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Is the Formal Sector too Large or too Small? A Reexamination of Minimum Wages in Developing Countries

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Abstract

This paper reexamines the issue of the division of the labor force between the two sub-markets (formal and informal) of a developing economy. The formal sector is represented by a matching model with vertically differentiated workers. Assuming that firms hire their best applicants, we state that the formal sector is too small in terms of its labor force but too large in terms of job creation. Next we show that introducing a minimum wage increases the size of the formal sector with respect both to its labor force and to job creation. In accordance with the well-known paradox of Harris and Todaro, the enlargement of the formal market is accompanied by a rise in unemployment. However, when associated with a tax on job creation, the introduction of a minimum wage in the formal sector improves the efficiency of the labor market by making the formal sector more attractive to workers.

Key words : Formal and informal labor markets, applicant ranking, efficiency, minimum wage.

JEL Classification numbers: O17, J64, J68.

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1 Introduction

In this paper we reexamine the efficiency of a dual labor market. As in Albrecht *et alii* (2009), workers are heterogeneous and search frictions generate unemployment in the formal labor market.

Since Harris and Todaro (1970) and possibly even earlier, economic theory represents the labor market in developing countries by distinguishing between two sub-markets: the informal (or traditional) sector and, the formal (or modern) sector. In these dual models workers are generally assumed to be homogeneous. This assumption clearly stands in the way of an explanation for one of the leading characteristics of developing economies: the formal sector employs workers who are more highly-skilled than those in the informal one. This is plainly a serious limitation.

Recently Albrecht *et alii* (2009) have moderated this limitation by developing a model where the labor force is heterogeneous. Workers are assumed to be vertically differentiated. Since there are search frictions in the formal sub-market only highly-skilled workers choose to join it. By contrast, low-skilled workers have a preference for the informal sector; the wages they expect from holding a job in the formal sector are not high enough to compensate for the risk of unemployment. In what could be seen as a companion paper, Albrecht *et alii* (2010) study the efficiency of a simplified but very similar model (see also Gavrel, 2011a). The authors present their analysis as a study of the efficiency of participation in developed countries. But as is now well known, the same model can often be used to study the distribution of the labor force between participating and non-participating individuals in developed economies as well as in studying the distribution of the labor force between the two sub-markets of developing countries. Analytically, non-participating individuals in a developed economy can be treated like workers in the informal sector of a developing economy. In terms of participation choices, Albrecht *et alii* (2010) find that too many individuals participate in the labor market. From the developmental perspective this indicates that the formal sector is too large. The intuition behind this finding is that when deciding to join the formal sector, lower-skilled workers are not aware of the (negative)

impact of their choices on the average output in the formal sub-market.

This result is obtained under what we consider to be a very strong assumption. Firms search in a random manner; hence all workers face the same probability of finding a job in the formal sector, independently of their productivity. This is a very unrealistic prediction.

We here relax this assumption noting that it appears to play a central role for Albrecht *et alii* (2010) in reaching their conclusions. We instead presume that firms are selective. They prefer good workers to bad ones. Under this more realistic assumption highly-skilled job-seekers in the formal sector have a greater chance of success than low-skilled ones. Hence the model that we develop does not contradict the observed facts. But this is not our main point. All models are of course unrealistic. What is really of importance is the influence this has on conclusions regarding the size of the formal sector. In this respect our findings are very different: when firms are selective too few workers enter the formal sector. In other words, the labor force in the formal sector is too small.

What could be done to improve the efficiency of the labor market in a developing country? First of all, the formal sector should be more attractive to workers. From a theoretical point a view a range of measures can be considered (such as for example an income tax credit). In a developing country however the chosen policy should be simple. We show that the introduction of a mandatory minimum wage in the formal sector is an effective instrument in rendering the sector more attractive. A minimum wage (provided it is not set at a too high level) increases the size of the formal sector with respect both to its labor force and to job creation. In accordance with the paradox identified by Harris and Todaro, the enlargement of the formal (modern) sector is accompanied by an increase in unemployment. But despite this increasing unemployment, aggregate income rises insofar as a tax on job creation stabilizes labor demand.

Many developing countries already have minimum wage legislation. Our analysis provides a rationale for the introduction of a mandatory minimum wage in developing countries.

Apart the treatment of the hiring process, the model we use is very close to Albrecht *et alii* (2010). The formal sector is represented by a matching model (Pissarides, 2000) with Nash bargaining and free-entry. The hiring process is represented by an extension of the urn-ball model in which firms rank their applicants and pick the best one: firms are choosy.

In the relevant literature we can distinguish between two subsets. One contains many articles dealing with the segmentation of the labor market in developing and developed economies. This literature is very extensive, even when restricted to theory (see Fields (2008) for a good survey). Apart from the path-breaking article of Harris and Todaro (1970) to which we have already referred, it is quite difficult to determine which contributions are of particular significance. Different papers incorporate the search-matching approach in a Harris-Todaro model (see for instance Zenou, 2008). But, contrary to ours, in those models workers are generally homogenous and firms do not rank their candidates. The other subset includes papers which attempt to derive the matching function from first principles. As Petrongolo and Pissarides (2001) put it, this literature seeks to throw some light into the black box. Many of these articles are, like the present one, based on the urn-ball model (following Hall, 1977). One interesting example is Albrecht *et alii* (2003) who extend the urn-ball model to the case where (homogeneous) job-seekers make multiple applications in each period. Stevens (2007) develops an elegant continuous-time model inspired by the urn-ball process. Two papers are based on an urn-ball model with heterogeneous workers and applicant ranking. Moen (1999) shows that applicant ranking is likely to create an over-education effect, while Gavrel (2009) argues that, where applicants are ranked, the technical skill bias can be seen as a consequence (and not as a cause) of a rise in unemployment.

2 A dual labor market with applicant ranking in the formal sector

2.1 Market structure

Apart from the matching process, the market structure used here is very close to Albrecht *et alii* (2010) and Gavrel (2011a). We study the efficiency of the division of the labor force between the two sectors in the following simplified static¹ environment. Total labor force has a positive mass n . When holding a job in the formal sector, workers' productivities y are distributed according to the (strictly) increasing distribution function $F(y)$ of support $[0, 1]$. The density $F'(y)$ is denoted by $f(y)$ ($f(y) > 0$). When belonging to the informal sector all workers yield the same positive output z . These workers are self-employed, earning the income z . One could object that the income z should grow with y . Our response is that the tasks required by informal jobs are assumed to be so simple that workers gain no advantage by virtue of their (cognitive) skills when holding such a job. We retain this (strong) assumption in order to emphasize the incidence of firms' selectiveness on the division of the labor force between both sub-markets.

There is no unemployment in the informal labor market. On the contrary, when participating in the formal labor market workers find a job with probability $p(., y)$. They are then unemployed with probability $[1 - p(., y)]$. In this case they all enjoy the same utility d ($0 \leq d < z$). All firms are identical and freely enter the (formal) market. When entering the (formal) labor market firms must incur some positive cost c to create a job. Each active firm creates a single vacancy.

In the formal sector, when matched with a worker of productivity y (an event which occurs with the probability $q(., y)$), a job generates the (private) surplus $(y - d)$. This surplus is divided between the two parties according to their bargaining strengths. The wage, $w(y)$, of an employed worker of the formal sector is then given by:

¹See Moen (1999) and Gavrel (2009, 2011b) for extensions of the matching model with applicant ranking to a dynamic setting.

$$w(y) = d + \beta(y - d)$$

with β being workers' bargaining strength ($0 < \beta < 1$).

Other arguments of functions $p(., y)$ and $q(., y)$ will be made explicit when we turn to the matching process.

2.2 Sectoral private choices

Ignoring corner points, one can surmise that high-skilled workers (workers whose productivities are higher than some cut off y^*) will choose to join the formal labor market, whereas low-skilled individuals (individuals whose productivities are lower than the cut off y^*) will prefer to participate in the informal sector. This cut off is thus determined by the following migration-equilibrium equation:

$$z = p(., y^*)w(y^*) + [1 - p(., y^*)]d$$

Or:

$$z = p(., y^*)[d + \beta(y^* - d)] + [1 - p(., y^*)]d = d + p(., y^*)\beta(y^* - d) \quad (1)$$

In the next paragraph, we shall show that in conformity with intuition, the probability $p(., y)$ increases with the output y . It transpires that:

$$p(., y)w(y) + [1 - p(., y^*)]d > (<)z \iff y > (<)y^*$$

Consequently, the labor force of the formal market has a mass equal to $([1 - F(y^*)]n)$ while the labor force of the informal market has a mass equal to $[F(y^*)n]$. Active firms offer a set of jobs of mass v . Hence the tightness of the formal labor market, θ , is given by the ratio:

$$\theta = \frac{v}{[1 - F(y^*)]n}$$

As already noted, the main innovative feature of our paper involves the matching process. Contrary to Albrecht *et alii* (2009, 2010) and Gavrel (2011a), (active) firms'

search is not random in the formal sector. As in Moen (1999) and Gavrel (2009), they rank their applicants and pick the best. As a consequence, workers in the formal sector do not face the same probability of finding a job. In accordance with empirical evidence, we find that high-skilled job seekers have a greater chance of success.

We now provide a detailed account of the modeling of meetings in the formal labor market.

2.3 Matching process in the formal labor market

Applying the usual urn-ball model, we assume that each job seeker draws one firm at random.² In general, firms will have several applicants of differing productivity. We assume that they have full knowledge of the sample of their applicants. They then pick the best one.

As regards the matching process, we state the following lemma:

Lemma 1. *In the formal labor market, the probability $q(\cdot, y)$ ($y^* \leq y \leq 1$) is given by:*

$$q(\cdot, y) = q((v/n), y) = \exp\left(-\frac{[1 - F(y)]n}{v}\right) \frac{f(y)n}{v} \quad (2)$$

Proof. For expositional simplicity, we consider countable sets. Appendix A extends the results to non-countable sets.

In order to compute the probability of hiring a worker of productivity y , let us first consider the probability for a firm not encountering any applicant of output higher than y ($y^* \leq y \leq 1$). This is given by: $[1 - \frac{1}{v}]^{[1-F(y)]n}$. Assuming that vacancies are numerous, this probability tends to: $[\exp(-\frac{[1-F(y)]n}{v})]$.

Let us now consider the probability that a sample of candidates contains at least one worker whose productivity remains within the range $[y, y + dy]$. This probability tends to $(\frac{f(y)n}{v})dy$ when dy tends to zero.

²Multiple applications are unlikely to affect the main results.

Finally, a firm will recruit a worker of productivity y only if the sample of its applicants does not contain a better worker. This proves Lemma 1.

Q.E.D.

Integrating $q((v/n), y)$ on the range $[y^*, 1]$ gives the probability of filling a vacancy, $Q((v/n), y^*)$. We get:

$$Q((v/n), y^*) = 1 - \exp\left(-\frac{[1 - F(y^*)]n}{v}\right) \quad (3)$$

From the probabilities $q(., y)$, we also deduce the probabilities of finding a job. It transpires that:

$$p(., y) = p((v/n), y) = \exp\left(-\frac{[1 - F(y)]n}{v}\right) \quad (4)$$

Integrating $[p((v/n), y)f(y)/(1 - F(y^*))]$ on the range $[y^*, 1]$ gives the average probability of finding a job, $P((v/n), y^*)$. We obtain:

$$P((v/n), y^*) = \theta Q((v/n), y^*)$$

$\rho((v/n), y, y^*)$ will denote the density of the output y among employed workers in the formal labor market (i.e. the set of occupied formal jobs). This density is defined as follows:

$$\rho((v/n), y, y^*) \equiv \frac{q((v/n), y)}{Q((v/n), y^*)} \quad (5)$$

3 Decentralized equilibrium and efficiency

We now define a labor market equilibrium, and will then study its welfare properties.

3.1 Job creation and equilibrium

As indicated above, firms freely enter the formal labor market. This means that the profit expected from a vacancy should be equal to the cost of its creation (c). Job creation is then formally deduced from the following equation:

$$-c + (1 - \beta) \int_{y^*}^1 q((v/n), y)(y - d)dy = 0 \quad (6)$$

Or:

$$-c + (1 - \beta)Q((v/n), y^*)(\bar{y} - d) = 0 \quad (7)$$

with \bar{y} being the expected output of a job when matched with a worker. Formally, \bar{y} is given by:

$$\bar{y} = \int_{y^*}^1 \rho((v/n), y, y^*)ydy$$

Consequently, a labor market equilibrium can be defined as follows:

Definition 1. *A laissez-faire equilibrium is a pair (y^*, v) which jointly satisfies equations(1) and (7).*

From the equilibrium pair (y^*, v) , one can deduce the equilibrium values of all other endogenous variables.

3.2 Efficiency

Our welfare criterion is the social surplus (also referred to as the aggregate income). Denoted by Σ , the social surplus per head is defined as follows:

$$\Sigma = (v/n)Q((v/n), y^*)\bar{y} + F(y^*)z + [1 - F(y^*) - Q((v/n), y^*)(v/n)]d - (v/n)c$$

In the previous expression, the quantity $[1 - F(y^*) - Q((v/n), y^*)(v/n)]$ measures the unemployment level (divided by the total labor force n).

A social optimum is then determined as follows:

Definition 2. *A social optimum is a pair (y^*, v) which maximizes the social surplus Σ .*

This definition requires that we compute the derivatives of Σ with respect to the two variables v and y^* .

Let us first study the efficiency of job creation.

One can show that the derivative of Σ with respect to v has the same sign as:

$$V \equiv Q(\cdot)(1 - \eta(\cdot))(\bar{y} - d) - c + vQ(\cdot)\frac{\partial \bar{y}(\cdot)}{\partial v}$$

where $\eta(\cdot)$ is the elasticity of the probability $Q(\cdot)$ with respect to v (in absolute values). Notice that this elasticity is a function of the pair (y^*, v) .

V depends on the derivative of the average output \bar{y} with respect to job creation v . We state the following Lemma with respect to this derivative:

Lemma 2. *In the formal labor market, an increase in job creation (v) lowers the output per job (\bar{y}).*

Proof. See Appendix B.

Lemma 2 illustrates a more general result. In the presence of applicant ranking, an increase in job creation compels firms to hire less efficient workers, leading to a decrease in expected output. This result holds whether workers' differentiation is vertical (Gavrel, 2009) or horizontal (Gavrel, 2011b).

Regarding the (partial) efficiency of job creation in the formal labor market, Lemma 2 allows us to state the following proposition:

Proposition 1. *In a laissez-faire equilibrium, job creation in the formal sector is inefficient under the Hosios rule. Firms create too many jobs.*

Proof. The expression V can be rewritten as follows:

$$V = Q(\cdot)(1 - \beta)(\bar{y} - d) - c + Q(\cdot)(\beta - \eta(\cdot))(\bar{y} - d) + vQ(\cdot)\frac{\partial \bar{y}(\cdot)}{\partial v}$$

Using equation (7) and applying the Hosios rule ($\beta = \eta$), we obtain:

$$V = vQ(\cdot) \frac{\partial \bar{y}(\cdot)}{\partial v} < 0$$

This proves that job creation is too high in the neighborhood of a decentralized equilibrium, whatever the value of the cut off y^* is.

Q.E.D.

In other words, *the formal sector is too large in terms of job creation.*

The intuition behind this finding is that, relative to the basic matching model, job creation acquires a new congestion effect in the presence of applicant ranking. An increase in the number of vacancies also lowers the expected profits of firms. The competition for skilled workers becomes tighter. Since firms do not internalize this negative externality labor demand is excessive.

Let us now turn to the efficiency of the output cut off y^* .

One can show that the derivative of Σ with respect to y^* has the same sign as:

$$Y \equiv z - [d + p((v/n), y^*)(y^* - d)]$$

We deduce the following proposition:

Proposition 2. *In a laissez-faire equilibrium, the cut off y^* is inefficient. The labor force of the formal sector $[1 - F(y^*)]n$ is necessarily too small.*

Proof. The expression Y can be rewritten as follows:

$$Y = z - d - \beta p((v/n), y^*)(y^* - d) - (1 - \beta)p((v/n), y^*)(y^* - d)$$

Using equation (1), we obtain:

$$Y = -(1 - \beta)p((v/n), y^*)(y^* - d) < 0$$

This proves that in the neighborhood of a laissez-faire equilibrium the cut off y^* is too high whatever the value of job creation v is.

Q.E.D.

In other words, *the formal labor market is too small in terms of labor force.*

Proposition 2 contradicts the results of Albrecht *et alii* (2010) and Gavrel (2011a). In these papers, where firms' search is random, the formal labor market is too large. This contradiction requires interpretation. The intuition behind it is that, with random search, a decrease in the cut off lowers the average output per filled job. As Albrecht *et alii* (2010) put it, *the marginal participant reduces the average productivity of matches in the [formal] market.* On the contrary, when firms rank their applicants, the marginal worker in the formal sector raises the aggregate income by the amount $[d + p((v/n), y^*)(y^* - d) - z]$. Because productivities are higher than wages ($\beta < 1$), the private return to search for a formal job is lower than the social return. Consequently the productivity of marginal workers in the formal sector is too high.

This market failure also explains why there is no value for the bargaining strength of workers which could make the laissez-faire equilibrium coincide with a social optimum. When β tends to 1, the entire output goes to workers. The cut off y^* becomes efficient, but job creation is reduced to zero.

Laissez-faire is inefficient. What should be done to palliate this market failure? Proposition 2 suggests that introducing a minimum wage is likely to improve the efficiency of the labor market.

4 A reexamination of minimum wages in developing countries

Many developing countries have a legal minimum wage. The introduction of a minimum wage is often viewed as a way of reducing poverty and inequality in these countries. But, as Fields and Kanbur (2007) pointed out, legal minimum wages only affect workers in the formal labor market. The situation of the poorest workers, those in the informal market, remains unaffected. As a consequence poverty does not decline. Furthermore, while inequalities between employed workers are reduced within the formal sector, overall they are increased.

In this section we provide another rationale for adopting a mandatory minimum wage in developing countries. We show that introducing a minimum wage in the formal labor market (the only sector where such a measure can be implemented) improves the efficiency of the economy insofar as this policy measure is accompanied by a tax on job creation in the formal sector (the only sector where such a tax can be levied).

4.1 The formal labor market in the presence of a binding minimum wage

In the presence of a minimum wage, \hat{w} , low-ability workers (workers whose productivities are lower than some trigger \hat{y}) now earn that minimum, whereas the wages of high-ability workers (workers whose productivities are greater than \hat{y}) are still subject to bargaining. Formally, we have:

$$w(\hat{y}) = d + \beta(\hat{y} - d) = \hat{w}$$

and:

$$y^* \leq y \leq \hat{y} \iff w(y) = \hat{w}$$

$$\hat{y} \leq y \leq 1 \iff w(y) = d + \beta(y - d)$$

Notice that the first equation determines the trigger \hat{y} .

The presence of a mandatory wage has three consequences. First, it clearly produces a spike in the distribution of wages in the formal labor market. Next, the equation which determines the cut off y^* is now written as follows:

$$z = p((v/n), y^*)\hat{w} + (1 - p((v/n), y^*))d \tag{8}$$

For a given value of job creation v , an increase in the minimum wage \hat{w} lowers the cut off y^* , consequently increasing the labor force of the formal sector. We will show that this intuitive result holds true when job creation is endogenous.

Finally, when matched with a worker whose productivity lies on the range $[y^*, \hat{y}]$, a job generates the profit $(y - \hat{w})$. Otherwise the profit remains equal to $[(1 - \beta)(y - d)]$. As a consequence, the job creation equation must be rewritten as follows:

$$-c + \int_{y^*}^{\hat{y}} q((v/n), y)(y - \hat{w})dy + (1 - \beta) \int_{\hat{y}}^1 q((v/n), y)(y - d)dy = 0 \quad (9)$$

This leads to the following definition:

Definition 3. *In the presence of a minimum wage, a labor market equilibrium is a pair (y^*, v) which jointly satisfies equations (8) and (9).*

It is worth noting that the latter definition is equivalent to Definition 2 when the minimum wage is not binding, that is:

$$y^* = \hat{y} \iff \hat{w} = d + \beta(y^* - d)$$

In what follows we examine the implications of introducing of a binding minimum in the neighborhood of an equilibrium where all wages are bargained. In other words, this amounts to studying the effects of a (small) increase in the minimum wage in a situation where the legal minimum \hat{w} is initially equal to the bargained wage: $[d + \beta(y^* - d)]$. This also means that in our benchmark, the cut off y^* coincides with the trigger \hat{y} .

4.2 Effects of a minimum wage increase

With respect to the effects of introducing a binding wage (\hat{w} becomes higher than $[d + \beta(y^* - d)]$), we first state the following proposition:

Proposition 3. *Introducing a minimum lowers the cut off (y^*) and raises job creation (v). Despite the increase in labor demand, the tightness of the formal labor market (θ) is reduced.*

Proof. See Appendix C.

Although the establishment of this result is lengthy, the intuition behind it is very

simple. Introducing a minimum wage makes lower-skilled workers (infra-marginal workers) join the formal labor market. Because their productivities remain higher than the wages they earn when holding a job, the expected profits of an offered job rise (in the neighborhood of laissez-faire the direct effect on profits is very small). This gives firms an incentive to create new jobs.

It is worth noting that the size of the formal market rises in terms both of offered jobs and labor force. Conversely, implementing a minimum wage lowers the market tightness, consequently increasing the unemployment rate in the formal sector. In other words, our model replicates the paradox of Harris and Todaro (1970): the enlargement of the formal labor market is associated with more unemployment. But rising unemployment means neither that employment falls in this sector nor that market efficiency deteriorates.

4.3 Minimum wage and market efficiency

We can now study the effect of implementing a minimum wage on the efficiency of the labor market of a developing economy.

Differentiating the social surplus Σ with respect to the minimum wage \hat{w} yields:

$$\frac{\partial \Sigma}{\partial \hat{w}} = \frac{\partial \Sigma}{\partial v} \frac{\partial v}{\partial \hat{w}} + \frac{\partial \Sigma}{\partial y^*} \frac{\partial y^*}{\partial \hat{w}}$$

From Propositions 1 and 2, we know that in the neighborhood of the laissez-faire regime (under the Hosios rule):

$$\frac{\partial \Sigma}{\partial v} < 0$$

and:

$$\frac{\partial \Sigma}{\partial y^*} < 0$$

We stated that the cut off y^* decreases with a minimum wage increase (Proposition 3). This increase in the labor force of the formal sector tends to improve market

efficiency. On the other hand job creation is enhanced in the formal labor market (Proposition 3). We have shown that with applicant ranking this increase in labor demand tends to deteriorate market efficiency under the Hosios rule. As a result the effect of introducing a minimum wage on the aggregate income is undeterminate.

This means that governments need a second instrument that allows them to control for job creation. A simple way of achieving this goal is to tax the firms of the formal sector. For the sake of simplicity we will assume that this tax, denoted by τ , is dedicated to the funding of neutral subsidies (see Holmlund, 1998). With the tax τ , the job creation equation (9) is rewritten as follows:

$$-c - \tau + \int_{y^*}^{\hat{y}} q((v/n), y)(y - \hat{w})dy + (1 - \beta) \int_{\hat{y}}^1 q((v/n), y)(y - d)dy = 0 \quad (10)$$

We can state the following proposition:

Proposition 4. *When associated with a positive tax that stabilizes job creation, the introduction of a minimum wage improves the efficiency of the labor market.*

Proof. The proof is straightforward. Since v holds as a constant, the derivative of the aggregate income with respect to the minimum wage reduces to:

$$\frac{\partial \Sigma}{\partial \hat{w}} = \frac{\partial \Sigma}{\partial y^*} \frac{\partial y^*}{\partial \hat{w}}$$

From equation (8) we obtain that:

$$\frac{\partial \theta}{\partial \hat{w}} = -\frac{\theta^2}{\beta(y^* - d)} < 0$$

On the other hand we have:

$$d\theta = \frac{vf(y^*)}{[1 - F(y^*)]^2 n} dy^*$$

It results that introducing a minimum wage lowers the cut off y^* , leading then to an increase in the social surplus.

Next, differentiating (10) in the neighborhood of laissez-faire gives:

$$\frac{\partial \tau}{\partial \hat{w}} = -q((v/n), y^*)(y^* - \hat{w}) \frac{\partial y^*}{\partial \hat{w}} > 0$$

This shows that the introduction of a minimum wage should be associated with a positive tax and proves the proposition.

Q.E.D.

Proposition 4 contradicts Gavrel (2011a) where the efficiency of the labor market is improved by subsidizing non-participants (i.e. making the informal sector more attractive to workers). The reason for this is that in this paper (as well as in Albrecht *et alii* 2009, 2010) random recruitment creates a mixing effect which tends to lower the social surplus when low-ability workers join the formal sub-market (see the comment of Proposition 2). On the contrary, with applicants ranking, the presence of bad workers in the (formal) market does not affect the probability for a firm to hire a good one. That only enhances the probability of filling the vacancies in a less productive but efficient manner.

5 Final comments

Using a matching model with vertically differentiated workers, we have shown that in the presence of applicant ranking, the formal labor market of a developing country is too small in terms of labor force, but too large in terms of job creation. Insofar it is not too large, a minimum wage (if accompanied by a tax on job creation) improves the efficiency of the economy by rendering the formal sector more attractive to workers. But does this model account for the main features of the labor market in developing countries? We believe that the formal sub-market has (at least) two key characteristics. On the one hand, the formal sector offers better jobs, giving high-skilled workers the opportunity of taking advantage of these skills. We accounted for this stylized fact and its consequences. This is where we are.

On the other hand, the formal sector has larger firms than the informal one. We did not take this characteristic into account and this aspect requires further investigation.

There are two reasons for this. Firstly, larger firms are likely to take advantage of increasing returns in the production process. We surmise that if we take account of increasing returns, the formal sector might become too small in both terms: labor force and job creation. Next, the analogy with participation models to which we referred in the introduction of our paper would no longer hold. This would be beneficial, since it would mean that the model really does describe the particularities of a developing economy.

At this stage we can still ask the following question: what are the results of our analysis when interpreted in terms of participation in a developed economy?

From this perspective we have shown that too few workers participate in the labor market. We also stated that a minimum wage (insofar as it is not too large) tends to enhance the efficiency of the labor market by increasing participation. This result holds despite an increase in unemployment, implying then that the effects of minimum wages on unemployment are not what really matters. Nevertheless, in developed countries other (and possibly better) policy measures can be implemented (such as an income tax credit). This is another line that can be studied in further research.

To conclude, we would like to emphasize that both (perfect) applicants ranking and random recruitment are strong assumptions. In the real world firms carry out a cost-benefit analysis when deciding on their recruitment process. In fact their selectiveness should be modeled as an endogenous variable that is likely to depend on market tightness as well as on the distribution of workers' productivities. To our knowledge such a study would be really innovative.

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Appendix A: Derivation of probabilities $q(\cdot, y)$ with non-countable sets

The purpose of this appendix is to show that the expressions of probabilities $q(\cdot, y)$ hold true when the sets of job seekers and vacancies are continuums (Lemma 1).

To that end, we define "pseudo" urn-ball processes where the set of vacancies is divided into M big urns and the set of job seekers into N big balls. Urns and balls have almost the same size and the true urn-ball process is obtained by making this size tend to zero. Each big ball is randomly sent to a single big urn.

From Lemma 1, we know that the probability $q(\cdot, y)$ is deduced from the probability for a firm not meeting any applicant of output higher than y ($y^* \leq y \leq 1$).

So let us divide the set of vacancies into M big urns of mass $h = v/M$ ($M > 1$).

The set of applicants whose productivities are greater than y has a mass equal to $[1 - F(y)]n$. This subset is divided into N big balls which have the same structure. N is the integer part $I([1 - F(y)](n/h))$.

Let us now compute the probability for a big urn not containing any big ball. It is given by:

$$\left[1 - \frac{1}{M}\right]^N = \exp\left(N \ln\left[1 - \frac{1}{M}\right]\right)$$

As

$$\frac{[1 - F(y)](n/h) - N}{M} < 1/M$$

this probability tends to:

$$\exp\left(-\frac{N}{M}\right) = \exp\left(-\frac{1 - F(y)}{v/n}\right)$$

when M tends to infinity.

From this result, we deduce that probabilities $q(., y)$ keep the same expressions by following the same line as Lemma 1.

Appendix B: Proof of Lemma 2

One can show that the derivative of $\rho((v/n), y, y^*)$ with respect to v has the same sign as the following expression:

$$[1 - F(y)]Q - [Q\frac{v}{n} - (1 - F(y^*))]$$

The previous expression is a decreasing function of y . As:

$$\int_{y^*}^1 \frac{\partial \rho((v/n), y, y^*)}{\partial v} dy = 0$$

it results that there exists some positive output \tilde{y} ($\tilde{y} > y^*$) such that:

$$\frac{\partial \rho((v/n), \tilde{y}, y^*)}{\partial v} = 0$$

and:

$$y < (>) \tilde{y} \iff \frac{\partial \rho((v/n), y, y^*)}{\partial v} > (<) 0$$

This implies that, $\forall y \in [y^*, \tilde{y} \cup \tilde{y}, 1]$:

$$\frac{\partial \rho((v/n), y, y^*)}{\partial v} y < \frac{\partial \rho((v/n), y, y^*)}{\partial v} \tilde{y}$$

Consequently we obtain that:

$$\frac{\partial \bar{y}}{\partial v} < \int_{y^*}^1 \frac{\partial \rho((v/n), y, y^*)}{\partial v} \tilde{y} dy = 0$$

Q.E.D.

6 Appendix C: Proof of Proposition 3

Let J denote the expected profit of an offered job:

$$J \equiv J((v/n), \hat{y}, y^*, \hat{w}) = \int_{y^*}^{\hat{y}} q((v/n), y)(y - \hat{w}) dy + (1 - \beta) \int_{\hat{y}}^1 q((v/n), y)(y - d) dy$$

In the neighborhood of the laissez-faire equilibrium (Definition 2), the result is obtained:

$$\frac{\partial J(\cdot)}{\partial y^*} = -(y^* - \hat{w})q((v/n), y^*) = -(1 - \beta)(y^* - d)q((v/n), y^*) < 0$$

$$\frac{\partial J(\cdot)}{\partial \hat{y}} = 0$$

$$\frac{\partial J(\cdot)}{\partial \hat{w}} = - \int_{y^*}^{\hat{y}} q((v/n), y) dy = 0$$

Let us now turn to the effect of job creation v on $J(\cdot)$. We will show that this effect is (strictly) negative. To that end we need to study the derivatives of the probabilities $q((v/n), y)$ with respect to v . We have:

$$\frac{\partial q((v/n), y)}{\partial v} = \frac{q((v/n), y)}{v} \left[\frac{1 - F(y)}{v/n} - 1 \right]$$

Thus, these derivatives have the same sign as:

$$H((v/n), y) \equiv \frac{1 - F(y)}{v/n} - 1$$

On the one hand, $H(\cdot)$ is a decreasing function of y . On the other hand, we already know that:

$$\int_{y^*}^1 \frac{\partial q((v/n), y)}{\partial v} dy = \frac{\partial Q((v/n), y^*)}{\partial v} < 0$$

From these two points, it follows that we must distinguish between two cases.

-i) $\frac{\partial q((v/n), y^*)}{\partial v} \leq 0$

Since $H(\cdot)$ is a decreasing function of y , it follows that in this first case, $\forall y \in]y^*, 1]$:

$$\frac{\partial q((v/n), y)}{\partial v} < 0$$

In the neighborhood of the laissez-faire equilibrium, the derivative of $J(\cdot)$ with respect to v is (very) close to:

$$\frac{\partial J(\cdot)}{\partial v} = (1 - \beta) \int_{y^*}^1 \frac{\partial q((v/n), y)}{\partial v} (y - d) dy$$

As $\forall y \in]y^*, 1]$

$$\frac{\partial q((v/n), y)}{\partial v} (y - d) < \frac{\partial q((v/n), y)}{\partial v} (y^* - d)$$

It follows that:

$$\frac{\partial J(\cdot)}{\partial v} < (1 - \beta) \int_{y^*}^1 \frac{\partial q((v/n), y)}{\partial v} (y^* - d) dy$$

Or:

$$\frac{\partial J(\cdot)}{\partial v} < (1 - \beta) \frac{\partial Q((v/n), y^*)}{\partial v} (y^* - d) < 0$$

-ii) $\frac{\partial q((v/n), y^*)}{\partial v} > 0$

In this second case there necessarily exists some output level \tilde{y} ($1 > \tilde{y} > y^*$) such that:

$$\frac{\partial q((v/n), y)}{\partial v} > 0, \forall y \in [y^*, \tilde{y}[$$

$$\frac{\partial q((v/n), \tilde{y})}{\partial v} = 0$$

$$\frac{\partial q((v/n), y)}{\partial v} < 0, \forall y \in]\tilde{y}, 1]$$

Thus, the derivative of $J(\cdot)$ can be rewritten as:

$$\frac{\partial J(\cdot)}{\partial v} = (1 - \beta) \int_{y^*}^{\tilde{y}} \frac{\partial q((v/n), y)}{\partial v} (y - d) dy + (1 - \beta) \int_{\tilde{y}}^1 \frac{\partial q((v/n), y)}{\partial v} (y - d) dy$$

As

$$\frac{\partial q((v/n), y)}{\partial v} (\tilde{y} - d) > \frac{\partial q((v/n), y)}{\partial v} (y - d), \forall y \in [y^*, \tilde{y}[\cup]\tilde{y}, 1]$$

it transpires that:

$$\frac{\partial J(\cdot)}{\partial v} < (1 - \beta) \int_{y^*}^1 \frac{\partial q((v/n), y)}{\partial v} (\tilde{y} - d) dy$$

Or:

$$\frac{\partial J(\cdot)}{\partial v} < (1 - \beta) \frac{\partial Q((v/n), y^*)}{\partial v} (\tilde{y} - d) < 0$$

To sum up, let us define the variable k as follows:

$$k = y^* \iff \frac{\partial q((v/n), y^*)}{\partial v} \leq 0$$

$$k = \tilde{y} \iff \frac{\partial q((v/n), y^*)}{\partial v} > 0$$

Notice that: $k \geq y^*$.

Using this notation, the derivative of $J(\cdot)$ with respect to v always satisfies:

$$\frac{\partial J(\cdot)}{\partial v} < (1 - \beta) \frac{\partial Q((v/n), y^*)}{\partial v} (k - d) < 0$$

Now, differentiating equation (9) yields:

$$-(1 - \beta)(y^* - d)q((v/n), y^*)dy^* + \frac{\partial J(\cdot)}{\partial v}dv = 0$$

From the two previous relations, we deduce:

$$0 > \frac{dv}{dy^*} > \frac{q((v/n), y^*)(y^* - d)}{\frac{\partial Q((v/n), y^*)}{\partial v}(k - d)}$$

As

$$\frac{\partial Q((v/n), y^*)}{\partial v} = -p((v/n), y^*)[1 - F(y^*)] \frac{n}{v^2}$$

and

$$q((v/n), y^*) = p((v/n), y^*)f(y^*) \frac{n}{v}$$

we obtain:

$$\frac{dv}{dy^*} > -\frac{y^* - d}{k - d} \frac{vf(y^*)}{1 - F(y^*)}$$

Next, differentiating equation (8) shows that the derivative of the cut off y^* with respect to the minimum wage \hat{w} has the same sign as:

$$-\left[f(y^*) + \frac{1 - F(y^*)}{v} \frac{dv}{dy^*}\right]$$

From the previous inequality, we deduce:

$$\frac{1 - F(y^*)}{v} \frac{dv}{dy^*} > -\frac{y^* - d}{k - d} f(y^*)$$

Consequently:

$$[f(y^*) + \frac{1 - F(y^*)}{v} \frac{dv}{dy^*}] > f(y^*) [1 - \frac{y^* - d}{k - d}]$$

As $k \geq y^*$, this proves that introducing a minimum wage lowers the cut off y^* . We already know that a decrease in y^* is associated with an increase in v ($\frac{dv}{dy^*} < 0$). Thus, introducing a minimum wage raises job creation.

From equation (8), it can be deduced that a binding minimum wage reduces the market tightness θ .

Q.E.D.