empirical evidence does not support the full insurance hypothesis. See Mace (1991), Cochrane (1991), Hayashi, Altonji and Kotlikoff (1996), Attanasio and Davis (1996).

[2] See also the survey of Fafchamps (2008) on the role of families and kinship networks in sharing risk.

a specific value of the variance of the distribution from which is drawn the idiosyncratic shock.[3] Individuals have the possibility to form a group in order to mutualize risk. We consider that individuals commit to sharing the random component of their income equally with members of their risk-sharing group. Given the membership, risk sharing is efficient. We examine the segmentation of society into such risk-sharing groups.[4] We show that the resulting core partition exists and is unique (under some mild assumption). It turns out that the key dimension of the coalition formation process is risk heterogeneity measured by ratios of variance between individuals. This leads the core partition to be homophily-based: coalitions gather together agents similar with respect to the variance of the idiosyncratic shock. In this sense, the extent of risk sharing is limited by the formation of coalitions due to heterogeneity. Two individuals belonging to the same society do not necessarily share risk in the same coalition.

We study the impact of specific variance schedules on the core partition and show thanks to these cases how the number and the size of coalitions belonging to the core partition are affected by the distribution of risk within society.

Defining an aggregate risk premium, we compare between two societies with equal number of individuals the amount of resources devoted to risk sharing. We prove that a *more* risky society (in the sense of second-order stochastic dominance) comprised of risk averse individuals may devote *less* resources to risk sharing than a less risky one. This implies that some individuals may prefer to live in the more risky society. This result relies on the fact that risk heterogeneity is the key determinant of the formation of coalitions.

We also discuss the empirical implications of our model. Most empirical studies have found evidence of partial risk sharing. This can be explained by the fact individuals sort themselves into risk-sharing groups smaller than the whole society. We show how the number and the size of risk-sharing coalitions can affect the values of the coefficients of the econometric specification of the consumption function. Following empirical works (see in particular, Jalan and Ravallion, 1999, and Suri, 2009), we consider that the average value of the coefficient on individual income measures the extent of risk sharing. One implication of the result stated above is that this coefficient is larger is larger, respectively smaller, when there is more, respectively less, discrepancy between idiosyncratic shocks variances.

These results highlight that these two dimensions (extent of risk sharing and aggregate risk premium) matter for the assessment of risk sharing in society when social segmentation endogenously

[3] The heterogeneity among individuals with respect to their variance could be explained by the fact that individuals do not use the same technology and are differently exposed to risks (see for instance Conley and Udry, 2010).

[4] Ghatak (1999), Chiappori and Reny (2005), Genicot (2006), Legros and Newman (2007), study risk-sharing groups formation but these works consider that the size of groups is exogenously given.

emerges. Take two societies one more risky than the other one. The more risky society may be characterized by a higher aggregate risk premium. According to this dimension, we may conclude that the insurance against risk is worse in this society. However, if this more risky society is more homogenous leading to larger risk-sharing coalitions then the extent of risk sharing would be larger which is commonly interpreted as a better insurance outcome. Therefore, we cannot rely on one indicator only to assess the risk sharing outcome.

In order to check the robustness of our approach, we discuss the alternative case where individuals differ in their aversion to risk. That is, we focus on heterogeneity affecting the individual utility functions. We prove that our propositions carry over to this case, even though the logic behind the formation of the core partition differs. In this case, agents are willing to cluster with the most risk tolerant agents.

The relationship between risk and group formation has already been studied by various authors. In particular, Genicot and Ray (2003) develop a group formation approach where one risk-sharing coalition must be robust to potential subgroup deviations. This stability condition may limit the size of the risk-sharing coalition. Bold (2009) solves for the optimal dynamic risk-sharing contract in the set of coalition-proof equilibria. We depart from these works from two respects. First, we focus on heterogeneity of individuals' attributes as the force limiting the size of risk-sharing coalition instead of the absence of commitment. Second, we study the partition of society into possibly multiple coalitions. Our work is also closely related to Taub and Chade (2002) who study in a dynamic setup whether the current core partition is immune to future individual defections. Our focus is different as we build a setup that allows us to characterize a relationship between (i) the risk characteristics of a society, (ii) the membership and size of risk-sharing groups and (iii) the extent of risk coping. Our paper bears also some similarities with Henriet and Rochet (1987) who develop a model of endogenous formation of mutuals using a cooperative game theoretical approach. The modelling strategy is different from ours as they assume a continuum of agents, the existence of congestion costs and a binomial distribution of shock. Further, they focus on formal insurance activity and do not address the issue of mutualization of risk under informal insurance schemes. Finally, Bramoullé and Kranton (2007) develop a model of network formation to tackle the risk sharing issue. As they consider identical individuals, they do not examine how heterogeneity shapes the architecture of networks.

The plan of the paper is as follows. In the following section, we present our coalition-formation framework with individuals differing with respect to the exposure to risk. We then characterize the partition that emerges and study the relationship between the risk distribution, the size of risk-sharing groups and the extent of risk sharing. Section 4 discusses the empirical implications of our theoretical setup. In section 5, we study a coalition-formation framework where individuals differ with respect to their risk aversion. Section 6 concludes.

2 The Model.

We consider a society $\mathbf{I}$ formed of N agents, indexed by $i = 1, ..., N$. These individuals live T periods. There is no production in this society and agents are endowed with quantities of a non-storable good. At each date t, the individual endowment y_{it} allotted to individual i has a deterministic component w_{it} and is affected by an idiosyncratic risk ε_{it} and a common shock ν_t :

$$y_{it} = w_{it} + \varepsilon_{it} + \nu_t$$

where ν_t is i.i.d across individuals and time and normally distributed: $\nu_t \rightsquigarrow \mathcal{N}(0, \sigma_\nu^2)$. Moreover, ε_{it} is i.i.d. across time and normally distributed: $\varepsilon_{it} \rightsquigarrow \mathcal{N}(0, \sigma_i^2)$.

Individuals have instantaneous CARA utility functions and, at date 0, agent i is characterized by the following expected utility function:

$$U_i(c_{it}) = -\mathbb{E}_0 \left[\frac{1}{\alpha_i} \sum_{t=1}^{T} \delta^{t-1} e^{-\alpha_i c_{it}} \right]$$

with $\mathbb{E}_0$ the mathematical expectation operator at date 0, α_i the absolute risk aversion, δ the discount factor and c_{it} the consumption of agent i at date t.

There is perfect information in the following sense: the various idiosyncratic variances are public information and the realised individual shocks are also perfectly observed by all agents when they occur. It is assumed that there are no financial markets allowing any agent to insure himself against his idiosyncratic risks. But agents have the possibility to form groups in order to cope with risk according to a particular informal risk-sharing rule that will be presented in the next section.

2.1 Risk-sharing Coalitions.

Without loss of generality, we index individuals as follows: for i and $i' = 1, .., N$ with $i < i'$ then $\sigma_i^2 < \sigma_{i'}^2$. We will thus say that a lower indexed individual is a "less risky agent" (strictly speaking, individual risk is associated with the law of motion of ε_i).

Given these differences among individuals, we define $\lambda_i \equiv \frac{\sigma_i^2}{\sigma_{i-1}^2}$ for $i = 2, ..., N$. λ_i is called the "risk ratio" between agents $i - 1$ and i. We will use the following

Definition 1 *Any society* $\mathbf{I}$ *can be characterized by a risk-ratio schedule* $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ *with* $\lambda_i \equiv \frac{\sigma_i^2}{\sigma_{i-1}^2}$ *for* $i = 2, ..., N$.

We consider that individuals have the possibility to group themselves in order to pool risk. We denote by S a group $S \subseteq \mathbf{I}$, formed of a finite number $n \leq N$ of agents. S is a subset of $\mathbf{I}$ whose membership is left undefined at this stage. The good being assumed non-storable, the time subscript is dropped in the sequel. We will also loosely refer to Λ as capturing society's "risk heterogeneity".

In order to illustrate the impact of heterogeneity with respect to risk, we assume full commitment:

(i) no agent is able to leave on her chosen coalition once the state of nature is realized;[5]

(ii) within each coalition, individuals commit to applying the following "mutual insurance rule":

$$c_i = w_i + \frac{\frac{1}{\alpha_i}}{\frac{1}{n}\sum\limits_{k\in S}\frac{1}{\alpha_z}}\left(\nu + \frac{\sum\limits_{k\in S}\varepsilon_k}{n}\right) \tag{1}$$

where n is the cardinal of S. The mutual insurance rule applied in each coalition is such that individual consumption is optimal within the risk-sharing group.[6] The consumption of agent i in a given risk-sharing group, based on the risk-sharing rule is a function of the ratio of individual absolute risk tolerance coefficient to the average absolute risk tolerance in the group. The lower the average risk tolerance with respect to the risk tolerance of agent i, the higher the consumption of agent i. This comes from the fact that the less risk tolerant agents in coalition S_j, the higher their transfers insuring against risk.

An alternative assumption would be that the sharing rule is negotiated after the formation of a coalition.[7] However various reasons (cultural and religious values, constitutional and political constraints, bargaining and information issues) may hamper the capacity to negotiate a sharing rule within a coalition. From this perspective looking at the case where the sharing rule is given provides a useful benchmark.

Let us first consider the case where individuals are characterized by the same risk aversion parameter but differ with respect to their risk. In this case, equation (1) becomes:

$$c_i = w_i + \frac{\sum_{k\in S}\varepsilon_k}{n} + \nu. \tag{2}$$

Notice that when the non-stochastic component is identical for all agents, this rule amounts to the equal sharing rule.[8] This rule has the crucial advantage of focusing on transfers among agents

[5] This is a key difference with for instance Genicot and Ray (2003).

[6] Notice that this rule corresponds to the optimal level of consumption when the Pareto weights denoted by μ_i are such that $\ln \mu_i = \alpha_i w_i$ (see in particular Equation (22) in Appendix 7.4 where we derive optimal allocations of resources).

[7] Chiappori and Reny (2005) address this issue in the case of a matching model.

[8] For the use of the equal sharing rule, see e.g. Bramoullé and Kranton (2007).

solely justified by the objective of sharing risk among individuals as these transfers relate to the random components of income. In other words, we abstract from any redistribution motive not related to risk sharing. All our results will be deduced from this sole rationale.

The expected utility of individual i in group S, $V_i(S)$, applying this insurance rule is:

$$V_i(S) = -\mathbb{E}\left[\frac{1}{\alpha}e^{-\alpha w_i - \alpha\frac{\sum_{k\in S}\varepsilon_k}{n} - \alpha\nu}\right].$$

As we assume a CARA utility function and normal distribution for each idiosyncratic shock, the Arrow-Pratt approximation is exact:

$$V_i(S) = -\frac{1}{\alpha}e^{-\alpha\left[w_i - \frac{\alpha}{2n^2}\sum_{k\in S}\sigma_k^2 - \frac{\alpha}{2}\sigma_\nu^2\right]}. \tag{3}$$

We define the certainty-equivalent income for individual i in group S, denoted by $\omega_i(S)$, as:

$$\omega_i(S) = w_i - \frac{\alpha}{2}\sum_{k\in S}\frac{\sigma_k^2}{n^2} - \frac{\alpha}{2}\sigma_\nu^2. \tag{4}$$

The risk premium for any individual i in group S, denoted by $\pi(S)$, is equal to $\frac{\alpha}{2}\sum_{k\in S}\frac{\sigma_k^2}{n^2} + \frac{\alpha}{2}\sigma_\nu^2$. It is immediate to remark that it is the same for every member of S.

The individual gain for agent i from membership to group S rather than to group S' amounts to the reduction in the risk premium:

$$\pi(S') - \pi(S).$$

In other words, an agent prefers joining a group (provided she is accepted in this group) in which her certainty-equivalent income is higher. The more risky an agent, the more he benefits from belonging to a given group (rather than remaining alone): individual gains from a group are differentiated and actually increasing with the riskiness of the agent. This is the core characteristics of a group functioning under our insurance rule.

Hence, the formation of a group relies on the following trade-off. Accepting a new member has two opposite effects: on the one hand, everything else equal, the higher its size, the lower the risk premium; on the other hand, accepting an individual increases the sum of individual risks leading members to pay a higher risk premium. Therefore when assessing the net benefit of accepting a given individual, characterized by a particular variance, an insider has to weigh these two effects.

2.2 The Core Partition.

From above, it is immediate that the characteristics and more specifically the size of a group matters for its members. In particular, as agents have different needs for risk sharing and expose the members of the group to their idiosyncratic risk, the membership of a group is a matter of concern. This leads to the question of the endogenous segmentation of society into risk-sharing groups.

We consider that a group is a coalition or club of individuals and a partition of the society is a set of coalitions. More formally, we use the following

Definition 2 *A non-empty subset S_j of $\mathbf{I}$ is called a coalition and $\mathcal{P} = \{S_1, ..., S_j, ..., S_J\}$ for $j = 1, ..., J$ is called a partition of $\mathbf{I}$ if (i) $\bigcup_{j=1}^{J} S_j = \mathbf{I}$ and (ii) $S_j \bigcap S_{j'} = \emptyset$ for $j \neq j'$.*

According to this definition, any individual belongs to one and only one coalition. The size of the $j-th$ coalition, $S_j \subseteq \mathbf{I}$, is denoted by n_j.

To address the issue of segmentation of society into risk-sharing coalitions, we consider the following sequence of events:

1. Agents form risk-sharing coalitions and a partition of society is obtained.
2. Individuals commit to paying transfers according to the insurance rule of Equation (2) in each coalition.
3. Idiosyncratic shocks are realized. Agents then consume their after-transfer income.

We solve this coalition-formation game by looking at a core partition defined as follows:

Definition 3 *A partition $\mathcal{P}^* = \left\{S_1^*, ..., S_j^*, ..., S_J^*\right\}$ belongs to the core of the coalition-formation game if:*

$$\nexists \pounds \subseteq \mathbf{I} \text{ such that } \forall i \in \pounds,\ V_i(\pounds) > V_i(\mathcal{P}^*)$$

where $V_i(\mathcal{P}^*)$ denotes the utility for agent i associated with partition $\mathcal{P}^*$.

According to this definition, a core partition is such that no subset of agents is willing to secede. It amounts to say that coalitions are formed according to a unanimity rule: (i) no one can be compelled to stay in a given group and (ii) to be accepted in a group, there must be unanimous consent by all existing members of this group. Given the mutual insurance rule, the core of the coalition-formation game we are looking for is Pareto-optimal.

3 Risk Exposure Heterogeneity and the Pattern of Risk-Sharing Coalitions.

In this section, we provide results on the impact of individual heterogeneity with respect to risk on the segmentation of society in multiple risk-sharing coalitions.

3.1 The Characteristics of the Core Partition.

First focusing on the outcome of endogenous formation of risk-sharing coalitions, we are able to offer the following:

Proposition 1 *A core partition* $\mathcal{P}^* = \left\{ S_1^*, ..., S_j^*, ..., S_J^* \right\}$ *exists and is characterized as follows:*

i/ *It is unique if*

$$\forall z = 2, ..., N-1, \quad \frac{\lambda_{z+1} - \lambda_z}{\lambda_{z+1} - 1} \geq -\frac{1}{z+1}. \tag{5}$$

ii/ *It is consecutive, that is, if* i *and* $\widetilde{i}$ *both belong to* S_j^* *then* $\forall i'$, $i > i' > \widetilde{i}$, $i' \in S_j^*$.

iii/ *For any two individuals* $i \in S_j^*$ *and* $i' \in S_{j'}^*$*, such that* $\sigma_i^2 < \sigma_{i'}^2$*, then* $\pi(S_j^*) \leqq \pi(S_{j'}^*)$.

Proof. See Appendix. ■

The existence of a core partition of **I** relies on the common ranking property proposed by Farrell and Scotchmer (1988) and Banerjee et al. (2001).[9]

The first result, *i/*, provides a sufficient condition for the core partition to be unique. The condition on uniqueness depends on the rank of individuals. If the risk ratios are increasing with the index z, this condition is always met. The condition may appear stringent when $\lambda_z > \lambda_{z+1}, \forall z = 2, ..., N-1$. The expression $\frac{-1}{z+1}$ is an increasing function of z which equals $\frac{-1}{3}$ when $z = 2$, $\frac{-1}{N}$ when $z = N-1$, and tending to 0 when N is sufficiently large.[10]

Turning to the characteristics of the core partition, the second result, *ii/*, is about consecutivity which captures the homophily feature. Coalitions belonging to the core partition include agents who are "close" in terms of exposure to risk. Take an individual who has to choose between two individuals in order to form a risk-sharing coalition. It is easy to check that he always prefers the less risky of the pair. This implies that if an agent i is willing to form a coalition with some other agent i', then all agents with a lower risk than i' are also accepted by i in the coalition.[11]

The third result, *iii/*, is in line with consecutivity. Take the less risky individual characterized by σ_1^2. He is accepted by any possible coalition and chooses the group that incurs the lowest risk

[9] Admittedly, the result does not rule out the existence of singletons within the core partition. Singletons are degenerate risk-sharing coalitions.

[10] Let us stress that the core partition is generically unique (see for instance Farrell and Scotchmer, 1988) but we need to provide a sufficient condition for uniqueness in order to proceed to our comparative static analysis.

[11] The consecutivity property is obtained in other models of risk-sharing agreements (see for instance Henriet and Rochet, 1987, and Legros and Newman, 2007).

premium. More risky individuals may not be accepted by agents characterized by low risks to pool resources in a same group. They thus pay a higher risk premium in other coalitions.[12]

Given the consecutivity property, from now on, we adopt the following convention that for any S_j^* and $S_{j'}^*$, $j' > j$ when $\sigma_i^2 < \sigma_{i'}^2$, $\forall i \in S_j^*, \forall i' \in S_{j'}^*$. Another way to express consecutivity is to say that a core partition can be characterized by a series of "pivotal agents", that is agents who are the most risky agents of the coalition they belong to:

Definition 4 *Given the coalition S_j^* of size n_j in the core-partition, the pivotal agent, defined by the integer $p_j \in \{1, ..., N\}$, associated with S_j^* and the next agent $p_j + 1$ are characterized by variances $\sigma_{p_j}^2$ and $\sigma_{p_j+1}^2$, respectively, such that:*

$$\pi(S_j^* \backslash \{p_j\}) \geq \pi(S_j^*) \text{ and } \pi(S_j^* \cup \{p_j + 1\}) > \pi(S_j^*).$$

Hence,

$$\sigma_{p_j}^2 \leq [2n_j - 1] \sum_{k \in S_j \backslash \{p_j\}} \frac{\sigma_k^2}{(n_j - 1)^2} \tag{6}$$

and

$$\sigma_{p_j+1}^2 > [2n_j + 1] \sum_{k \in S_j} \frac{\sigma_k^2}{{n_j}^2}. \tag{7}$$

A pivotal agent, associated with the $j - th$ coalition S_j^*, is by the consecutivity property, the most risky agent belonging to this club. He is the ultimate agent for which the net effect of his inclusion in the club is beneficial for all other (less risky) agents belonging to the club. Even though he increases the sum of risks in the club (i.e. the numerator of the risk premium), thus inflicting a loss to their welfare, his addition also increases its size (the denominator of the risk premium). Actually, his inclusion decreases the risk premium paid by each member of the coalition S_j^*. But if this coalition were to include the next agent, $p_j + 1$, as he is more risky than p_j, the net effect of his inclusion would be negative for all other agent of S_j^*. Therefore they prefer not to let him in. In the brief, adding the pivotal agent p_j generates the lowest possible risk premium paid by each member of the coalition S_j^*.

[12] Let us remark that the CARA specification is not crucial for the results obtained. If we assume an increasing and concave utility function $u(c)$ and infinitesimal shocks, then using the Arrow-Pratt approximation would yield the following risk premium for any individual i in group S

$$\pi_i(S) = -\frac{u''(w_i)}{u'(w_i)} \frac{\sum_{k \in S} \sigma_k^2}{|S|^2}.$$

Hence, the purpose for each individual remains to obtain the lowest ratio $\frac{\sum_{k \in S} \sigma_k^2}{|S|^2}$. However, when it is not assumed that $u(.)$ is CARA our mutual insurance rule is no more optimal.

Let us remark that the definition of a pivotal agent depends neither on the level of the variance nor on the degree of risk aversion. The conditions (6) and (7) can be rewritten as:

$$1 \leq \frac{[2n_j - 1]}{(n_j - 1)^2} \sum_{k \in S_j \backslash \{p_j\}} \prod_{z=k+1}^{p_j - 1} \frac{1}{\lambda_z} \tag{8}$$

and

$$1 > \frac{[2n_j + 1]}{n_j^2} \sum_{k \in S_j} \prod_{z=k+1}^{p_j} \frac{1}{\lambda_z}. \tag{9}$$

What matters in the formation of a coalition, is the heterogeneity of the exposure to risk measured by risk ratios. Consider the less risky agent, characterized by σ_1^2. If he forms a coalition, it is necessarily with a more risky agent. The best choice for him is agent 2 who adds the lowest increase in the common risk premium:

$$\begin{aligned} \pi(\{1,2\}) = \frac{\alpha}{8}(\sigma_1^2 + \sigma_2^2) = \frac{\alpha\sigma_1^2}{8}(1 + \lambda_2) \\ < \pi(\{1,i\}) = \frac{\alpha}{8}(\sigma_1^2 + \sigma_i^2) = \frac{\alpha\sigma_1^2}{8}(1 + \prod_{k=2}^{i} \lambda_k), \forall i > 2. \end{aligned}$$

This formula makes clear that agent 1 prefers to form a coalition with agent 2 than with any other agent in society, because he is relatively closer to him in terms of risk. Eventually, what matters for agent 1, is the sequence of risk ratios, that is the individual variances relative to his own. This reasoning can then be generalized to any n-agent coalition so as to obtain the core partition.

Given the consecutivity property of the core partition, the coalition S_j^* is fully defined by the two agents whose indices are $p_{j-1} + 1$ and p_j. In other words, the core partition is defined by the set of pivotal agents. Then we are able to offer the following:

Proposition 2 *The core partition is characterized by a set of J pivotal agents indexed by p_j satisfying (6) - (7) for $j = 1, ..., J - 1$ and $\sigma_{p_J}^2 = \sigma_N^2$.*

Remark that the last coalition is peculiar. Its pivotal agent is *per force* agent N who satisfies condition (6) and not condition (7). We refer to this ultimate coalition as the "residual" risk-sharing coalition.

Finally, Proposition 2 highlights that, depending on the risk-ratio schedule, the mutual insurance rule may lead to various risk-sharing groups. We could obtain the grand coalition belonging to the core if the risk heterogeneity was sufficiently small.[13]

[13] Again, our result relies on the insurance rule we adopt. Let us stress that if we take the insurance rule that gives individual i the following level of optimal consumption:

$$c_i = w_i + \frac{\sum_{k \in S} \varepsilon_k}{n} + \nu + \frac{\alpha}{2n}\left(\frac{\sum_{k \in S} \sigma_k^2}{n} - \sigma_i^2\right)$$

3.2 Particular Risk-Ratio Schedules.

We have just emphasized the importance of the risk-ratio schedule Λ characterizing a society $\mathbf{I}$ in the endogenous determination of the core partition of this society. In this subsection, we explore the link between patterns of the risk-ratio schedule and the characteristics of the core partition. This allows us to better understand how heterogeneity affects the way individuals congregate so as to share risk. Formally, we want to assess the impact of Λ on the series of pivotal agents, i.e. on the number and size of risk-sharing coalitions.

We restrict the analysis to risk-ratio schedules with simple monotonicity properties: either the sequence of λ_i increases, decreases or remains constant. We then offer the following

Proposition 3 *If the risk-ratio schedule* $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ *is such that:*

i/ $\lambda_i = \lambda$, $\forall i = 2, ..N$ *then* $n_j^* = n, \forall j = 1, ..., J-1.$

ii/ $\lambda_i \leq \lambda_{i+1}$, $\forall i = 2, ..N$ *then* $n_j^* \geq n_{j+1}^*, \forall j = 1, ..., J-1.$

iii/ $\lambda_i \geq \lambda_{i+1}$, $\forall i = 2, ..N$ *then* $n_j^* \leq n_{j+1}^*, \forall j = 1, ..., J-1.$

Proof. See Appendix. ■

This proposition makes clear that risk heterogeneity affects the core partition, that is the way agents collectively cope with risk. To understand this proposition, each individual makes his decision about membership with several principles in mind that we have previously uncovered. First, he prefers joining the least risky coalition; second, he prefers being joined by the less risky agents among those who are more risky than himself; third, when selecting (approving the admission of) members in his coalition, he cares about the risk ratios. Consecutivity, the ordering of coalition-risk premia, and the impact of risk ratios in determining the pivotal agent of any coalition are the key elements for understanding how a core partition relates to the risk ratio schedule.

First, consider that the risk ratios are constant and equal to λ. From (8) and (9), we see that inequalities determining the pivotal agent are identical for any club S_j. It turns out that coalitions in the core partition have the same size. In fact, it amounts to say that with constant risk

then the grand coalition belongs to the core of the coalition formation game *for any* risk-ratio schedule (see Theorem 2 of Baton and Lemaire, 1981). This rule corresponds to Pareto weights equal to $\ln \mu_i = \alpha w_i - \frac{\alpha^2}{2n}\sigma_i^2$ (see Equation (22) in Appendix 7.4). Indeed, this rule yields the following certainty-equivalent income

$$\omega_i(S) = w_i - \frac{\alpha}{2}\frac{\sigma_i^2}{n} - \frac{\alpha}{2}\sigma_\nu^2$$

which monotonously decreases with the size of the risk-sharing group. Hence, every individual i wishes to form a club encompassing the whole society.

ratios individuals, while deciding to form a risk-sharing group, individuals face the same trade-off whatever the level of their exposure to risk.

Second, consider that the risk ratios are increasing with the rank of individuals. The condition determining p_2 implies higher values of the risk ratios than the one determining p_1 (remember that the absolute values of variances of the first agents do not matter). Hence, pondering the benefit of increasing size and the cost of bearing risk with agents further in the distribution of risk, the size of S_2^* turns out to be smaller than the size of S_1^*. Repeating the argument, we find that the succeeding club sizes decrease.

Third, the case where the risk ratios are decreasing with the rank of individuals is easily understood by using a similar argument. Now the cost of forming the second risk-sharing group is lower yielding its size to be higher than for the first group.

3.3 Risk-sharing Partitions and Aggregate Risk Premium.

We aim to study the impact of an increase in risk, on the pattern of risk-sharing coalitions and on the resource cost of dealing with risk. We first define the aggregate risk premium.

Definition 5 *The aggregate risk premium associated with the core partition $\mathcal{P}$ is defined as:*

$$\begin{aligned} \overline{\pi}(\mathcal{P}) &= \frac{1}{N}\sum_{i=1}^{N}\pi_i = \frac{1}{N}\left(\sum_{j=1}^{J} n_j \pi(S_j)\right) \\ &= \frac{1}{N}\frac{\alpha}{2}\left(\sum_{j=1}^{J}\frac{1}{n_j}\sum_{k\in S_j}\sigma_k^2\right) + \frac{\alpha}{2}\sigma_\nu^2. \end{aligned} \tag{10}$$

The aggregate risk premium is an indicator of the willingness to pay for risk coping, at the society level. From equation (10), it clearly depends on the core partition.

We should expect that an increase in individual risk should lead to a higher aggregate risk premium. This is obviously true if the coalition formation is taken as given. However this is not necessarily true when agents form their risk-sharing coalitions. It may happen that the change in the whole core partition leads to different risk-sharing arrangements, the outcome of which is to decrease the average risk premium.

This counter-intuitive result is proven in the following

Proposition 4 *Consider two societies $\mathbf{I}$ and $\mathbf{I}'$ with ε_i, respectively ε_i', the idiosyncratic risk of any individual i in $\mathbf{I}$, respectively $\mathbf{I}'$. Assuming that ε_i SS-Dominates ε_i' for every $i = 1, ..., N$, then society $\mathbf{I}$ may be characterized by a higher aggregate risk premium than $\mathbf{I}'$:*

$$\overline{\pi}(\mathcal{P}') < \overline{\pi}(\mathcal{P}) \tag{11}$$

where $\mathcal{P}$ (resp. $\mathcal{P}'$) is the core partition associated with $\mathbf{I}$ ($\mathbf{I}'$).

Proof. See Appendix. ■

Proposition 4 highlights the fact that if endogenous formation of risk-sharing groups is taken into consideration than we cannot claim that all individuals pay a higher risk premium in a more risky society. We consider the case where society $\mathbf{I}'$ is associated with a lower number of risk-sharing coalitions than $\mathbf{I}$, even though agents face more risk (higher idiosyncratic variances) in $\mathbf{I}'$ than in $\mathbf{I}$, because as stressed in Proposition 3, this society is characterized by less risk heterogeneity. Hence, in society $\mathbf{I}'$, risk may be allocated in larger coalitions. In other words, in society $\mathbf{I}'$, individuals have the possibility to mutualize risk on a larger scale. This leads that the sum of these risk premia may be lower in the more risky society and some individuals will pay lower risk premium and consume more in this society.

4 Empirical implications of Risk-Sharing Groups Formation.

In this section, we link the evidence of partial risk sharing obtained by most empirical studies to the presence of risk-sharing groups. Partial risk sharing corresponds to the simultaneous rejection of perfect risk sharing and autarky (cf. Dercon and Krishnan, 2003).

Most empirical studies test for efficient risk-sharing by considering that the conditional expectation of individual consumption equals:

$$\mathbb{E}(c_{it}|\,\frac{Y_t^{\mathbf{I}}}{N}, y_{it}) = \kappa_i + \beta_i \frac{Y_t^{\mathbf{I}}}{N} + \zeta_i y_{it} \tag{12}$$

with $Y_t^{\mathbf{I}} \equiv \sum_{i=1}^{N} y_{it}$, and where β_i and ζ_i obtain using properties of conditional expectations of multivariate normal distributions (Ramanathan, 1993):

$$\beta_i = \frac{cov\left(\frac{Y_t^{\mathbf{I}}}{N}, c_{it}\right) var\left(y_{it}\right) - cov\left(y_{it}, c_{it}\right) cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)}{var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) var\left(y_{it}\right) - \left[cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)\right]^2} \tag{13a}$$

$$\zeta_i = \frac{cov\left(y_{it}, c_{it}\right) var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) - cov\left(\frac{Y_t^{\mathbf{I}}}{N}, c_{it}\right) cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)}{var\left(y_{it}\right) var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) - \left[cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)\right]^2}. \tag{13b}$$

Equation (12) builds on the well known result that with CARA utility function, individual consumption at the optimum is a linear function of both global resources and individual income (see for instance Tonwsend, 1994).[14]

Denoting by $\overline{\beta}_{\mathbf{I}} \equiv \frac{\sum_{i\in\mathbf{I}}\beta_i}{N}$ and $\overline{\zeta}_{\mathbf{I}} \equiv \frac{\sum_{i\in\mathbf{I}}\zeta_i}{N}$, we then offer the following

[14]See Appendix 7.4 for a formal presentation of the maximization program for delivering Pareto-optimal allocations.

Lemma 1 *Given a partition of the society, if we assume that individuals share risk optimally within coalitions, whatever the Pareto-oprimal risk-sharing rule, we get*

$$\lim_{N \longrightarrow +\infty} \overline{\beta}_{\mathbf{I}} = 1 - \frac{1}{\overline{n}_J}$$

$$\lim_{N \longrightarrow +\infty} \overline{\zeta}_{\mathbf{I}} = \frac{1}{\overline{n}_J}$$

with $\overline{n}_J$ the average size of risk-sharing groups.

Proof. See Appendix. ■

It turns out that risk-sharing group membership is a crucial determinant of the value of β_i and ζ_i. We immediately see that if the grand coalition is formed then $\overline{n}_J = N$ implying that $\lim_{N \longrightarrow +\infty} \overline{\beta}_{\mathbf{I}} = 1$ and $\lim_{N \longrightarrow +\infty} \overline{\zeta}_{\mathbf{I}} = 0$. While if individuals decide to pool risk in smaller groups than the whole society $\mathbf{I}$ then $\overline{n}_J < N$ implying that $\lim_{N \longrightarrow +\infty} \overline{\beta}_{\mathbf{I}} \neq 1$ and $\lim_{N \longrightarrow +\infty} \overline{\zeta}_{\mathbf{I}} \neq 0$. Most empirical studies assume that the relevant unit to test for efficient risk sharing is the grand coalition. This assumption may be inaccurate and may explain why the null hypothesis $\overline{\zeta}_{\mathbf{I}} = 0$ is rejected.

We will thus consider that $\overline{\zeta}_{\mathbf{I}}$ measures the extent of risk sharing in this society. A higher $\overline{\zeta}_{\mathbf{I}}$ means that an individual on average benefits from lower risk sharing. This is congruent with the interpretation of the estimated value of ζ as a measure of the extent of risk sharing (see for instance Jalan and Ravallion, 1999, and Suri, 2009).

Characterizing the relationship between heterogeneity in risk exposure and the size of risk-sharing coalitions, our setup helps to understand the impact of heterogeneity on the extent of risk sharing. Lemma 1 allows us to prove the following

Proposition 5 *For two societies $\mathbf{I}$ and $\mathbf{I}'$, $\mathbf{I}$ being characterized by $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ and $\mathbf{I}'$ being characterized by $\Lambda' = \{\lambda_2', \lambda_3', ..., \lambda_N'\}$, if $\lambda_i < \lambda_i'$, $\forall i = 2, ..., N$, then the extent of risk sharing is higher in society $\mathbf{I}$ than in society $\mathbf{I}'$.*

Proof. See Appendix. ■

This proposition highlights the crucial impact of risk heterogeneity on the allocation of risk in any society. In the more heterogenous society $\mathbf{I}'$, individuals share risk in smaller coalitions, thus diminishing the extent of risk sharing.

Let us provide an intuition for the proof of Proposition 5 by taking the special case where $\lambda_i = \lambda_{i+1} = \lambda$, $\lambda_i' = \lambda_{i+1}' = \lambda'$ and $\lambda < \lambda'$ whatever $i = 2, ..., N$. Consider agent 1 in society $\mathbf{I}$. Taking into account that the decision for membership only depends on the risk ratios, and pondering the trade-off between the benefit of size and the cost of higher marginal relative risk, agent 1 prefers being included in a (weakly) larger risk-sharing coalition in society $\mathbf{I}$ than in society

$\mathbf{I}'$. From Proposition 3, the agent following the first pivotal agent faces the same trade-off as agent 1. Hence the second club is of the same size than the first club, and consequently is of a larger size in society $\mathbf{I}$ than in society $\mathbf{I}'$. Repeating the argument, we find that the number of non-residual clubs is (weakly) reduced in the core partition of society $\mathbf{I}$ compared to the core partition of society $\mathbf{I}'$. The case of decreasing and increasing λ_is can similarly be dealt with.

Propositions 4 and 5 highlight that both aggregate risk premium and the extent of risk sharing must both be considered for the assessment of risk sharing when risk-sharing coalitions are endogenous. Take two societies $\mathbf{I}$ and $\mathbf{I}'$ with any ε_i SS-Dominating ε_i' and the risk-ratio schedule in $\mathbf{I}$ being characterized by higher risk ratios than the one in $\mathbf{I}'$ such that the aggregate risk premium and the extent of risk sharing may both be higher in society $\mathbf{I}'$. A larger extent of risk sharing is commonly considered as desirable when it is assumed that agents are risk averse. However, a higher aggregate risk premium is viewed as worse insurance outcome as it would lead to a lower certainty-equivalent income. Hence, reasoning on the variation of the aggregate risk premium or the extent of risk sharing separately would draw contradictory conclusions about the impact of an increase in risk on risk sharing performances, at the society level.

5 Heterogenous Risk Aversion.

Recent advances in the empirical literature support the evidence of heterogenous risk preferences (see for instance Ogaki and Zhang, 2001, Mazzocco, 2004, Dubois, 2006, Mazzocco and Saini, 2009, Chiappori *et alii,* 2011). In this section, we study the impact of heterogenous risk aversion on the formation of risk-sharing groups and show how the properties of the partition previously found remain valid.

The model we study is similar to the above setting with the two following modifications: First, individuals face the same exposure to risk σ_ε^2; second, individuals differ with respect to risk aversion. Without loss of generality, we index individuals as follows: for i and $i' = 1, .., N$ with $i > i'$ then $\frac{1}{\alpha_i} > \frac{1}{\alpha_{i'}}$. The inverse of risk aversion $\frac{1}{\alpha}$ being defined as risk tolerance, a lower indexed individual is characterized by a lower risk tolerance.

Considering the consumption function given by equation (1), under these assumptions, the indirect utility function for individual i in S_j obtains:

$$V_i(S_j) = -\frac{1}{\alpha_i} e^{-\alpha_i [w_i - \frac{1}{2}\frac{1}{\alpha_i}\frac{n_j^2}{(\sum_{k\in S}\frac{1}{\alpha_k})^2}\left(\sigma_\nu^2 + \frac{\sigma_\varepsilon^2}{n_j}\right)]}.$$

We denote by $\pi_i(S_j)$ the risk premium evaluated by individual i when belonging to S_j:

$$\pi_i(S_j) = \frac{\alpha_i}{2}\left(\frac{\frac{1}{\alpha_i}}{\frac{1}{n_j}\sum_{z\in S_j}\frac{1}{\alpha_z}}\right)^2\left(\sigma_\nu^2 + \frac{\sigma_\varepsilon^2}{n_j}\right). \tag{14}$$

The risk premium paid by agent i in her risk-sharing group expresses the trade-off on which the partition of society is based. Accepting a new member in a group pools risk among more individuals and therefore reduces the risk associated with the group (which is equal to $\sigma_\nu^2 + \frac{\sigma_\varepsilon^2}{n_j}$ in equation 14). However accepting a new member affects the coalition average risk tolerance: if this new member is less tolerant to risk than the average agent in the group, it will decrease the average risk tolerance and thus increase the transfer in order to be insured against risk. This is captured by the ratio $\frac{1}{\frac{1}{n_j}\sum_{z\in S_j}\frac{1}{\alpha_z}}$ in equation (14).

Defining the risk tolerance ratio between agent i and agent $i-1$ as $\chi_i = \frac{1/\alpha_i}{1/\alpha_{i-1}}$ for any $i = 2, ..., N$, the following proposition characterizes the core partition:

Proposition 6 *A core partition* $\mathcal{P}^* = \left\{S_1^*, ..., S_j^*, ..., S_J^*\right\}$ *exists and is characterized as follows:*

i/ *It is unique if*

$$\forall z = 2, ..., N-1, \quad \frac{\chi_{z+1} - \chi_z}{\chi_{z+1} - 1} \geq -\frac{1}{(z+1)}. \tag{15}$$

ii/ *It is consecutive, that is, if* i *and* $\widetilde{i}$ *both belong to* S_j^* *then* $\forall i'$, $i > i' > \widetilde{i}$, $i' \in S_j^*$.

iii/ *For any two individuals* $i \in S_j^*$ *and* $i' \in S_{j'}^*$*, such that* $\frac{1}{\alpha_i} > \frac{1}{\alpha_{i'}}$*, then* $\pi_i(S_j^*) \leqq \pi_{i'}(S_{j'}^*)$.

Proof. See Appendix. ■

This proposition is quite similar to Proposition 1. The sufficient condition for uniqueness parallels equation (5). Consecutivity is a characteristic of the core partition: two individuals are more likely to congregate the closer they are in terms of risk tolerance. Finally, the less risk tolerant an individual, the higher the risk premium that this individual is ready to pay. In other words, we can index coalitions according to the ordering of risk premia.

Notice however that the rationale behind the formation of coalitions and therefore of the core partition is different than in the case of heterogeneity with respect to risk. In the case of risk aversion heterogeneity, each agent wants to join a coalition formed by the most risk tolerant agents: This decreases her risk premium. But the most risk tolerant agents deny membership to agents with sufficiently high risk aversion who would demand a too high transfer. Hence the formation of the core partition is obtained by clustering the most risk tolerant agents into the first coalition, and proceeding sequentially for the other coalitions.

The core partition can be characterized by a set of pivotal agents defined as follows:[15]

$$\sum_{k\in S_j\backslash\{p_j\}}\frac{1}{\alpha_k}\left(-1+\frac{1}{(n_j-1)}\sqrt{n_j^2\frac{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)}{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j-1}\right)}}\right)\leq\frac{1}{\alpha_{p_j}}$$

and

$$\sum_{k\in S_j}\frac{1}{\alpha_k}\left(-1+\frac{1}{n_j}\sqrt{(n_j+1)^2\frac{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j+1}\right)}{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)}}\right)>\frac{1}{\alpha_{p_j+1}}.$$

As inequalities defining pivotal agents are homogenous of degree 0 with respect to $\frac{1}{\alpha_i}$, we also derive a relationship between the heterogeneity of risk tolerance and coalition's size:

Proposition 7 *If the risk-tolerance ratio* $\chi=\{\chi_2,\chi_3,...,\chi_N\}$ *is such that:*

i) $\chi_i=\chi$, $\forall i=2,...,N$, *then* $n_j^*=n,\forall j=1,...,J-1$.

ii) $\chi_i\leq\chi_{i+1}$, $\forall i=2,...,N$, *then* $n_j^*\leq n_{j+1}^*,\forall j=1,...,J-1$.

iii) $\chi_i\geq\chi_{i+1}$, $\forall i=2,...,N$, *then* $n_j^*\geq n_{j+1}^*,\forall j=1,...,J-1$.

Proof. See Appendix. ■

This proposition is similar to Proposition 3 and can be explained along the same line. What matters for the shaping of the partition is the heterogeneity with respect to risk aversion, captured by the risk tolerance ratios, which plays the same role as the risk ratios.[16]

In brief, our various propositions are robust to the nature of heterogeneity. This heterogeneity may apply to the utility functions of agents or to the stochastic environment they face, yet it will trigger identical behaviors which will lead agents to sort themselves into distinct risk-sharing groups (even though it may happen that the grand coalition forms). Importantly, the segmentation of society into different risk-sharing coalitions depend not on the *levels* of idiosyncratic characteristics such as the exposures to risk or risk aversion coefficients, but on heterogeneity as expressed by the risk ratios or the risk-tolerance ratios.

[15] Let us consider $\Omega(\sigma_\nu^2)=\left(-1+\frac{1}{n_j}\sqrt{(n_j+1)^2\frac{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j+1}\right)}{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)}}\right)$. It is easy to check that it monotonously increases with σ_ν^2, $\forall n_j\geq1$ and $0<\Omega(0)=\left(-1+\frac{1}{n_j}\sqrt{(n_j+1)n_j}\right)<1$ and $\lim_{\sigma_\nu^2\longrightarrow+\infty}\Omega(\sigma_\nu^2)=\frac{1}{n_j}$.

[16] It is irrelevant to search for a proposition equivalent to Proposition 4. Comparing aggregate risk premia obtained in societies which are not similar in risk aversion and thus value differently the protection with respect to risk has no economic meaning.

6 Conclusion.

Non-financial risk-sharing arrangements are widely used in developing economies. In the absence of proper and well-functioning financial markets, agents rely on informal insurance schemes, often based on a social or geographical (the “village”) proximity. Hence it is legitimate to ask how are designed the risk-sharing mechanisms in a society and what are their properties and consequences.

This justifies an inquiry into the endogenous formation of risk-sharing coalitions. Considering a society without financial markets and relying on a particular insurance rule, we study the endogenous formation of risk-sharing coalitions. Agents can form any possible group but commit to remaining in their chosen group whatever the realization of idiosyncratic shocks.

Heterogeneities in the variances of the idiosyncratic shocks, and in risk aversion are successively studied. First we obtain a characterization of the core partition of society with respect to risk, depending on the differentiated idiosyncratic risks born by individuals. It is unique (under some mild assumption), and consecutive: a coalition integrates agents of relatively similar risks. There is perfect risk-sharing within a coalition. However, there is no full insurance across society. In other words, the amplitude of risk sharing cannot be studied without precisely taking into account the memberships of risk-sharing groups and their differences.

Turning to the discussion of the role of risk heterogeneity on the segmentation of society and focusing on three special cases, we characterize the relationship between the characteristics (i.e. number, sizes and memberships) of risk-sharing coalitions and the distribution of risk across society.

When the segmentation in risk-sharing coalitions of societies differing in their risk heterogeneity are compared, we prove that the extent of risk sharing assimilated to the average size of coalitions decreases with this heterogeneity. The link between partial risk sharing and risk heterogeneity comes from the partition of society into different risk-sharing coalitions shaped by risk heterogeneity. Finally a more risky society (in the sense of second-order stochastic dominance) may devote less resources to risk sharing than a less risky one as it may be less heterogenous, thus less segmented, and therefore better able to pool individual risks. This illustrates a tension between the levels of individual variances and their ratios (which express risk heterogeneity). When heterogeneity with respect to risk aversion is considered, similar propositions obtain.

The present research proves how coalition theory tools can be applied to study the functioning of an economy in the presence of uncertainty when agents are risk-averse. It can be extended along several lines, where these tools are also of potential interest.

First, reversing the present logic which takes as given the risk-sharing rule and then look for the endogenous membership of risk-sharing groups, it would be valuable to address the issue of recontracting the risk-sharing rule once risk-sharing groups are formed.

Second, the assumption of full commitment could be relaxed so as to assess the impact of defection on the number and the size of risk-sharing coalitions forming the core partition.

Third, our paper shows that sorting individuals into risk-sharing coalitions affects the extent of risk sharing over the whole society. This suggests to empirically search for boundaries of groups and their impacts on the relationship between society's heterogeneity and the degree of partial risk sharing.

References

[1] Arcand, J.-L., and M. Fafchamps, "Matching in Community-Based Organizations", *Journal of Development Economics*, forthcoming.

[2] Arrow, K., 1964, "The Role of Securities in the Optimal Allocation of Risk Bearing", *Review of Economic Studies*, 31, 91-96.

[3] Attanasio, O., and S. Davis, 1996, "Relative Wage Movements and the Distribution of Consumption", *Journal of Political Economy*, 104, 1227-1262.

[4] Banerjee, S., H. Konishi and T. Sonmez, 2001, "Core in a Simple Coalition Formation Game", *Social Choice and Welfare*, 18, 135-153.

[5] Baton, B., and J. Lemaire, 1981, "The Core of a Reinsurance Market", *Astin Bulletin*, 12, 57-71.

[6] Bold, T., 2009, "Implications of Endogenous Group Formation for Efficient Risk Sharing", *The Economic Journal*, 119, 562-591.

[7] Borch, R., 1962, "Equilibrium in a Reinsurance Market", *Econometrica*, 30, 424-444.

[8] Bramoullé, Y., and R. Kranton, 2007, "Risk-Sharing Networks", *Journal of Economic Behavior and Organization*, 64, 275-294.

[9] Chiappori, P.-A., and Ph. Reny, 2005, "Matching to Share Risk", mimeo.

[10] Chiappori, P.-A., Samphantharak, K., Schulhofer-Wohl, S., and R. Townsend, 2011, "Heterogeneity and Risk Sharing in Village Economies", Federal Reserve Bank of Minneapolis, Working Paper 683.

[11] Cochrane, J., 1991, "A Simple Test of Consumption Insurance", *Journal of Political Economy*, 99, 957-976.

[12] Conley, T., and Ch. Udry, 2010, "Learning About a New Technology: Pineapple in Ghana", *American Economic Review*, 100, 35-69.

[13] Dercon, S., 2004, "Income Risk, Coping Strategies and Safety Nets", in S. Dercon (Ed.), *Insurance against Poverty*, Oxford University Press, Oxford: U.K..

[14] Dercon S., and P. Krishnan, 2003,"Risk Sharing and Public Transfers", *The Economic Journal*, 113, C86-C94.

[15] Dercon, S., J. De Weerdt, T. Bold, and A. Pankhurst, 2006, “Group-Based Funeral Insurance in Ethiopia and Tanzania”, *World Development*, 34, 685-703.

[16] Dubois, P., 2002, “Consommation, partage de risque et assurance informelle : développements théoriques et tests empiriques récents”, *L'Actualité économique*, 116-149.

[17] Dubois, P., 2006, “Heterogeneity of Preferences, Limited Commitment and Coalitions: Empirical Evidence on the Limits to Risk Sharing in Rural Pakistan”, mimeo.

[18] Dubois, P., B. Jullien, and Th. Magnac, 2008, “Formal and Informal Risk Sharing in LDCs: Theory and Empirical Evidence”, *Econometrica*, 76, 679-725.

[19] Fafchamps, M., 2008, “Risk Sharing Between Households”, in J. Benhabib, A. Bisin and M. Jackson eds, *Handbook of Social Economics*, Elsevier.

[20] Fafchamps, M., and S. Lund, 2003, “Risk-Sharing Networks in Rural Philippines”, *Journal of Development Economics*, 71, 261-287.

[21] Farrell, J., and S. Scotchmer, 1988,“Partnerships”, *Quarterly Journal of Economics*, 103, 279-297.

[22] Genicot, G., 2006, “Does Wealth Inequality Help Informal Insurance?”, mimeo.

[23] Genicot, G., and D. Ray, 2003, “Group Formation in Risk-Sharing Arrangements”, *Review of Economic Studies*, 40, 87-113.

[24] Ghatak, M., 1999, “Group Lending, Local Information and Peer Selection”, *Journal of Development Economics*, 60, 27-53.

[25] Hayashi, F., J. Altonji and L. Kotlikoff, 1996, “Risk Sharing between and within Families”, *Econometrica*, 64, 261-294.

[26] Henriet, D., and J.-Ch. Rochet, 1987, “Some Reflections on Insurance Pricing”, *European Economic Review*, 31, 863-885.

[27] Jalan, J., and M. Ravallion, 1999, “Are the Poor Less Well Insured? Evidence on Vulnerability to Income Risk in Rural China”, *Journal of Development Economics*, 58, 61-81.

[28] Kazianga, H., and Ch. Udry, 2005, “Consumption Smoothing ? Livestock, Insurance and Drought in Rural Burkina Faso”, mimeo.

[29] Legros, P., and A. Newman, 2007, "Beauty is a Beast, Frog is a Prince: Assortative Matching with Nontransferabilities", *Econometrica*, 75, 1073-1102.

[30] Mace, B., 1991,"Full Insurance in the Presence of Aggregate Uncertainty", *Journal of Political Economy*, 99, 928-956.

[31] Mazzocco, M., 2004, "Savings, Risk Sharing and Preferences for Risk", *American Economic Review,* 94, 1169-1182.

[32] Mazzocco, M., and S. Saini, 2009, "Testing Efficient Risk Sharing with Heterogenous Risk Preferences", mimeo.

[33] Ogaki, M., and Q. Zhang, 2001, "Decreasing Relative Risk Aversion and Tests of Risk Sharing", *Econometrica*, 69, 515-526.

[34] Ramanathan, R., *Statistical Methods in Econometrics*, Academic Press, 1993.

[35] Ray, D., *Development Economics*, Princeton University Press, 1998.

[36] Suri, T., 2009, "Spillovers in Village Consumption: Testing the Extent of Partial Insurance", mimeo.

[37] Taub, B., and H. Chade, 2002, "Segmented Risk in a Continuous-Time Setting", *Economic Theory*, 20, 645-675.

[38] Townsend, R., 1994, "Risk and Insurance in Village India", *Econometrica*, 62, 539-591.

[39] Townsend, R., 1995, "Consumption Insurance: An Evaluation of Risk-Bearing Systems in Low-Income Economies", *Journal of Economic Perspectives*, 9, 83-102.

[40] Wilson, R., 1968, "The Theory of Syndicates", *Econometrica*, 36, 119-132.

7 Appendix.

7.1 Proof of Proposition 1.

Existence. Given the value of $V_i\left(S_j\right)$, if for the two groups S_j and $S_{j'}$, we have:

$$V_i\left(S_j\right) \geq V_i\left(S_{j'}\right) \Longleftrightarrow \frac{\alpha^2}{2\left(n_j^2\right)} \sum_{k \in S_j} \sigma_k^2 \leq \frac{\alpha^2}{2\left(n_{j'}^2\right)} \sum_{k \in S_{j'}} \sigma_k^2$$

then we have:

$$V_{i'}\left(S_j\right) \geq V_{i'}\left(S_{j'}\right), \forall i' \in \mathbf{I}.$$

This implies that the common ranking property is satisfied, that is:

$$\forall i, k \in \mathbf{I},\ V_i\left(S_j\right) \geq V_i\left(S_{j'}\right) \Leftrightarrow V_k\left(S_j\right) \geq V_k\left(S_{j'}\right).$$

According to Banerjee *et al.* (2001), the common ranking property implies that a core partition exists.

Proof of (ii): Consecutivity.

By contradiction, let us consider a core-partition $\mathcal{P}^*$ characterized by some non consecutive groups, that is, there exist individuals $i, \widetilde{i} \in S_j^*$ and $i' \in S_{j'}^*$ with $i < i' < \widetilde{i}$.

Suppose first that $\pi(S_j^*) \geq \pi(S_{j'}^*)$. As $i < i' < \widetilde{i} \Longleftrightarrow \sigma_i^2 < \sigma_{i'}^2 < \sigma_{\widetilde{i}}^2$, we have $\pi(S_{j'}^*) > \pi((S_{j'}^* \backslash \{i'\}) \cup \{i\})$, which leads to

$$\forall z \in (S_{j'}^* \backslash \{i'\}) \cup \{i\},\ V_z((S_{j'}^* \backslash \{i'\}) \cup \{i\}) > V_z(\mathcal{P}^*).$$

Second, assume that $\pi(S_{j'}^*) \geq \pi(S_j^*)$. We have $\pi(S_j^*) > \pi((S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\})$, which leads to

$$\forall z \in (S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\},\ V_z((S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\}) > V_z(\mathcal{P}^*).$$

Hence a contradiction with the fact that $\mathcal{P}^*$ is assumed to be a core-partition.

Proof of (i): Uniqueness.

Let us define p_j the most risky agent of the consecutive group $S_j \backslash \{p_j\}$ with size $\widetilde{n}_j = n_j - 1$ satisfying the two following inequalities:

$$\sigma_{p_j}^2 \leq [2\widetilde{n}_j + 1] \sum_{k \in S_j \backslash \{p_j\}} \frac{\sigma_k^2}{\widetilde{n}_j^2}$$

and

$$\sigma_{p_j+1}^2 > [2\widetilde{n}_j + 3] \sum_{k \in S_j} \frac{\sigma_k^2}{(\widetilde{n}_j + 1)^2}.$$

Let us consider the consecutive group S_j whose lowest-individual-risk agent is i. Given the definition of the most risky agent, we can introduce the two following functions: $\Gamma(\widetilde{n}) = \frac{\widetilde{n}}{2\widetilde{n}+1}$ and $\Theta(i,\widetilde{n}) = \frac{1}{\widetilde{n}} \frac{\sum_{k=i}^{i+\widetilde{n}-1} \sigma_k^2}{\sigma_{i+n}^2}$ with $\widetilde{n} = 1, ..., N-i+1$. Let us denote by $\widetilde{n}^*(i)+1$ the size of group S_j such that:

$$\Gamma(\widetilde{n}^*(i)) \leq \Theta(i, \widetilde{n}^*(i))$$

and

$$\Gamma(\widetilde{n}^*(i)+1) > \Theta(i, \widetilde{n}^*(i)+1).$$

It is easy to check that $\Gamma(\widetilde{n})$ is an increasing function of $\widetilde{n}$ and $\Gamma(1) = \frac{1}{3}$. Given $\Theta(i,1) = 1 > \Gamma(1)$, if $\Theta(i,\widetilde{n})$ is decreasing with respect to $\widetilde{n}$ whatever $i \in \mathbf{I}$ and $\widetilde{n} \leq N-i$, then $\widetilde{n}^*(i)$ is unique as $\Gamma(\widetilde{n}) \leq \Theta(i,\widetilde{n})$ for $\widetilde{n} \leq \widetilde{n}^*(i)$ and $\Gamma(\widetilde{n}) > \Theta(i,\widetilde{n})$ for $\widetilde{n} > \widetilde{n}^*(i)$.

The function $\Theta(i,\widetilde{n}(i))$ is decreasing if and only if:

$$\Delta\Theta(i,\widetilde{n}) \equiv \Theta(i,\widetilde{n}(i)+1) - \Theta(i,\widetilde{n}(i)) = \frac{1}{\widetilde{n}+1} \frac{\sigma_{i+\widetilde{n}}^2 + \sum_{k=i}^{i+\widetilde{n}-1} \sigma_k^2}{\sigma_{i+\widetilde{n}+1}^2} - \frac{1}{\widetilde{n}} \frac{\sum_{k=i}^{i+\widetilde{n}-1} \sigma_k^2}{\sigma_{i+\widetilde{n}}^2} < 0 \Longleftrightarrow$$

$$\psi(i,\widetilde{n}) = \widetilde{n}\sigma_{i+\widetilde{n}}^2 - \left((\widetilde{n}+1) \frac{\sigma_{i+\widetilde{n}+1}^2}{\sigma_{i+\widetilde{n}}^2} - \widetilde{n} \right) \left(\sum_{k=i}^{i+\widetilde{n}-1} \sigma_k^2 \right) < 0.$$

Let us consider the function $\psi(i,\widetilde{n})$. It is negative for all i, $\widetilde{n} \leq N-i$ if

$$\psi(i,1) = \sigma_{i+1}^2 - \left(2\frac{\sigma_{i+2}^2}{\sigma_{i+1}^2} - 1 \right) \left(\sigma_i^2 \right) \leq 0 \text{ and } \Delta\psi(i,\widetilde{n}) \equiv \psi(i,\widetilde{n}+1) - \psi(i,\widetilde{n}) \leq 0.$$

Defining $\lambda_{i+1} = \frac{\sigma_{i+1}^2}{\sigma_i^2}$, the inequality $\psi(i,1) \leq 0$ is equivalent to

$$\frac{\left(\frac{\sigma_{i+1}^2 - \sigma_i^2}{\sigma_i^2} \right) - \left(\frac{\sigma_{i+2}^2 - \sigma_{i+1}^2}{\sigma_{i+1}^2} \right)}{\left(\frac{\sigma_{i+2}^2 - \sigma_{i+1}^2}{\sigma_{i+1}^2} \right)} = \frac{\lambda_{i+1} - \lambda_{i+2}}{\lambda_{i+2} - 1} \leq 1. \tag{16}$$

Moreover, $\forall \widetilde{n} \geq 1$, $\Delta\psi(i,\widetilde{n}) \leq 0$ is equivalent to

$$\Delta\psi(i,\widetilde{n}) = ((\widetilde{n}+1)\lambda_{i+\widetilde{n}+1} - (\widetilde{n}+2)\lambda_{i+\widetilde{n}+2} + 1)(\sigma_{i+\widetilde{n}}^2 + \left(\sum_{k=i}^{i+\widetilde{n}-1} \sigma_k^2 \right)) \leq 0 \Leftrightarrow$$

$$\frac{\lambda_{i+\widetilde{n}+1} - \lambda_{i+\widetilde{n}+2}}{\lambda_{i+\widetilde{n}+2} - 1}(\widetilde{n}+1) \leq 1.$$

Defining $z \equiv i + \widetilde{n} + 1$, we can rewrite this inequality as follows:

$$\frac{\lambda_z - \lambda_{z+1}}{\lambda_{z+1} - 1}(z+1)(\frac{z-i}{z+1}) \leq 1.$$

As $0 \leq \frac{(z-i)}{(z+1)} \leq 1$, we deduce that if for all $z = 3, ..., N-1$, $\frac{\lambda_z - \lambda_{z+1}}{\lambda_{z+1} - 1}(z+1) \leq 1$, then $\triangle\psi(i,\widetilde{n}) \leq 0$.

Given equation (16), we deduce that if for all $z = 2, ..., N-1$, $\frac{\lambda_z - \lambda_{z+1}}{\lambda_{z+1}-1}(z+1) \leq 1$ then $\triangle\psi(i, \widetilde{n}) \leq 0$ and $\psi(i, 1) \leq 0$, $\forall i = 1, ..., N$.

Hence, when for all $z = 2, ..., N-1$, $\frac{\lambda_z - \lambda_{z+1}}{\lambda_{z+1}-1}(z+1) \leq 1$, we deduce that there is a unique size n_j for the club S_j.

Proof of (iii): Risk premium ordering.

Consider the first group S_1^*. Let us define the group $\pounds_j = \left\{1, ..., n_j^*\right\}$ which is consecutive, comprised of the lowest-individual-risk agents and has the same size as group S_j^*. From the definition of the core-partition, we know that, $\forall \pounds \subset \mathbf{I}$, $\forall z \in S_1^*$ and $\pounds$, $V_z(S_1^*) \geq V_z(\pounds)$ and in particular $\forall z \in S_1^*$ and $\pounds_j$, $\forall j = 2, ..., J$, $V_z(S_1^*) > V_z(\pounds_j)$ which means that $\forall \pounds_j$, $\pi(S_1^*) < \pi(\pounds_j)$. Moreover, given the consecutivity property, it is easy to show that $\pi(\pounds_j) < \pi(S_j^*)$, $\forall j > 1$. Hence, $\pi(S_1^*) < \pi(S_j^*)$. Considering the subset $\mathbf{I} \backslash (S_1^* \cup S_2^* \cup ... \cup S_j^*)$, the same argument can be applied for S_{j+1}^* leading to the result $\pi(S_1^*) < \pi(S_2^*) < \pi(S_3^*) < ... < \pi(S_j^*) < ... < \pi(S_{J-1}^*)$.

7.2 Proof of Proposition 3.

Let us first denote by $S^c(i)$ any consecutive group whose less risky individual is i. We will denote by $\widehat{n}(i)$ the size of $S^c(i)$ such that $\widehat{n}(i) = \arg\max V_i(S^c(i))$ in the subset $\mathbf{I} \backslash \{1, 2, ..., i-1\}$, for a risk ratio schedule Λ. Hence, $\widehat{n}(i)$ satisfies inequalities characterizing a pivotal agent:

$$\Gamma(\widehat{n}(i) - 1) \leq \Theta(i, \widehat{n}(i) - 1) \tag{17}$$

and

$$\Gamma(\widehat{n}(i)) > \Theta(i, \widehat{n}(i)). \tag{18}$$

From Proof of Proposition 1, we know that $\Gamma(n)$ is an increasing function of n and, under some condition, $\Theta(i, n)$ decreases with respect to n. We can rewrite $\Theta(i, n)$ as follows:

$$\Theta(i, n) = \frac{1}{n} \sum_{v=i}^{i+n-1} \prod_{z=v+1}^{i+n} \frac{1}{\lambda_z}.$$

$\Theta(i, n)$ is a function of i such that:

(i) When $\lambda_z = \lambda$, $\forall z = 2, ..., N$, then $\Theta(i, n) = \Theta(i', n)$ $\forall i, i'$.

(ii) When $\lambda_z \leq \lambda_{z+1}$, $\forall z = 2, ..., N$, then $\Theta(i, n) \geq \Theta(i', n)$ for $i < i'$.

(iii) When $\lambda_z \geq \lambda_{z+1}$, $\forall z = 2, ..., N$, then $\Theta(i, n) \leq \Theta(i', n)$ for $i < i'$.

Hence (i), (ii), (iii) and inequalities (17) and (18) lead to Proposition 3.

7.3 Proof of Proposition 4.

Let us consider the two following societies. In society $\mathbf{I'}$, there are N individuals characterized with $\sigma_i'^2 = 1$. Hence, $\mathcal{P}' = \{I'\}$. In society $\mathbf{I}$, n_1 individuals are characterized with σ_1^2 and n_2 individuals are characterized with σ_2^2 such that $1 > \sigma_2^2 > \sigma_1^2$. Let us choose σ_1^2 and σ_2^2 such that $\mathcal{P} = \{S_1^*, S_2^*\}$ with S_1^* (respectively S_2^*) comprised of the n_1 (respectively n_2) individuals with σ_1^2 (respectively σ_2^2). Hence, σ_1^2, σ_2^2, n_1, n_2 and x are such that

$$\frac{n_1\sigma_1^2 + x\sigma_2^2}{(n_1+x)^2} > \frac{n_1\sigma_1^2}{(n_1)^2} \text{ for all } x \in \{1, ..., n_2\}$$

which is equivalent to

$$\sigma_2^2 > \sigma_1^2 \frac{2n_1 + x}{n_1}.$$

As the RHS is an increasing function of x, a sufficient condition for this inequality to hold is

$$\sigma_2^2 > \sigma_1^2 \frac{2n_1 + n_2}{n_1}.$$

Thus, given both core partitions, we deduce that

$$\overline{\pi}(\mathcal{P}) = \frac{\alpha}{2}\frac{1}{N}\left(n_1\frac{n_1\sigma_1^2}{(n_1)^2} + n_2\frac{n_2\sigma_2^2}{(n_2)^2}\right) + \frac{\alpha}{2}\sigma_\nu^2 \text{ and } \overline{\pi}(\mathcal{P}') = \frac{\alpha}{2}\frac{1}{N} + \frac{\alpha}{2}\sigma_\nu^2.$$

In order to have $\overline{\pi}(\mathcal{P}) > \overline{\pi}(\mathcal{P}')$, σ_1^2 and σ_2^2 must be such that:

$$\sigma_1^2 + \sigma_2^2 > 1.$$

Clearly there exist σ_1^2 and σ_2^2 that satisfy the following inequalities:

$$1 > \sigma_1^2;\ 1 > \sigma_2^2;\ \sigma_1^2 + \sigma_2^2 > 1;\ \sigma_2^2 > \sigma_1^2\frac{2n_1 + n_2}{n_1}.$$

For example, take $\sigma_1^2 < \frac{n_1}{3n_1+n_2}$ which satisfies $1 > \sigma_1^2$. As $\sigma_1^2 > 0$, we have $1 > 1 - \sigma_1^2$. Notice that $\sigma_1^2 < \frac{n_1}{3n_1+n_2}$ is equivalent to $1 - \sigma_1^2 > \sigma_1^2\frac{2n_1+n_2}{n_1}$. So that for any σ_2^2 such that $1 > \sigma_2^2 > 1 - \sigma_1^2$, the four inequalities are satisfied.

7.4 Proof of Lemma 1.

We first present as a benchmark the case of risk-sharing within a given society when resources are allocated by a benevolent planner. Let us mention that we assume that individuals differ with respect to exposure to risk and also risk aversion.

Let the state of nature be denoted by $\epsilon_t^{\mathbf{I}} = (\nu_t, \varepsilon_{1t}, \ldots, \varepsilon_{jt}, \ldots \varepsilon_{Nt})$. We will denote by $Y_t^{\mathbf{I}}(\epsilon_t^{\mathbf{I}})$ the aggregate level of resources at date t:

$$Y_t^{\mathbf{I}}(\epsilon_t^{\mathbf{I}}) = \sum_{i=1}^{N} w_{it} + \sum_{i=1}^{N} \varepsilon_{it} + N\nu_t$$

Following the seminal paper by Townsend (1994), the planner's program can be expressed as follows:

$$\max_{\{c_{it}\}} U = \sum_{i=1}^{N} \mu_i \left(-\mathbb{E}_0\left[\frac{1}{\alpha_i}\sum_{t=1}^{T} \delta^{t-1} e^{-\alpha_i c_{it}}\right]\right)$$

subject to the following feasibility constraint at each date t:

$$c_t^{\mathbf{I}}(\epsilon_t^{\mathbf{I}}) \equiv \sum_{i=1}^{N} c_{it} \leq Y_t^{\mathbf{I}}(\epsilon_t^{\mathbf{I}})$$

where μ_i, $i = 1, ..., N$, denote the non-negative Pareto weights attached on the consumers. It turns out that Pareto-optimal consumptions can be written as follows[17]:

$$c_{it} = \frac{1}{\alpha_i}\left[\ln \mu_i - \frac{\sum_{k\in\mathbf{I}} \frac{\ln \mu_k}{\alpha_k}}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}}\right] + \frac{\frac{1}{\alpha_i} N}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}} \frac{Y^{\mathbf{I}}(\epsilon_t^{\mathbf{I}})}{N}, \ \forall i \in \mathbf{I}. \tag{19}$$

Given (19) the conditional expectation of individual consumption is used by econometricians when testing for the perfect risk sharing hypothesis:

$$\mathbb{E}(\, c_{it} |\, \frac{Y_t^{\mathbf{I}}(\epsilon_t^{\mathbf{I}})}{N}, y_{it}) = \kappa_i + \beta_i \frac{Y_t^{\mathbf{I}}}{N} + \zeta_i y_{it} \tag{20}$$

where the formulas of β_i and ζ_i are obtained by using properties of conditional expectations of multivariate normal distributions (Ramanathan, 1993):

$$\beta_i = \frac{cov\left(\frac{Y_t^{\mathbf{I}}}{N}, c_{it}\right) var\left(y_{it}\right) - cov\left(y_{it}, c_{it}\right) cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)}{var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) var\left(y_{it}\right) - \left[cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)\right]^2} \tag{21a}$$

$$\zeta_i = \frac{cov\left(y_{it}, c_{it}\right) var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) - cov\left(\frac{Y_t^{\mathbf{I}}}{N}, c_{it}\right) cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)}{var\left(y_{it}\right) var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) - \left[cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)\right]^2}. \tag{21b}$$

Given (19), some straightforward computations lead to the following

$$\beta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}} \frac{\left(N\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}} \sigma_m^2}{N}\right)\left(\sigma_\nu^2 + \sigma_i^2\right) - \left(N\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}} \sigma_m^2}{N^2}\right) - \left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)^2}$$

$$\zeta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}} \frac{\left(N\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}} \sigma_m^2}{N^2}\right) - \left(N\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}} \sigma_m^2}{N}\right)\left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}} \sigma_m^2}{N^2}\right) - \left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)^2}.$$

Hence,

$$\beta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}} N$$

and

$$\zeta_i = 0.$$

[17] If we considered a production sector and leisure choice, formulas of Pareto-optimal consumptions would not be affected if separable utility functions are assumed (see Townsend, 1994).

Second, let us now assume that optimal risk sharing takes place in subset $S \subset \mathbf{I}$, with $n \equiv card(S) < N$. It turns out that Pareto-optimal consumptions can now be written as follows

$$c_{it} = \frac{1}{\alpha_i}\left[\ln \mu_i - \frac{\sum_{k\in S}\frac{\ln \mu_k}{\alpha_k}}{\sum_{k\in S}\frac{1}{\alpha_k}}\right] + \frac{\frac{1}{\alpha_i}n}{\sum_{k\in S}\frac{1}{\alpha_k}}\frac{\sum_{k\in S}(w_{kt}+\varepsilon_{kt}+\nu_t)}{n}, \text{ for } i \in S. \tag{22}$$

If we still consider (20) and use the latter expression of c_{it} to compute (21a) and (21b), it turns out that coefficients β_i and ζ_i are equal to:

$$\beta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}\frac{(n\sigma_\nu^2 + \frac{\sum_{k\in S}\sigma_k^2}{N})\left(\sigma_\nu^2+\sigma_i^2\right) - \left(n\sigma_\nu^2+\sigma_i^2\right)\left(\sigma_\nu^2+\frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2+\sigma_i^2\right)\left(\sigma_\nu^2+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right) - \left(\sigma_\nu^2+\frac{\sigma_i^2}{N}\right)^2} \tag{23}$$

$$\zeta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}\frac{\left(n\sigma_\nu^2+\sigma_i^2\right)\left(\sigma_\nu^2+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right) - (n\sigma_\nu^2 + \frac{\sum_{k\in S}\sigma_k^2}{N})\left(\sigma_\nu^2+\frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2+\sigma_i^2\right)\left(\sigma_\nu^2+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right) - \left(\sigma_\nu^2+\frac{\sigma_i^2}{N}\right)^2} \tag{24}$$

Dividing by $(\sigma_i^2)^2$ both the numerator and denominator in (23) and (24) leads to:

$$\beta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}\frac{(n\frac{\sigma_\nu^2}{\sigma_i^2} + \frac{\sum_{k\in S}\sigma_k^2}{\sigma_i^2 N})\left(\frac{\sigma_\nu^2}{\sigma_i^2}+1\right) - \left(n\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}+\frac{1}{N}\right)}{\left(\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{\sigma_i^2 N^2}\right) - \left(\frac{\sigma_\nu^2}{\sigma_i^2}+\frac{1}{N}\right)^2} \tag{25}$$

$$\zeta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}\frac{\left(n\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{\sigma_i^2 N^2}\right) - (n\frac{\sigma_\nu^2}{\sigma_i^2} + \frac{\sum_{k\in S_j}\sigma_k^2}{\sigma_i^2 N})\left(\frac{\sigma_\nu^2}{\sigma_i^2}+\frac{1}{N}\right)}{\left(\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{\sigma_i^2 N^2}\right) - \left(\frac{\sigma_\nu^2}{\sigma_i^2}+\frac{1}{N}\right)^2}. \tag{26}$$

If we assume that $\lim_{N\to\infty}\frac{\sigma_N^2}{\sigma_1^2} < \infty$, this implies that

$$\lim_{N\to\infty}\frac{\sigma_N^2}{N\sigma_1^2} = 0.$$

Further, as $\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2\sigma_i^2} \leq \frac{\sigma_N^2}{\sigma_1^2 N}$ $\forall i = 1, ..., N$, we thus easily deduce that when $\lim_{N\to\infty}\frac{\sigma_N^2}{\sigma_1^2} < \infty$, then

$$\lim_{N\to\infty}\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2\sigma_i^2} = 0, \forall i = 1, ..., N.$$

If N tends to infinity, the following equalities obtain:

$$\beta_i \simeq \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}\frac{(n\frac{\sigma_\nu^2}{\sigma_i^2} + \frac{\sum_{k\in S}\sigma_k^2}{\sigma_i^2 N})\left(\frac{\sigma_\nu^2}{\sigma_i^2}+1\right) - \left(n\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}\right)}{\left(\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}\right) - \left(\frac{\sigma_\nu^2}{\sigma_i^2}\right)^2} \tag{27}$$

$$\zeta_i \simeq \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}\frac{\left(n\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}\right) - (n\frac{\sigma_\nu^2}{\sigma_i^2} + \frac{\sum_{k\in S}\sigma_k^2}{\sigma_i^2 N})\left(\frac{\sigma_\nu^2}{\sigma_i^2}\right)}{\left(\frac{\sigma_\nu^2}{\sigma_i^2}+1\right)\left(\frac{\sigma_\nu^2}{\sigma_i^2}\right) - \left(\frac{\sigma_\nu^2}{\sigma_i^2}\right)^2}. \tag{28}$$

As $\lim_{N\to\infty}\frac{\sigma_N^2}{\sigma_1^2}<\infty$ implying that $\lim_{N\to\infty}\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2\sigma_i^2}=0$ and $\lim_{N\to\infty}\frac{\sum_{k\in S}\sigma_k^2}{\sigma_i^2 Nn}=0$ $\forall i=1,...,N$, we get:

$$\beta_i \simeq \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}(n-1) \tag{29}$$

$$\zeta_i \simeq \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}}. \tag{30}$$

If we assume that each individual belongs to one risk-sharing coalition only, and society $\mathbf{I}$ is organized into J risk-sharing coalitions, denoting by $\overline{\beta}_{\mathbf{I}}\equiv\frac{\sum_{i\in\mathbf{I}}\beta_i}{N}$ and $\overline{\zeta}_{\mathbf{I}}\equiv\frac{\sum_{i\in\mathbf{I}}\zeta_i}{N}$ we immediately get:

$$\lim_{N\to+\infty}\overline{\beta}_{\mathbf{I}} = 1-\frac{1}{\overline{n}_J} \tag{31}$$

and

$$\lim_{N\to+\infty}\overline{\zeta}_{\mathbf{I}} = \frac{1}{\overline{n}_J} \tag{32}$$

with $\overline{n}_J\equiv\frac{N}{J}$.

7.5 Proof of Proposition 5.

We will denote by $S^c(i)$ the consecutive club whose lowest risky agent is individual i. Let us denote by $\widehat{n}(i|\,\Lambda)$ the size of $S^c(i)$ such that $\widehat{n}(i|\,\Lambda)=\arg\max V_i(S^c(i))$, for a risk ratio schedule Λ.

We first offer the following Lemma

Lemma 2 *For two societies $\mathbf{I}$ and $\mathbf{I}'$ characterized respectively by $\Lambda=\{\lambda_2,\lambda_3,...,\lambda_N\}$ and $\Lambda'=\{\lambda_2',\lambda_3',...,\lambda_N'\}$ with $\lambda_z<\lambda_z'$ for $z=2,...,N$, we have $\widehat{n}(i|\,\Lambda)\geq\widehat{n}(i|\,\Lambda')$.*

Proof. Let us denote by $\Theta(\overrightarrow{\lambda}_{i,n})\equiv\Theta(i,n)=\frac{1}{n}\sum\limits_{v=i}^{i+n-1}\prod\limits_{z=v+1}^{i+n}\frac{1}{\lambda_z}$ with $\overrightarrow{\lambda}_{i,n}=(\lambda_{i+1},\lambda_{i+2},...,\lambda_{i+n-1})$. Hence for two vectors $\overrightarrow{\lambda}_{i,n}$ and $\overrightarrow{\lambda'}_{i,n}$ where $\lambda_z'>\lambda_z$, $\forall z=i+1,...,i+n-1$, we have $\Theta(\overrightarrow{\lambda}_{i,n})>\Theta(\overrightarrow{\lambda'}_{i,n})$, $\forall i\in\mathbf{I}$ and $\forall n=1,...,N-i+1$. Given inequalities (17) and (18) and that $\Theta(\overrightarrow{\lambda}_{i,n})>\Theta(\overrightarrow{\lambda'}_{i,n})$, it is thus easy to deduce that the optimal size of the consecutive group beginning with agent i is larger under $\Lambda=\{\lambda_2,\lambda_3,...,\lambda_N\}$ than under $\Lambda'=\{\lambda_2',\lambda_3',...,\lambda_N'\}$. Hence, Lemma 1. ■

Lemma 3 *Let us denote by $p_{S^c(i)}$ the pivotal agent of any consecutive club $S^c(i)$. For any society $\mathbf{I}$, any $i'<i$ we have $p_{S^c(i)}>p_{S^c(i')}$.*

Proof. We know that $\sigma^2_{p_{S^c(i)}}$ satisfies

$$\sigma^2_{p_{S^c(i)}} \leq \left[2n_j^c-1\right]\sum_{k=i}^{p_{S^c(i)}-1}\frac{\sigma_k^2}{(n_j^c-1)^2} \tag{33}$$

and

$$\sigma^2_{p_{S^c(i)}+1} > \left[2n_j^c + 1\right] \sum_{k=i}^{p_{S^c(i)}} \frac{\sigma_k^2}{n_j^{c2}}. \tag{34}$$

Let us consider the consecutive club $S^c(i') = \{i', ..., p_{S^c(i)}+1\}$. By assumption on the individuals ordering, we have

$$\sum_{k=i}^{p_{S^c(i)}} \frac{\sigma_k^2}{n_j^c} > \sum_{k=i'}^{p_{S^c(i)}} \frac{\sigma_k^2}{n_j'^c} \text{ for any } i' < i.$$

Hence as $\frac{2n'+1}{n'} < \frac{2n+1}{n}$ for any $n' > n$, we thus have

$$\sigma^2_{p_{S^c(i)}+1} > \left[2n_j^c + 1\right] \sum_{k=i}^{p_{S^c(i)}} \frac{\sigma_k^2}{n_j^{c2}} > \left[2n_j'^c + 1\right] \sum_{k=i'}^{p_{S^c(i)}} \frac{\sigma_k^2}{n_j'^{c2}}, \text{ for any } i' < i.$$

We easily deduce that $p_{S^c(i)} > p_{S^c(i')}$ for any $i' < i$. ■

Let us now define $p_j^*(\Lambda)$ the pivotal agent of club S_j in the core partition associated to Λ. Let us consider individual 1. Using Lemma 2, for $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ and $\Lambda' = \{\lambda_2', \lambda_3', ..., \lambda_N'\}$ with $\lambda_z < \lambda_z'$ for $z = 2, ..., N$, we deduce that $p_1^*(\Lambda) \geq p_1^*(\Lambda')$. Using Lemma 3, we thus deduce that $p_2^*(\Lambda) \equiv p^*_{S^c(p_1(\Lambda)+1)}(\Lambda) > p_{S^c(p_1(\Lambda')+1)}(\Lambda)$. Using again Lemma 2 allows us to say that $p_{S^c(p_1(\Lambda')+1)}(\Lambda) \geq p^*_{S^c(p_1(\Lambda')+1)}(\Lambda') \equiv p_2^*(\Lambda')$. Hence $p_2^*(\Lambda) \geq p_2^*(\Lambda')$. Iterating this reasoning until $j = J$ allows us to say that $p_j^*(\Lambda) \geq p_j^*(\Lambda')$ for any $j = 1, ..., J$. Hence for any $i = 1, ..., N$ we thus deduce that the number of pivotal agents associated with Λ such that $p_j^*(\Lambda) \leq i$ compared to the number of pivotal agents associated with Λ' such $p_j(\Lambda') \leq i$ is higher for Λ than Λ'. This ends proof of Proposition 5.

7.6 Proof of Proposition 6.

Existence. It is easy to see that in our case the common ranking property is also satisfied. Hence, a core partition exists.

Proof of (ii): Consecutivity. By contradiction, let us consider a core-partition $\mathcal{P}^*$ characterized by some non consecutive groups, that is, there exist individual $i, \widetilde{i} \in S_j^*$ and $i' \in S_{j'}^*$ with $i < i' < \widetilde{i}$.

Suppose first that $\pi^z(S_j^*) \geq \pi^z(S_{j'}^*)$ for any $z = 1, ..., N$. As $i < i' < \widetilde{i} \Longleftrightarrow \frac{1}{\alpha_i} > \frac{1}{\alpha_{i'}} > \frac{1}{\alpha_{\widetilde{i}}}$, we have

$$\forall z \in (S_{j'}^* \backslash \{i'\}) \cup \{i\},\ V_z((S_{j'}^* \backslash \{i'\}) \cup \{i\}) > V_z(\mathcal{P}^*).$$

Second, assume that $\pi^z(S_{j'}^*) \geq \pi^z(S_j^*)$ for any $z = 1, ..., N$. We have

$$\forall z \in (S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\},\ V_z((S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\}) > V_z(\mathcal{P}^*).$$

Hence a contradiction with the fact that $\mathcal{P}^*$ is assumed to be a core-partition.

Proof of (i): Uniqueness.

Knowing that core partition satisfies the consecutivity property, an individual z is accepted if and only if

$$-\frac{n_j^2}{(\sum_{k\in S}\frac{1}{\alpha_k})^2}\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)\geq -\frac{(n_j+1)^2}{(\sum_{k\in S}\frac{1}{\alpha_k}+\frac{1}{\alpha_z})^2}\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{(n_j+1)}\right)$$

which amounts to

$$-\left(\frac{1}{\alpha_z}\right)^2 n_j^2\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)-\left(\frac{1}{\alpha_z}\right)2n_j^2(\sum_{k\in S}\frac{1}{\alpha_k})\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)+(\sum_{k\in S}\frac{1}{\alpha_k})^2\left((2n_j+1)\sigma_\nu^2+\sigma_\varepsilon^2\right)\geq 0.$$

For positive α_z, the LHS of the above inequality is positive if and only if

$$\frac{\sum_{k\in S_j}\frac{1}{\alpha_k}}{\frac{1}{\alpha_z}n_j}\left(-n_j+\sqrt{(n_j+1)^2\frac{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{(n_j+1)}\right)}{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)}}\right)\leq 1.$$

The aim is to show that the LHS of this inequality is monotonously increasing with the size of the coalition.

First, the expression $\left(-n_j+\sqrt{(n_j+1)^2\frac{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{(n_j+1)}\right)}{\left(\sigma_\nu^2+\frac{\sigma_\varepsilon^2}{n_j}\right)}}\right)$ is increasing with respect to n (we omit j for convenience) if and only if:

$$\begin{aligned}&2\left[n(n+1)+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}n\right]\left[n+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}\right]+\left[2n+1+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}\right]\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}\\&>2\left[n+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}\right]\sqrt{\left[n(n+1)+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}n\right]\left[n(n+1)+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}(n+1)\right]}.\end{aligned}\tag{35}$$

Using the fact that

$$\left[n(n+1)+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}(n+\frac{1}{2})\right]^2>\left[n(n+1)+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}n\right]\left[n(n+1)+\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}(n+1)\right]$$

we can show that inequality (35) is satisfied whatever $\frac{\sigma_\varepsilon^2}{\sigma_\nu^2}$ and n.

Second, we offer a sufficient condition such that the ratio $\frac{\sum_{k\in S_j}\frac{1}{\alpha_k}}{\frac{1}{\alpha_z}n_j}$ increases with respect to the size of the coalition S_j, i.e.

$$\frac{\sum_{k\in S_j}\frac{1}{\alpha_k}+\frac{1}{\alpha_z}}{\frac{1}{\alpha_{z+1}}(n_j+1)}-\frac{\sum_{k\in S_j}\frac{1}{\alpha_k}}{\frac{1}{\alpha_z}n_j}\geq 0$$

which is equivalent to

$$\left(n_j-\frac{\frac{1}{\alpha_{z+1}}}{\frac{1}{\alpha_z}}(n_j+1)\right)\left(\sum_{k\in S_j}\frac{1}{\alpha_k}\right)+n_j\frac{1}{\alpha_z}\geq 0.$$

Let us define $\Phi(i,n) = \left(n - \frac{\frac{1}{\alpha_{i+n+1}}}{\frac{1}{\alpha_{i+n}}}(n+1)\right)\left(\sum_{k=i}^{i+n-1}\frac{1}{\alpha_k}\right) + n\frac{1}{\alpha_{i+n}}$ and show that $\Phi(i,n) \geq 0$, $\forall n \geq 1$.

We have $\Phi(i,1) = (1 - \frac{\frac{1}{\alpha_{i+2}}}{\frac{1}{\alpha_{i+1}}}(2))\left(\frac{1}{\alpha_i}\right) + \frac{1}{\alpha_{i+1}}$. With $\chi_i = \frac{1/\alpha_i}{1/\alpha_{i-1}} < 1$ whatever i, $\Phi(i,1)$ is positive if and only if

$$1 \geq \frac{\chi_{i+2} - \chi_{i+1}}{1 - \chi_{i+2}}, \text{ whatever } i = 1, ..., N.$$

Let us show that $\Phi(i,n)$ is monotonously increasing with respect to n, that is,

$$\begin{aligned}\Phi(i,n+1) - \Phi(i,n) = &\left(n + 1 - \frac{\frac{1}{\alpha_{i+n+2}}}{\frac{1}{\alpha_{i+n+1}}}(n+2)\right)\left(\sum_{k=i}^{i+n-1}\frac{1}{\alpha_k} + \frac{1}{\alpha_{i+n}}\right)\\ &+ (n+1)\frac{1}{\alpha_{i+n+1}} - \left(n - \frac{\frac{1}{\alpha_{i+n+1}}}{\frac{1}{\alpha_{i+n}}}(n+1)\right)\left(\sum_{k=i}^{i+n-1}\frac{1}{\alpha_k}\right) - n\frac{1}{\alpha_{i+n}}\end{aligned}$$

which is equivalent to

$$\begin{aligned}\Phi(i,n+1) - \Phi(i,n) = &\left(\sum_{k=i}^{i+n-1}\frac{1}{\alpha_k}\right)\left(1 - \frac{\frac{1}{\alpha_{i+n+2}}}{\frac{1}{\alpha_{i+n+1}}}(n+2) + \frac{\frac{1}{\alpha_{i+n+1}}}{\frac{1}{\alpha_{i+n}}}(n+1)\right)\\ &+ \left(1 - \frac{\frac{1}{\alpha_{i+n+2}}}{\frac{1}{\alpha_{i+n+1}}}(n+2)\right)\frac{1}{\alpha_{i+n}} + (n+1)\frac{1}{\alpha_{i+n+1}}\\ &\Leftrightarrow\end{aligned}$$

$$\begin{aligned}\Phi(i,n+1) - \Phi(i,n) = &\left(\sum_{k=i}^{i+n-1}\frac{1}{\alpha_k}\right)(1 - \chi_{i+n+2}(n+2) + \chi_{i+n+1}(n+1))\\ &+ (1 - \lambda_{i+n+2}(n+2))\frac{1}{\alpha_{i+n}} + (n+1)\frac{1}{\alpha_{i+n+1}}.\end{aligned}$$

With $\chi_{i+n+1} \equiv \frac{\alpha_{i+n}}{\alpha_{i+n+1}}$, we have

$$\Phi(i,n+1) - \Phi(i,n) = \left(\left(\sum_{k=i}^{i+n-1}\frac{1}{\alpha_k}\right) + \frac{1}{\alpha_{i+n}}\right)(1 - \chi_{i+n+2}(n+2) + \chi_{i+n+1}(n+1)).$$

Hence,

$$\Phi(i,n+1) - \Phi(i,n) \geq 0 \Leftrightarrow 1 \geq (n+1)\frac{(\chi_{i+n+2} - \chi_{i+n+1})}{(1 - \chi_{i+n+2})}.$$

With $z \equiv i + n + 1$, we can rewrite this inequality as follows

$$1 \geq (\frac{z-i}{z+1})(z+1)\frac{(\chi_{z+1} - \chi_z)}{(1 - \chi_{z+1})}$$

As $0 \leq \frac{(z-i)}{(z+1)} \leq 1$, we deduce that if for all $z = 3, ..., N-1$, $\frac{\chi_z - \chi_{z+1}}{\chi_{z+1} - 1}(z+1) \leq 1$, then $\triangle\Phi(i,n) \geq 0$. We deduce that if for all $z = 2, ..., N-1$, $\frac{\chi_z - \chi_{z+1}}{\chi_{z+1} - 1}(z+1) \leq 1$ then $\triangle\Phi(i,n) \geq 0$ and $\Phi(i,1) \geq 0$, $\forall i = 1, ..., N$.

Proof of (iii): The proof is identical to Proof of (iii) of Proposition 1 except with the fact that the risk premium $\pi(S_j)$ must now be replaced by $\pi^z(S_j)$.

7.7 Proposition 7.

Let us define $\widetilde{\Gamma}(n) = \left(-n + \sqrt{(n+1)^2 \frac{\left(\sigma_\nu^2 + \frac{\sigma_\varepsilon^2}{n+1}\right)}{\left(\sigma_\nu^2 + \frac{\sigma_\varepsilon^2}{n}\right)}} \right)^{-1}$ and $\widetilde{\Theta}(i,n) = \frac{1}{n} \frac{\sum_{k=i}^{i+n-1} \frac{1}{\alpha_k}}{\frac{1}{\alpha_{i+n}}}$. We will denote by $\widehat{n}(i)$ the size of $S^c(i)$ such that $\widehat{n}(i) = \arg\max V_i(S^c(i))$ in the subset $\mathbf{I} \backslash \{1, 2, ..., i-1\}$, for a risk ratio schedule Λ. Hence, $\widehat{n}(i)$ satisfies inequalities characterizing a pivotal agent:

$$\widetilde{\Gamma}(\widehat{n}(i) - 1) \geq \widetilde{\Theta}(i, \widehat{n}(i) - 1) \tag{36}$$

and

$$\widetilde{\Gamma}(\widehat{n}(i)) < \widetilde{\Theta}(i, \widehat{n}(i)). \tag{37}$$

From Proof of Proposition 6, we deduce that $\widetilde{\Gamma}(n)$ is a decreasing function of n and, under some condition, $\Theta(i,n)$ increases with respect to n. We can rewrite $\Theta(i,n)$ as follows: $\frac{1}{n} \frac{\sum_{k=i}^{i+n-1} \frac{1}{\alpha_k}}{\frac{1}{\alpha_{i+n}}}$

$$\Theta(i,n) = \frac{1}{n} \sum_{v=i}^{i+n-1} \prod_{z=v+1}^{i+n} \frac{1}{\chi_z}.$$

$\Theta(i,n)$ is a function of i such that:

(i) When $\chi_z = \chi$, $\forall z = 2, ..., N$, then $\Theta(i,n) = \Theta(i',n)$ $\forall i, i'$.

(ii) When $\chi_z \leq \chi_{z+1}$, $\forall z = 2, ..., N$, then $\Theta(i,n) \geq \Theta(i',n)$ for $i < i'$.

(iii) When $\chi_z \geq \chi_{z+1}$, $\forall z = 2, ..., N$, then $\Theta(i,n) \leq \Theta(i',n)$ for $i < i'$.

Hence, (i), (ii), (iii) and inequalities (36) and (37) lead to Proposition 7.