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Endogenous Coalitions Formations Through Technology Transfers and Fair Prices

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ENDOGENOUS COALITIONS FORMATIONS THROUGH TECHNOLOGY TRANSFERS AND FAIR PRICES

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Abstract. We consider a situation in which members of an oligopoly have different technologies, which allow them to produce at different costs. Members may license their technology to other members. Using the Aumann-Drèze modification of the Shapley value, we compute fair prices for these licenses. We also study the problem of stability for these "licensing coalitions."

1. Introduction

The objective of this paper is to provide a model of coalition formation where firms have technologies that are perfectly replicable. Hence, within an asymmetric oligopoly, we are interested in technology transfers between firms. The profitability of technology transfers is a two-edged sword. Faced by a possible technology transfer between any of its competitors and resulting loss of its profit, a firm will try to disrupt such transfer by offering a more attractive deal to either firm[1].

Even though endogenous coalition formation is a central theme in cooperative game theory, much of this literature is of limited applicability to the study of oligopoly markets since the standard definition of the characteristic function ignores externalities among coalitions[2]. In these games, it is implicitly assumed that the payoffs levels members of a coalition can attain, are independent of the actions chosen by the players outside this coalition: the characteristic function gives the same value irrespective of how the other players are partitioned. By contrast, in this paper, the model explicitly describes a procedure in which individual players, when deciding to form a coalition, consider the consequences of their actions on the behavior of the other players.

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[1]See for example, La Manna [1993].

[2]Greenberg [1995] emphasizes that the characteristic function ignores the possibility of externalities.

In order to model this cooperative behavior, we shall determine the gains each coalition can obtain by the cooperation of its members and according to the behavioral assumption that every possible coalition of firms makes about firms that are not in the coalition. This is done by defining two different games in characteristic function form, as in Hart and Kurz [1983], each game corresponding to a different assumption about how the opponents of a particular coalition will respond. However, unlike Hart and Kurz, we do not assume that players evaluate the payoffs they receive in each coalition structure according to an extension to the Shapley Value, first analyzed by Owen [1977]. We rather assume that each coalition only gets its own worth, as in Aumann and Maschler [1964], Aumann and Drèze [1975] and Shenoy [1979].

For our purpose, we consider here an oligopoly, producing as homogeneous product. Different producers have different technologies, giving rise to different cost structures. The more efficient producers may (for a price) license their technology to others. If this happens, the purchaser will then have the same cost structure as the seller. The question is as to what price one or another of the firms may pay for these licenses.

There are two possible models to consider here:

(1) Several firms form a coalition. They then act as though they were a single firm, sharing the most efficient (least-cost) technology, and pooling their profits. In other words, we have here a *merger*.
(2) Several firms form a coalition. In fact, however, all they do is share technologies. Each one of them now uses the most efficient technology, but otherwise acts independently of the other firms in the coalition. The problem is then to determine the price the several firms must pay for the use of this technology.

While interpretation 1 above can reasonably be made, this is precisely the sort of behavior that anti-trust laws forbid. It is in fact possible that some mergers might be allowed, but mergers are complicated procedures, requiring all sorts of approval (from the shareholders? from the authorities?) and thus, even if approval is ultimately granted, the game would have to be modified to take into account the one-time costs of the merger. We will therefore study the situation of coalition formation where firms adopt a competitive behavior inside the coalition.

2. The Model

To simplify matters, we will assume both the demand and the cost functions are linear. By changing the units if necessary, we can assume that the demand function is given by

$$p = A - \sum_i q_i \tag{1}$$

where p is price, and q_i is the amount produced by firm i.

Assume next that each firm has a linear cost structure, and that firm i has unit costs c_i. Without loss of generality, we will assume that the producers are listed in order of efficiency, with the most efficient firms listed first, i.e.,

$$c_1 \le c_2 \le \ldots \le c_n$$

Then, for a Cournot-Nash equilibrium [1927, 1950], the general rule is that (at equilibrium) firm i will produce a quantity

$$q_i = \frac{A + \sum_j c_j - (n+1)c_i}{n+1} \tag{2}$$

(though admittedly some care should be taken to consider the case that some of the firms – the least efficient ones, i.e., those with highest costs – will produce nothing at all[3]).

Let us suppose a coalition S forms. The members of S will then all use the most efficient technology available to them; i.e., all will produce with unit costs

$$c_{\min} = \min \{c_i \mid i \in S\} \tag{3}$$

Each member of S will then produce the quantity

$$q_i = \frac{A + \sum_j c_j - (n+1)c_{\min}}{n+1} \tag{4}$$

Total production for a coalition S will then be

$$Q(S) = \frac{s\left(A + \sum_j c_j - (n+1)c_{\min}\right)}{n+1} \tag{5}$$

[3]The rule is that, if the q_n given by equation (2) is negative, then firm n is effectively driven out of the market. The firm will then produce 0, and the quantities given by (2) must be recalculated, replacing n by $n-1$, etc.

where s is the cardinality of S. We note, however, that in the summation term, $\sum_j c_j$, each firm j's cost c_j should be replaced by the lowest c_k among all k which belong to the same coalition as j does. Thus coalition S's production, and also its profits, will depend on the behavior of firms outside S. We therefore distinguish two special cases. Case 1 assumes that the firms outside S all act together, and case 2 assumes that they all act independently.

We choose to consider case 1 only: if coalition S forms, the complementary coalition, N-S, also forms. Here we consider the minimum a coalition S can guarantee that is we consider the worth possible scenario for the coalition S whatever the behavior of the N-S firms.

In that case, the s firms in our coalition will all be producing with the same unit costs, $c = c_{\min}$, while the other n-s firms will all be producing with unit costs k, given by

$$k = \min \{c_j \mid j \in (N - S)\} \tag{6}$$

i.e., the lowest unit costs among the members of N-S. Recall, now, that they are all producing independently.

Then, by the above, each member of S would be producing

$$q_i = \frac{A + (n - s)\, k - (n - s + 1)c}{n + 1} \tag{7}$$

while each member of N-S would produce

$$q_j = \frac{A + sc - (s + 1)k}{n + 1} \tag{8}$$

(assuming of course that these quantities are non-negative). Thus total production would be

$$Q = sq_i + (n - s)q_j = \frac{nA - (n - s)k - sc}{n + 1} \tag{9}$$

The price would then be

$$p = A - Q = \frac{A + (n - s)k + sc}{n + 1} \tag{10}$$

and profits, per unit, to each member firm of S would be

$$\pi_i = p - c = \left(\frac{A + (n-s)k - (n-s+1)c}{n+1} \right)^2 \tag{11}$$

(which turns out to be the same as q_i), and so profits to a single player i are ${q_i}^2$, and total profits to the coalition S will be

$$v(S) = sq_i^2 = \frac{s\left[A + (n-s)\,k - (n-s+1)\,c\right]^2}{(n+1)^2} \tag{12}$$

We can now treat this as the characteristic function of our (semi-cooperative) game.

3. The Results

We consider a 4-person game, with $A = 10$, and $c_i = 0$, 1, 2, and 3 respectively. If no cooperation is allowed, then we find

$$q_1 = \frac{10 + 6 - 5(0)}{5} = 3.2$$

Similarly, $q_2 = 2.2$, $q_3 = 1.2$, and $q_4 = 0.2$. Total production is then $Q = 6.8$, so $p = 3.2$, and the several players' profits are 10.24, 4.84, 1.44, and 0.04 respectively. We note parenthetically that firm 4 is essentially moribund because of its high costs, and really needs some technological help if it is to survive.

Suppose, now, that some cooperation is allowed, in the form of technology sharing. In that case, we find that, if the coalitions $\{1\}$ and $\{2, 3, 4\}$ form, then 2, 3, and 4 all produce with unit cost 1, so

$$q_1 = \frac{10 + 3 - 5(0)}{5} = 2.6$$

$$q_2 = \frac{10 + 3 - 5(1)}{5} = 1.6$$

And $q_3 = q_4 = 1.6$. Total production is $Q = 7.4$, so $p = 2.6$, and profits are 6.76, 2.56, 2.56, and 2.56. Thus $v(\{1\}) = 6.76$, $v(\{2, 3, 4\}) = 7.68$.

In a similar way, we compute $v(S)$ for the remaining coalitions, to obtain the characteristic function:

In each case, $Q(S)$ denotes the production by S assuming the structure $\{S,\ N\text{-}S\}$ forms.

Table 1

S	$Q(S)$	p	$v(S)$		S	$Q(S)$	p	$v(S)$
$\emptyset$	0		0		{2, 3}	2.8	2.4	3.92
{1}	2.6	2.6	6.76		{2, 4}	2.8	2.4	3.92
{2}	1.2	2.2	1.44		{3, 4}	1.6	2.8	1.28
{3}	0.4	2.4	0.16		{1, 2, 3}	7.5	2.5	18.75
{4}	0	2.5	0		{1, 2, 4}	7.2	2.4	17.28
{1, 2}	5.6	2.8	15.68		{1, 3, 4}	6.6	2.2	14.52
{1, 3}	4.8	2.4	11.52		{2, 3, 4}	4.8	2.6	7.68
{1, 4}	4.8	2.4	11.52		N	8	2	16

As may be seen, the function v is not superadditive. The reason is that, for example, coalition {1, 2, 3} would find it better to drive firm 4 out of the market than to license to it the powerful technology owned by firm 1. Nevertheless the function can be used for the usual game-theoretic purposes. For example, assume the coalition {1, 2} forms. We see that, following the Aumann-Drèze model, value to firm 1 is (15.68+6.76-1.44)/2 = 10.5, and to firm 2 it is (15.68-6.76+1.44)/2 = 5.18. In fact, using firm 1's technology, they would each produce 2.8 units, and obtain a profit of 7.84. Thus, in this case, firm 2 should pay firm 1 a licensing fee of 2.66 units for the right to use this technology.

Suppose next that coalition {1, 2, 4} forms. In that case, the value of the restricted game is (11, 4.55, 1.73) to 1, 2, and 4 respectively. Now if all three firms use 1's technology, they will each produce 2.4 units and obtain a profit of 5.76. The value can therefore be obtained if firm 2 pays 1.21 units, and firm 4 pays 4.03 units, for the license to this technology.

For {1, 2, 3}, the restricted game has value (11.46, 5.00, 2.28). In this case, each of the firms would produce 2.5 units, with profits of 6.25. Then 2 and 3 would pay 1.25 and 3.97 to firm 1 for the license.

Definition 1. *We call "fair price", a price given by the difference between the Aumann-Drèze value and the profit that each firm could obtain if each benefits from the technology transfer i.e.* $\forall j \in S$, *with* $j \neq i$, $F := \varphi_j \left[v \,|S\right] - \dfrac{v\,(S)}{s}$, *where i is the most efficient firm of the coalition S.*

Take this analysis one step further. Assume {1, 2} has formed, receiving (as discussed above) payoffs (10.5, 5.18). If firm 4 were now to approach the coalition, asking for membership, the eventual payoffs would be (11, 4.55, 1.73). Firm 1 would be happy enough about this, but 2 would reject the deal. Similarly, if firm 3 were to approach and ask for membership, 2 would reject the deal. Finally, each of 1 and 2 would lose if it were to leave the coalition and try to go it alone. We conclude that the coalition $\{1, 2\}$ is stable in the sense that (a) one of the members would reject any new prospective member, and (b) each of the current members would lose, should it decide to leave the coalition.

This leads us to the following definition:

Definition 2. *A coalition S is stable if the following hold:*

(1) *for any $k \notin S$, <u>either</u> (i) there is at least one $j \in S$ such that $\varphi_j[v|S \cup \{k\}] < \varphi_j[v|S]$, <u>or</u> (ii) $\varphi_k[v|S \cup \{k\}] < v(\{k\})$*
(2) *for each $j \in S$, $\varphi_j[v|S] \geq v(\{j\})$.*

Example 1 (continued).

We compute the possibilities for each of the coalitions. Still following the Aumann-Drèze model, values to the several firms are:

Table 2

Coalition	Firm 1	Firm 2	Firm 3	Firm 4
{1,2}	10.5	5.18		
{1,3}	9.06		2.46	
{1,4}	9.14			2.38
{2,3}		2.6	1.32	
{2,4}		2.68		1.24
{3,4}			0.72	0.56
{1,2,3}	11.46	5	2.28	
{1,2,4}	11	4.54		1.74
{1,3,4}	10.48		2.06	1.98
{2,3,4}		3.893	1.933	1.854
{1,2,3,4}	10.32	3.73	1.25	0.70

We note here that coalition $\{1, 2, 3\}$ is also stable, as (a) its members would lose, should 4 join the group, and (b) any one of the three members would lose should it decide to leave and go it alone. On closer analysis, all the three-person coalitions are stable, as are the two-person coalitions that include firm 1. The other two-person coalitions are unstable, as is the grand coalition N.

We turn now to consider a 5-person game, with $A = 15$, and $c_i = 0$, 1, 2, 2.5 and 3 respectively. If no cooperation is allowed, then we find

$$q_1 = \frac{15 + (1 + 2 + 2.5 + 3) - 5(0)}{6} = \frac{23.5}{6} = 3.917$$

Similarly, $q_2 = 2.917$, $q_3 = 1.917$, $q_4 = 1.417$ and $q_5 = 0.9167$. Total production is then $Q = 66.5/6 = 11.083$, so $p = 23.5/6 = 3.917$, and the several players' profits are 15.34, 8.507, 3.674, 2.007 and 0.8403 respectively. We note that in this case, firm 5 is essentially moribund because of its high costs.

We assume again that some cooperation is allowed, in the form of technology sharing. In that case, we find that, if the coalitions {1} and {2, 3, 4, 5} form, then 2, 3, 4 and 5 all produce with unit cost 1, so

$$q_1 = \frac{15 + 4(1) - 5(0)}{6} = \frac{19}{6} = 3.167$$

$$q_2 = \frac{15 + 0 - 2(1)}{6} = \frac{13}{6} = 2.167$$

And $q_2 = q_3 = q_4 = q_5 = 13/6$. Total production is Q = 71/6, so $p = 19/6$, and profits are 10.028, 4.694, 4.694, 4.694, and 4.694. Thus $v(\{1\}) = 10.028$, $v(\{2, 3, 4, 5\}) = 18.776$.

In a similar way, we compute $v(S)$ for the remaining coalitions, to obtain the characteristic function:

TABLE 3

S	$Q(S)$	p	$v(S)$		S	$Q(S)$	p	$v(S)$
$\emptyset$	0		0		{1, 2, 3}	10	3.333	33.333
{1}	3.167	3.167	10.028		{1, 2, 4}	9.5	3.167	30.083
{2}	1.667	2.667	2.778		{1, 2, 5}	9.5	3.167	30.083
{3}	0.833	2.833	0.694		{1, 3, 4}	8.5	2.833	24.083
{4}	0.417	2.917	0.174		{1, 3, 5}	8.5	2.833	24.083
{5}	0	3	0		{1, 4, 5}	8.5	2.833	24.083
{1, 2}	7	3.5	24.5		{2, 3, 4}	6	3	12
{1, 3}	6	3	18		{2, 3, 5}	6	3	12
{1, 4}	6	3	18		{2, 4, 5}	6	3	12
{1, 5}	6	3	18		{3, 4, 5}	4.5	3.5	6.75
{2, 3}	3.667	2.833	6.722		{1, 2, 3, 4}	12	3	36
{2, 4}	3.667	2.833	6.722		{1, 2, 3, 5}	11.667	2.917	34.028
{2, 5}	3.667	2.833	6.722		{1, 2, 4, 5}	11.333	2.833	32.111
{3, 4}	2.333	3.167	2.722		{1, 3, 4, 5}	10.668	2.667	28.445
{3, 5}	2.333	3.167	2.722		{2, 3, 4, 5}	8.667	3.167	18.776
{4, 5}	1.667	3.333	1.389		N	12.5	2.5	31.25

Following the Aumann-Drèze model, values to firm 1, firm 2, firm 3, firm 4 and firm 5 are:

Table 4

Coalition	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
{1, 2}	15.875	8.625			
{1, 3}	13.667		4.333		
{1, 4}*	13.927			4.073	
{1, 5}*	14.014				3.986
{2, 3}		4.403	2.319		
{2, 4}		4.663		2.059	
{2, 5}		4.75			1.972
{3, 4}			1.621	1.101	
{3, 5}			1.708		1.014
{4, 5}				0.7815	0.6075
{1, 2, 3}*	18.718	9.454	5.162		
{1, 2, 4}*	17.721	8.457		3.905	
{1, 2, 5}*	17.75	8.486			3.847
{1, 3, 4}*	16.318		4.012	3.752	
{1, 3, 5}*	16.347		4.041		3.694
{1, 4, 5}*	16.878			3.646	3.559
{2, 3, 4}		6.115	3.073	2.813	
{2, 3, 5}*		6.144	3.102		2.755
{2, 4, 5}		6.675		2.706	2.619
{3, 4, 5}			2.897	1.97	1.883
{1, 2, 3, 4}*	19.189	8.986	4.541	3.284	
{1, 2, 3, 5}*	18.710	8.507	4.062		2.748
{1, 2, 4, 5}*	18.115	7.911		3.071	3.013
{1, 3, 4, 5}*	17.810		3.830	3.433	3.375
{2, 3, 4, 5}*		7.740	3.962	3.566	3.508
N	17.260	7.190	3.106	2.115	1.579

We note here that starlit-coalitions are stable. For example, the coalition {1, 2, 4, 5} is stable as its member would lose, should 3 join the group, and any one of the four members would lose should it decide to leave and go it alone. For the three-player coalitions case, we may remark that all coalitions that include the more efficient firm are stable. Moreover, all four-person coalitions are stable.

The question we can now ask is the following: what are the fair prices firms 2, 4 and 5 ought to pay to firm 1 in order to be allowed to use the most efficient technology? In fact, using firm 1's technology, they would each produce 2.833 units, and obtain a profit of 8.03. Hence, in this case, firm 2 should pay firm 1 a fair price that corresponds to the licensing fee of 0.12 units to be allowed to use this technology. In the same way, firm 4 should pay 4.96 units to use the technology and finally firm 5 should pay 5.02 units.

Finally, we give an existence proof for stable coalitions:

Theorem 1. *Every $i \in N$ belongs to at least one stable coalition.*

Proof. Essentially, we use a process, which starts with the singleton coalition $\{i\}$ and adds new members until a stable coalition is obtained.

Let then $S_1 = \{i\}$, and note that $\varphi_i[v|S_1] = v(\{i\})$. Thus, for this coalition, condition (b) holds.

Now either S_1 is stable, in which case our process stops, or S_1 is unstable. Since (b) holds, this means (a) does not hold. Thus there is some $k \neq i$ for which neither (a-i) nor (a-ii) holds; i.e., $\varphi_i[v|\{i, k\} \geq \varphi_i[v|S_1] = v(\{i\})$ and $\varphi_k[v|\{i, k\} \geq v(\{k\})$

Let now $S_2 = \{i, k\}$. Again note (b) holds. Now, if S_2 is stable, the process stops. If not, there is a firm $l \neq i, k$ such that neither (a-i) nor (a-ii) holds. Thus, letting $S_3 = \{i, k, l\}$

$\varphi_i[v|S_3] \geq \varphi_i[v|S_2] \geq v(\{i\})$

$\varphi_k[v|S_3] \geq \varphi_k[v|S_2] \geq v(\{k\})$

$\varphi_l[v|S_3] \geq v(\{l\})$.

We continue in this way, adding a new member to the coalition at each step, so that S_{m+1} is simply S_m plus one new member. At each step, $\varphi_k[v|S_{m+1}] \geq \varphi_k[v|S_m] \geq v(\{k\})$ for each $k \in S_m$, and $\varphi_l[v|S_{m+1}] \geq v(\{l\})$ for the new member, l, of S_{m+1}.

Thus condition (b) holds at each step. Eventually this process of adding new firms must stop as there is only a finite number of firms. The final coalition, S_M, satisfies condition (a) and is therefore stable. □

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