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On the Inefficiency of Matching Models of Unemployment with Heterogeneous Workers and Jobs when Firms Rank their Applicants

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Abstract

In a circular matching model, firms rank their applicants and pick the best suited one. Job creation appears to lower the average output. As firms do not internalize this effect, jobs are too many in the *laissez-faire* equilibrium under the Hosios condition. Due to similar externalities firms' search intensities are too strong whereas workers' search intensities are too weak.

Key words : Matching, Differentiation of skills, Applicant ranking, Labor market efficiency.

JEL Classification numbers : D8, J6.

1 Introduction

It is now well known that in the matching model with homogenous workers and jobs, job creation, search intensities as well as workers' participation decisions are efficient if and only if firms internalize the so-called congestion effect. Hosios (1990) stated

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that in the basic matching model, this imposes that the elasticity of the matching function with respect to unemployment coincides with workers' bargaining strength: the famous Hosios condition. Pissarides (2000) also showed that this condition ensures that job rejection is efficient in a model (referred to as the stochastic job matching model) where the output of a match is an exogenous random variable.

Over the past two decades, labor economists have proposed a lot of matching models with heterogeneous workers or (and) jobs. Most of them have vertically differentiated workers and jobs. Workers are more or less skilled while jobs are more or less complex. These models have often been built with the aim of explaining the skill-biased technical change (Acemoglu, 1999, Mortensen and Pissarides, 1999, Albrecht and Vroman, 2002, Gautier, 2002). Other ones study the consequences of on-the-job search (Pissarides 1994, Kennes, 2006) or the efficiency of individual participation decisions (Albrecht et al., 2010, Gavrel, 2011). Models with horizontally differentiated agents are fewer. Marimon and Zilibotti (1999)' circular model which transposes the analysis of Salop (1979) to the labor market is a main contribution to this literature. It gave rise to different extensions as Gautier *et al.* (2010) who use the circular model to study the effects of on-the-job search on the efficiency of the labor market.

All these articles assume that firms' search is random. Firms randomly pick one worker among the pool of their applicants. Except the (unrealistic) case in which the labor market is divided into as many sub-markets as worker types (as in Mortensen and Pissarides, 1999), this assumption is very strong. This is all the more heroic as firms usually have perfect knowledge of their applicants' productivities.

This short paper studies an economy where workers and jobs are differentiated in the same (circular) way as Marimon and Zilibotti (1999). Contrary to this paper and other related ones, firms do not draw one candidate at random; they rank their applicants and pick the best suited one. In order to account for this recruitment behavior, the technology of contacts is described by an extension of the urn-ball model. We show that, in the presence of applicant ranking, more vacancies deteriorate the assignment of workers to jobs, lowering then the average output. As firms do not

internalize this effect, job creation is too high when the Hosios condition is satisfied. We first present a simple static model. This static model helps us to explain with clarity how the ranking of applicants creates a new externality of job creation. We then extend the analysis to a continuous-time dynamic setting with endogenous search intensities. Due to similar external effects, firms create too many vacancies and devote too much effort to search whereas workers' search intensities are too weak.

As already mentioned, matching models usually assume that firms' search is random. Two exceptions are Moen (1999) and Gavrel (2009); but in those papers, workers are vertically differentiated while firms are identical. Moen (1999) shows that applicant ranking tends to create an over-education effect while Gavrel (2009) argues that in the presence of applicant ranking, the technical bias can be viewed as a response of firms to a general rise in unemployment.

The second section presents the static model and exposes our main argument. In section 3, we extend the analysis to a dynamic setting. Section 4 studies the efficiency of a decentralized equilibrium and states the main results.

2 An introductory static model

We first use a static search-matching model with bargained wages and free-entry. Regarding the differentiation of the agents, we follow Salop (1979). Workers and jobs are uniformly distributed along a circle of unit length. The distance between two locations on the circle measures the mismatch x ($0 \leq x \leq 1/2$) between a job type and a worker type. The output of a filled job is a decreasing function $y(x)$ of the mismatch x . As we focus on the efficiency of job creation, the output of the worst matches ($y(1/2)$) is assumed to be greater than the value of leisure d^1 . Each active firm offers a single vacancy.

Relative to the literature, the innovative feature of our analysis concerns the hiring process.

¹Results extend to the case where some matches are not viable.

2.1 Hiring process, mismatch and market tightness

Firms try to fill v vacancies while n workers apply for jobs. The ratio (v/n) is denoted by θ and referred to as the tightness of the labor market. Applying the usual urn-ball model, we assume that each job seeker draws one firm at random². In general, firms will have several applicants of different types. Firms are assumed to have full knowledge of the sample of their applicants. They then pick the best suited one.

Let $q(x, \theta)$ denote the “probability” (in fact, it is a density) for a firm to hire a worker of mismatch x .

Regarding the matching process, we state the following lemma:

Lemma 1. *In the circular matching model with applicant ranking, the density $q(x, \theta)$ is given by:*

$$q(x, \theta) = \exp\left(-\frac{2x}{\theta}\right) \frac{2}{\theta} \quad (1)$$

Proof. In order to compute the probability of hiring a worker of mismatch x , let us first consider the probability for a firm not encountering any applicant of mismatch lower than x ($x \leq 1/2$)³. It is given by: $[1 - \frac{1}{v}]^{2xn}$. Assuming that vacancies are numerous, this probability tends to: $[\exp(-\frac{2x}{\theta})]$.

Let us now consider the probability that a sample of candidates contains at least one worker whose mismatch remains within the range $[x, x + dx]$. This probability tends to $(\frac{2dx}{\theta})$ when dx tends to zero.

Finally, a firm will recruit a worker of mismatch x only if the sample of its applicants does not contain any better-suited worker. This proves Lemma 1.

Q.E.D.

Integrating $q(x, \theta)$ on the range $[0, 1/2]$ gives the probability of filling a vacancy, called $Q(\theta)$. We get:

²Multiple applications are unlikely to affect the main results.

³For simplicity, we consider countable sets. However, the results extend to continuums.

$$Q(\theta) = 1 - \exp\left(-\frac{1}{\theta}\right) \quad (2)$$

Consequently, the number of job-worker matches per period, M , is given by the CRTS matching function: $M = vQ(\theta)$. Notice that the derivative $Q'(\theta)$ is negative. We also deduce the probability of finding a job: $P(= \theta Q(\theta))$. The probability P can also be obtained from the following integral:

$$P = \int_0^{1/2} 2 \exp\left(-\frac{2x}{\theta}\right) dx$$

In this expression, the term $2 \exp(-2x/\theta)dx$ represents the probability that an unemployed worker finds a job with which the mismatch would lie on the range $[x, x + dx]$.

We will denote by $\rho(x, \theta)$ the density of mismatch x among employed workers (i.e. the set of occupied jobs). We have:

$$\rho(x, \theta) = \frac{q(x, \theta)}{Q(\theta)} \quad (3)$$

2.2 Nash bargaining and job creation

Let $\bar{y}$ denote the expected output of a filled job, that is:

$$\bar{y} = \bar{y}(\theta) = \int_0^{1/2} \rho(x, \theta) y(x) dx$$

According to the Nash rule, the surplus of a match is divided between both parties according to their bargaining strength. As there is free-entry, the value of a vacancy reduces to zero. We then obtain:

$$-c + (1 - \beta)[\bar{y}(\theta) - d] = 0 \quad (4)$$

where c is the cost to create a vacancy and β denotes the workers' bargaining strength.

2.3 Equilibrium and efficiency

An equilibrium of the labor market can be defined as follows:

Definition 1. *An equilibrium of the labor market is a scalar (θ) which satisfies equation (4).*

One interesting issue is the efficiency of such a decentralized equilibrium. To address this issue, we first need to study the effect of vacancy creation on the average output.

2.3.1 Average output and market tightness

Regarding the effect of an increase in market tightness on the expected output of a filled job, one can state the following lemma:

Lemma 2. *An increase in market tightness (θ) lowers the average output $(\bar{y})$.*

Proof. See Appendix A.

The intuition behind Lemma 2 is as follows. Given the fact that firms pick the best suited worker they meet, a firm's expected candidate is an order statistic which is the expected maximum of the queue of applicants. As the expected maximum of a sample from a given distribution rises with the sample size, an increase in the number of applicants per firm (*i.e.* a decrease in market tightness) raises the expected quality of the best application. In other words, when the market becomes tighter, firms are led to hire ill-suited workers. This raises the average mismatch leading then to a decrease in the average output

2.3.2 Market efficiency

Following Hosios (1990) and Pissarides (2000), our welfare analysis is based on the social surplus (*per head*) called Σ :

$$\Sigma = \theta Q(\theta) \bar{y}(\theta) + (1 - \theta Q(\theta))d - \theta c \quad (5)$$

In the following, we will assume that the usual condition for market efficiency is

satisfied. The elasticity $\eta(= -\theta Q'(\theta)/Q)$ is then equal to β . Using equation (4), it comes that the derivative of Σ with respect to θ in the neighborhood of a decentralized equilibrium satisfies:

$$\frac{d\Sigma}{d\theta} = \theta Q(\theta) \frac{d\bar{y}}{d\theta} < 0$$

This proves that:

Proposition 1. *Under the Hosios condition, firms create too many vacancies in a decentralized equilibrium.*

This result contradicts Marimon and Zilibotti (1999) in which job creation is efficient as long as the Hosios rule is satisfied. Indeed, as firms choose the best applicant they meet, job creation gets a new externality in this model. More vacancies also compel firms to recruit ill-suited workers, leading then to a decrease in the expected output (see the interpretation of Lemma 2). As firms do not internalize this effect, they create too many vacancies in the *laissez faire* situation.

3 A dynamic model with endogenous search intensities

In order to test the robustness of our results, we now extend the analysis to a continuous-time dynamic setting with endogenous search intensities.

3.1 Analytical framework

Workers are infinitely lived and all agents discount future income flows at the common rate r . The destruction rate of jobs, s , is assumed not to depend on the match quality. The market tightness θ is now defined as the ratio of vacancies v , to unemployment, u ($\theta = v/u$). As the measure of the labor force is normalized to one, u also represents the rate of unemployment. On-the-job search is ruled out.

Following Moen (1999), we obtain a continuous-time matching process by assuming

that during a time interval of length dt , firms advertise their vacancies with probability bdt while (unemployed) workers randomly send an application to one of the (advertised) vacancies with probability adt . The rates a and b can be seen as search efforts which generate costs. The search cost of a worker, $\gamma(a)$, is an increasing and convex function of his/her effort a ($\gamma'(\cdot) > 0, \gamma''(\cdot) > 0$). Accordingly, the search cost of a firm, $\kappa(b)$, is also an increasing and convex function of its effort b .

Contrary to Moen (1999), the search intensities a and b are made endogenous here. The rate a is obtained by maximizing the lifetime utility of an unemployed worker and the rate b , by maximizing the asset value of a vacancy (see Pissarides, 2000).

For the sake of simplicity, we restrict ourselves to a stationary equilibrium where workers and jobs are uniformly distributed on the circle of skills⁴. As they face the same situation, workers (firms) devote the same effort a (b) to search.

We first extend the results about the hiring process to this dynamic setting.

3.2 Applicant ranking in continuous time with variable search intensities

During the time interval dt , $(audt)$ unemployed workers send an application to one of the $(bvd t)$ advertised vacancies at random. Let $q(x, \cdot)dxdt$ denote the probability for an advertised vacancy to be filled with a worker whose mismatch stays on the range $[x, x + dx]$ during the delay dt . In other words, $bq(x, \cdot)$ is the rate at which firms with a vacancy hire workers whose mismatch (with the offered job) x belongs to the subset $[x, x + dx]$.

Let λ denote the ratio a/b . In this dynamic setting, Lemma 1 is transposed as follows:

Lemma 4. *In the circular matching model with applicant ranking and continuous time, the rate $q(x, \cdot)$ is given by:*

$$q(x, (\theta/\lambda)) = \exp\left(-\frac{2x\lambda}{\theta}\right) \frac{2\lambda}{\theta} \quad (6)$$

⁴In fact the circular model is built so as to generate such an equilibrium

Proof. The proof of Lemma 4 is obtained by replacing (v) with (bv) and n with (au) in the proof of Lemma 1.

Q.E.D.

At first glance this result might look counterintuitive since one could object that when the time interval dt tends to zero, the measure of the set of applicants (advertised vacancies) also tends to zero. But this does not mean that the "number" of applicants (advertised vacancies) is reduced to zero. The two sets remain continuums, hence containing "very large numbers" of elements.

As pointed out by Moen (1999), the ratio $\pi = \lambda/\theta$ can be viewed as the degree of workers' competition for jobs. Conversely, the ratio $bv/au = \theta/\lambda$ is an index of firms' competition for workers. Is this competition degree an optimum? Our welfare analysis can be seen as an answer to this question.

From the rate $q(x, (\theta/\lambda))$, we deduce the density $\rho(x, .)$ of the mismatch among the subset of the filled jobs:

$$\rho(x, .) = \frac{q(x, (\theta/\lambda))}{Q}$$

where Q is the rate at which advertised vacancies are filled (whatever the mismatch is). Q is then obtained by integrating $q(x, .)$ on the range $[0, 1/2]$:

$$Q = Q(\theta/\lambda) = 1 - \exp\left(-\frac{\lambda}{\theta}\right)$$

From the rate Q , we deduce the rate $P = Q\theta/\lambda$ at which applicants find a job (whatever the mismatch is).

3.3 Average output and competition for workers

We now turn to the relation between the degree of competition for workers $\pi = \theta/\lambda$ and the average output $\bar{y}$, hence between the market tightness θ and the average output.

Lemma 3 can be transposed as follows:

Lemma 5. *An increase in the degree of competition for workers (π) lowers the average output ($\bar{y}$).*

Proof. Lemma 5 is stated by substituting π for θ in the proof of Lemma 3 (see Appendix A).

In steady state the unemployment is derived from flow-equilibrium. We then have $u = s/(s + aP)$. As P grows with π , this implies that, like Lemma 3, Lemma 5 means that the lower the unemployment rate is, the higher the average mismatch is. Beyond the results of the static model, Lemma 5 also means that an increase in firms' search intensity (b) lowers the average output in the same way as an increase in the tightness of the labor market. On the contrary, an increase in workers' search intensity (a) raises the average output by lengthening the expected queue of applicants in front of each advertised vacancy, providing firms with the opportunity of hiring a better-suited worker. In sum, a higher degree of competition for workers makes that jobs are less likely to be held by the right man.

3.4 Asset values and search intensities

Asset values are deduced from the usual Bellman equations. They all depend on the (private) surplus $S(x)$ of a match of quality x ($0 \leq x \leq 1/2$).

When holding a job, a worker earns a wage $w(x)$ which depends on the mismatch x . His/her lifetime utility, $W(x)$, satisfies:

$$rW(x) = w(x) - s\beta S(x) \quad (7)$$

A worker is either employed or unemployed. When unemployed, his/her expected lifetime utility, U , verifies:

$$rU = d - \gamma(a) + aP\beta\bar{S} \quad (8)$$

where $\bar{S}$ is the expected value of a match. This expected value is defined by:

$$\bar{S} = \int_0^{1/2} \rho(x, \cdot) S(x) dx$$

A job is either filled or vacant. When vacant, its value V satisfies:

$$rV = -\kappa(b) + bQ(1 - \beta)\bar{S} \quad (9)$$

When held by a worker of mismatch x , a job yields a profit $(y(x) - w(x))$ per period. The value of a (filled) job, $J(x)$, is then obtained from:

$$rJ(x) = y(x) - w(x) - s(1 - \beta)S(x) \quad (10)$$

Workers and firms decide on their search intensities by maximizing the corresponding asset values (respectively U and V). As the agents are very small, the rates Q and P are taken as exogenous. Search intensities a and b are then set by equalizing the marginal cost with the marginal return to search; that is:

$$\gamma'(a) = P\beta\bar{S} \quad (11)$$

and,

$$\kappa'(b) = Q(1 - \beta)\bar{S} \quad (12)$$

As search costs are assumed to be convex, the second-order optimality conditions are fulfilled.

4 Equilibrium and efficiency

We can now derive a stationary equilibrium of the labor market.

Combining equations (7), (8) and (10) gives the expected surplus of a match $\bar{S}$. We obtain:

$$\bar{S} = \frac{\bar{y}(\pi) - d + \gamma(a)}{r + s + \beta a P(\pi)} \quad (13)$$

Under the assumption of free-entry, substituting (13) into (9) yields a first equilibrium equation:

$$-\kappa(b) + bQ(\pi)(1 - \beta)\frac{\bar{y}(\pi) - d + \gamma(a)}{r + s + \beta aP(\pi)} = 0 \quad (14)$$

The previous equilibrium equation is similar to the reduced form of the basic matching model (Pissarides, 2000). For a given pair (a, b) , it determines the tightness of the labor market $\theta = a\pi/b$. We can refer to it as a job creation equation.

The two other equilibrium equations are obtained by substituting (13) into (11) and (12). We obtain:

$$-\gamma'(a) + P(\pi)\beta\frac{\bar{y}(\pi) - d + \gamma(a)}{r + s + \beta aP(\pi)} = 0 \quad (15)$$

and,

$$-\kappa'(b) + Q(\pi)(1 - \beta)\frac{\bar{y}(\pi) - d + \gamma(a)}{r + s + \beta aP(\pi)} = 0 \quad (16)$$

The latter equations can be seen as determining the search intensities a and b .

To summarize, a stationary equilibrium of the labor market can be defined as above.

Definition 2. *A stationary equilibrium of the labor market is a set of variables (π, a, b) which jointly satisfy equations (14), (15) and (16).*

From variables (π, a, b) , one deduces the market tightness θ as well as the equilibrium rate of unemployment by using the condition of flow-equilibrium:

$$u = \frac{s}{s + aP(\pi)} \quad (17)$$

We will now show that the three variables θ , a and b are inefficient in the *laissez-faire* regime. As in the static model, job creation is too high because firms do not internalize the (negative) effect of the creation of new vacancies on the average mismatch. For

similar reasons, firms' search intensity is too strong whereas workers' search effort is too weak.

4.1 Efficiency

In the welfare analysis, the interest rate is assumed to be zero⁵. This assumption allows us to compare steady states according to the social surplus per period. Denoted by σ , the social surplus per period (and per head) is defined by:

$$\sigma = (1 - u)\bar{y}(\pi) + u(d - \gamma(a)) - \theta u\kappa(b) \quad (18)$$

A social maximum is obtained by maximizing the aggregate income σ with respect to the variables θ , a and b subject to the constraint of flow-equilibrium, equation (17).

This leads to compute the derivatives of σ with respect to these three variables.

Let η denote the elasticity of the rate $Q(\pi) = Q(\theta/\lambda)$ in absolute value. The derivative of σ with respect to θ has the same sign as:

$$\Theta \equiv -\kappa + (1 - \eta)bQ\frac{\bar{y} - d + \gamma}{s + \eta aP} + \frac{aP}{(s + \eta aP)^2}\bar{y}'(\pi)\frac{1}{\lambda} \quad (19)$$

On the other hand, the derivative with respect to a has the sign of:

$$A \equiv -\gamma'(a) + \eta P\frac{(\bar{y} - d + \gamma) + \theta\kappa}{s + aP} - \frac{aP}{s}\bar{y}'(\pi)\frac{\theta b}{a^2} \quad (20)$$

Finally, the sign of the derivative of σ with respect to b is the same as the sign of:

$$B \equiv -\theta\kappa'(b) + (1 - \eta)\lambda P\frac{(\bar{y} - d + \gamma) + \theta\kappa}{s + aP} + \frac{aP}{s}\bar{y}'(\pi)\frac{\theta}{a} \quad (21)$$

Notice that, contrary to the equilibrium equations (Definition 2), the three previous expressions include the derivative of the social surplus with respect to firms' competition for workers π . A social planner takes into account this external effect.

⁵The extension to a positive interest rate is straightforward but lengthy. It is available upon request from the author.

As we want to put the stress on this new externality, we assume that firms internalize the well-known congestion effect. In other words, according to the Hosios condition, the elasticity η is equal to the bargaining strength of workers β .

Under this condition, we obtain the following result:

Proposition 2. *A decentralized equilibrium is inefficient in terms of job creation and search intensities. Job creation (θ) as well as firms' search intensity (b) are too strong whereas workers' search intensity (a) is too weak.*

Proof. Remember that the interest rate r is zero and that η is equal to β . Let us first consider the effect of an increase in θ in the neighborhood of a decentralized equilibrium (Definition 2). From the equilibrium equation (14), we deduce that the quantity Θ (see equation (19)) reduces to:

$$\Theta = \frac{aP}{(s + \eta aP)^2} \bar{y}'(\pi) \frac{1}{\lambda} < 0$$

This states that θ is too high in a decentralized equilibrium.

Next, equation (14) can be rewritten as:

$$\theta\kappa = aP(1 - \eta) \frac{\bar{y} - d + \gamma}{s + \eta aP}$$

Substitution of the latter expression of $(\theta\kappa)$ into (20) yields:

$$A = -\gamma'(a) + \eta P \frac{\bar{y} - d + \gamma}{s + \eta aP} - \frac{aP}{s} \bar{y}'(\pi) \frac{\theta b}{a^2}$$

From the equilibrium equation (15), we deduce that the quantity A satisfies:

$$A = -\frac{aP}{s} \bar{y}'(\pi) \frac{\theta b}{a^2} > 0$$

This means that a is too low in a decentralized equilibrium.

Finally, substituting the previous expression of $(\theta\kappa)$ into (21) yields:

$$B = -\theta\kappa'(b) + (1 - \eta)\lambda P \frac{\bar{y} - d + \gamma}{s + \eta a P} + \frac{aP}{s} \bar{y}'(\pi) \frac{\theta}{a}$$

or,

$$B = \theta[-\kappa'(b) + (1 - \eta)Q \frac{\bar{y} - d + \gamma}{s + \eta a P}] + \frac{aP}{s} \bar{y}'(\pi) \frac{\theta}{a}$$

From (16), we deduce that B satisfies:

$$B = \frac{aP}{s} \bar{y}'(\pi) \frac{\theta}{a} < 0$$

This states that b is too high in a decentralized equilibrium.

Q.E.D.

We already explained why job creation is inefficient (see Section 2). The intuition behind the results about search intensities is similar. An increase in firms' search effort (b) reduces the sample size of applicants for each advertised vacancy, lowering then the expected output of filled jobs (see our comment of Lemma 2). For obvious reasons an increase in workers' search intensity (a) acts in the opposite way. As firms (workers) do not internalize this external effect, the search intensity b is too high whereas the search intensity a is too low.

What can be done to restore the efficiency of the labor market? In line with intuition, one can show that the workers' bargaining strength β should be higher than the elasticity η . But, as Flinn (2006) points out, the parameter β is not a policy instrument.

5 Conclusion

In this short paper, we have shown that the usual condition for market efficiency in matching models of unemployment does not apply any longer when firms rank their applicants and pick the best one. Applicant ranking creates a new externality

which leads firms to create too many vacancies. Due to similar reasons, firms' search intensities are too strong whereas workers' search effort is deficient.

For the sake of simplicity, we used the circular matching model; but this externality is likely to hold in any model with heterogeneous agents. With fewer job seekers per vacancy, firms will find it more difficult to recruit the right man for their jobs.

One important limitation of our work is that on-the-job search was excluded. Although we surmise that it would only reduce the impact of job creation on the average output, introducing on-the-job search is an interesting line for further research.

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Appendix A: Proof of Lemma 2

Step 1. One can show that for any θ , there exists a mismatch level $\tilde{x}$ such that the derivative $\frac{\partial \rho(\cdot)}{\partial \theta}$ is negative for $x \in [0, \tilde{x}[$, equal to zero for $\tilde{x}$ and positive for $x \in]\tilde{x}, 1/2]$.

Step 2. As $y(\cdot)$ is decreasing, it comes that for all x different from $\tilde{x}$:

$$\frac{\partial \rho(x, \theta)}{\partial \theta} y(x) < \frac{\partial \rho(x, \theta)}{\partial \theta} y(\tilde{x})$$

It results that:

$$\frac{d\bar{y}(\theta)}{d\theta} = \int_0^{1/2} \frac{\partial \rho(x, \theta)}{\partial \theta} y(x) dx < \int_0^{1/2} \frac{\partial \rho(x, \theta)}{\partial \theta} y(\tilde{x}) dx = 0$$

This proves Lemma 2.

Q.E.D.